BSCPH202

B. Sc. II YEAR
Optics

DEPARTMENT OF PHYSICS
SCHOOL OF SCIENCES
UTTARAKHAND OPEN UNIVERSITY

Programme Coordinator and Board of Studies

Board of Studies

Prof. P. D. Pant
Director I/C School of Sciences
Uttarakhand Open University, Haldwani

Prof Awadhesh Kumar Rai,
Department of Physics,
University of Allahabad, Allahabad

Dr. Kamal Devlal (Programme Coordinator)
Department of Physics
School of Sciences, Uttarakhand Open University,
Haldwani

Prof. C.K. Ghosh,
Regional Director, Regional Services Division,
I.G.N.O.U., New Delhi

Prof. R. C. Srivastva,
Professor and Head, Department of Physics,
CBSH, G.B.P.U.A.&T. Pantnagar, India

Unit writing and Editing

Editing

Dr. M. Sharma
Department Physics
Pt. L.M.S. Govt. P.G. College.
Rishikesh, Uttarakhand

Writing

1. **Dr. Piyush Sinha**
Department Physics
HNB University Campus, Pauri, Uttarakhand

2. **Dr. Kamal Devlal**
Department of Physics
School of Sciences, Uttarakhand Open University,
Haldwani

3. **Dr. M. Sharma**
Department Physics
Pt. L.M.S. Govt. P.G. College. Rishikesh, Uttarakhand

4. **Dr. Santosh K. Joshi**
Department Physics
University of Petroleum and Energy Studies,
Prem Nagar, Dehradun-248007, Uttarakhand

Dr. C. C. Dhondiyal
Department Physics
M.B. G.P.G.C. Haldwani, Nainital, Uttarakhand

Dr. B S Tiwari
Department Physics
University of Petroleum and Energy Studies,
Prem Nagar, Dehradun-248007, Uttarakhand

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OPTICS

DEPARTMENT OF PHYSICS
SCHOOL OF SCIENCES
UTTARAKHAND OPEN UNIVERSITY
Phone No. 05946-261122, 261123
Toll free No. 18001804025
Fax No. 05946-264232, E. mail info@uou.ac.in
htpp://uou.ac.in

Contents

Course 5: Optics

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UNIT 1: FERMAT'S PRINCIPLE AND ITS APPLICATION

CONTENTS

1.1 INTRODUCTION

Light is a form of energy that enables us to see and perceive objects with our eyes. Scientifically, light is an electromagnetic wave of wavelength belonging to visible part (wavelength of 400 nm to 750 nm) of electromagnetic spectrum. We see objects either by the light they produce or by the light they reflect from other objects. Objects that produce their own light are said to be luminous. Examples are the sun, electric bulbs, candle light etc., whereas, non luminous objects do not produce their own light. We can see these objects only when light fall on them from other sources and it is thrown back or reflected into our eyes. An important example is that of the moon, which shines in the night because it reflects light coming from the sun and not because it is luminous.

Optics is the science or more specifically a branch of physics in which we study the behavior and properties of light. The study also includes the interaction of light with matter and construction of instruments that use or detect it. For the sake of convenience the subject of optics can be divided into two parts: (i) physical or wave optics, which deals with the wave nature of light. It accounts for optical effects such as diffraction and interference etc., and (ii) geometrical or ray optics, which deals with the formation of images by lenses and mirrors and their combinations on the basis of certain geometrical laws obeyed by light.

The present block of the course 'optics' is dedicated to the geometrical optics only hence, in the following sections, we will concentrate on its learning in detail.

Geometrical optics describes light propagation in terms of rays. The rays are the approximate paths along which light propagates under certain circumstances. The basic assumptions of geometrical optics include, that light rays:

- Propagate in straight line paths in a homogeneous medium, called as rectilinear propagation.
- Bend, and in particular circumstances may split into two, at the interface between two dissimilar media
- Follow curved paths (iterative bending) in a medium in which refractive index changes
- May be absorbed or reflected at glossy surfaces

There are certain laws which explain the above assumptions. These laws form the basis of geometrical optics and are called *fundamental laws*. The fundamental laws are

1. The laws of rectilinear propagation of light
2. The laws of reflection of light
3. The laws of refraction of light

A general principle which covers all these laws is known as Fermat's principle of least time.

1.2 OBJECTIVES

After studying this unit, you will be able to:

- know Fermat's principle of least time
- familiarize with incident ray, reflected ray and refracted ray
- familiarize with angle of incidence, reflection and refraction
- state laws of reflection
- state laws of refraction- Snell's law
- define refractive index
- explain total internal reflection as a special case of refraction
- understand Gauss general theory of image formation

Before we discuss the Fermat's principle of least action, let us know about the terms – optical path and optical path time interval.

1.3 OPTICAL PATH

Optical path is the path taken by light ray through an optical system. It is also known as the product of refractive index (see section 1.5.3) of the medium, i.e. μ and the distance travelled S by the light ray in medium, i.e.,

$$\mu\, S = \text{Optical path}$$

It is the path travelled by the light ray in air, in the same time, it takes to traverse the distance S of medium or we can say it to be the equivalent air path.

If v is the velocity of light and t is the time taken for covering the distance S, then,

$$S = v\, t \qquad \ldots\ldots\ (1.1)$$

But we know that the refractive index (section 1.5.3)

$$\mu = \frac{c}{v} = \frac{Velocity\ of\ light\ in\ vacuum}{Velocity\ of\ light\ in\ medium} \qquad \ldots\ldots\ (1.2)$$

Thus

$$v = \frac{c}{\mu} \qquad \ldots\ldots\ (1.3)$$

Substituting equation (1.3) in equation (1.1), we get,

$$\text{S} = \frac{c}{\mu}\ t \qquad \ldots\ldots\ (1.4)$$

or

$$\mu\, S = c\, t = \Delta = optical\ path \qquad \ldots\ldots\ (1.5)$$

1.3.1 Optical Path Time Interval

When a ray travels S_1, S_2, S_3, S_4 etc. distances in media of refractive indices $\mu_1, \mu_2, \mu_3, \mu_4$ etc., then the optical path is given by

$$\Delta = \mu_1 S_1 + \mu_2 S_2 + \mu_3 S_3 + \mu_4 S_4 + \ldots..$$

$$= \sum_i \mu_i S_i \qquad \ldots\ldots\ (1.6)$$

If dS is the small distance covered by light between two points P and Q in a medium and v is its velocity in that particular medium, then the mathematical form of Fermat's principle of extreme path is defined as

$$\int_P^Q \frac{dS}{v} = \text{maximum or minimum or stationary}$$

or

$$\int_P^Q \frac{\mu dS}{c} = \text{maximum or minimum or stationary} \qquad \left[Here\ \mu = \frac{c}{v}\right]$$

Since velocity of light in vacuum (c) is constant, the Fermat's principle of extreme path takes the form

$$\int_P^Q \mu\, dS = \text{maximum or minimum or stationary} \qquad \ldots\ldots\ (1.7)$$

Where μdS is the optical path in a medium of refractive index μ.

1.4. FERMAT'S PRINCIPLE OF LEAST TIME

In 1658 Pierre De Fermat, a French mathematician enunciated the principle of least time in the following way:

A ray of light in passing from one point to another through a set of media by any number of reflections or refractions chooses a path along which the time taken is minimum or the least.

Based on this principle, the laws of rectilinear propagation, the laws of reflection and refraction can be derived (see section 1.5). However in some cases, it has been found that the time taken by light is not minimum but maximum or else it is neither maximum nor minimum but it is stationary. This is found in case of image formation by lenses, in which all rays starting from an object point, reaching to the image point; choose the path of maximum or minimum time. Therefore, the modified form of Fermat's principle of least time is known as Fermat's principle of stationary time or Fermat's principle of extreme path, which may be stated as follows:

A ray of light in passing from one point to another through a set of media by any number of reflections or refractions chooses a path for which the time taken is either minimum or maximum or stationary. The mathematical verification of this law is provided in the later sections.

1.5 APPLICATION OF FERMAT'S PRINCIPAL

On the basis of Fermat's principle you can derive laws of reflection and refraction.

1.5.1. Laws of Reflection

When light ray falls on a smooth polished surface separating two media, it comes back in the same medium, the phenomenon is called reflection and the boundary is called reflecting surface. The light obeys following two laws of reflection.

First law:

The incident ray, reflected ray and the normal to the surface at the point of incidence all lie in one plane. You can prove this law in the following way.

Let the plane ABCD be normal to the plane mirror shown in figure 1.1. P is point object imaged by mirror as P'. Consider a point M' on the plane mirror; but not on plane ABCD. Let a ray PM' be reflected as $M'P'$. Draw a normal $M'M$ on plane ABCD. Point M is the foot of the normal on ABCD.

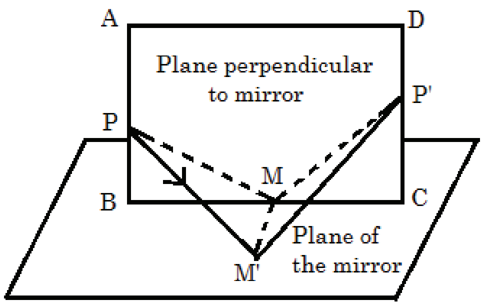

Fig. 1.1

Now, PMM' and $P'MM'$ are right angle triangles. PM' and $P'M'$ are respective hypotenuse. Therefore, we have

$$PM' > PM \quad \text{and} \quad P'M' > P'M$$

But Fermat's principle demands that the path followed must be the shortest, i.e., the light would not travel along $PM'P'$. As we shift M' towards M the path of light ray becomes shorter. It is seen that the shortest possible path is $PM\,P'$, where the point of incidence M lies on plane ABCD. PM and MP' are the incident and reflected rays. This proves the first law of reflection.

Second law:

For a smooth surface, the angle of incidence is equal to the angle of reflection. You can prove the second law as follows.

Assuming DD' is a reflecting plane shown in figure 1.2. Object P is imaged as P' and M is the point of incident. The normal to the plane at this point is MN and is shown by dotted line.

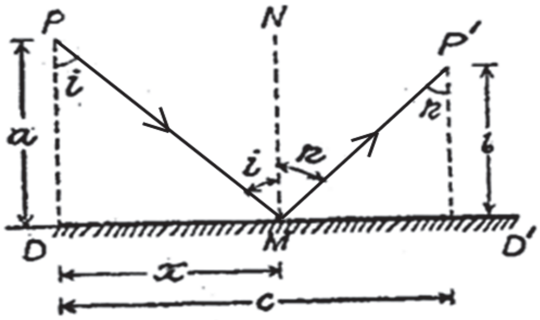

Fig. 1.2

PM and *MP'* are the incidence and reflected rays. Let *I* and *r* are the angles of incidence and reflection respectively. Let us suppose distances

$$PD = a,\ P'D' = b,\ DM = x,\ DD' = c.$$

The ray of light travels in air from *P* to *P'*. Let the path *PMP'* be *s*, then the total distance covered by light ray be

$$s = P\,MP' = PM + MP'$$

$$= (PD^2 + DM^2) + (D'M^2 + D'P'^2)$$

$$= (a^2 + x^2) + \{(c - x)^2 + b^2\} \qquad \ldots\ldots (1.8)$$

It is evident that the path from P to P' remains the same even if the point of incidence M shifts. Shifting of M changes *x* only. According to Fermat's principle the path *PMP'* must be either minimum or maximum. It means that the differential coefficient of *s* with respect to *x* must be zero, i.e.

$$\frac{ds}{dx} = \frac{1}{2}\frac{2x}{\sqrt{(a^2 + x^2)}} - \frac{1}{2}\frac{2\,(c-x)}{\sqrt{\{(c-x)^2 + b^2}} = 0$$

or
$$\frac{x}{\sqrt{(a^2+x^2)}} = \frac{(c-x)}{\sqrt{\{(c-x)^2 + b^2\}}}$$

From figure 1.2, we have,

$$\frac{x}{\sqrt{(a^2+x^2)}} = \sin i,\ \ (c - x)\,/\sqrt{\{(c - x)2 + b2\}} = \sin r$$

$\therefore$
$$\sin i = \sin r \qquad \ldots\ldots (1.9)$$

or
$$i = r$$

Hence you can see that the second law of reflection is derived from Fermat's principle. Further the second differential co-efficient of s, i.e., $\frac{d^2s}{dx^2}$ comes out to be positive, which proves that the path is minimum (or path of least time).

1.5.2. Laws of Refraction

When a ray of light passes from one homogenous medium to another, the phenomenon of bending of light ray towards or away from the normal is called refraction. Again there are following two laws of refraction.

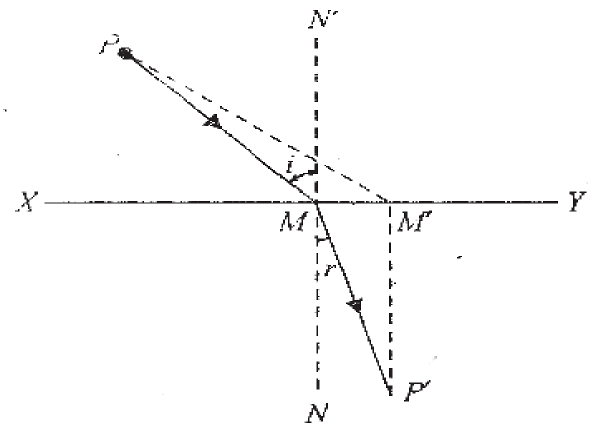

Fig. 1.3

First Law:

The incident ray, refracted ray and the normal at the point of incidence all lie in one plane.

Let us assume XY be a plane surface dividing two media shown in figure 1.3. A ray starting from point P is incident on M. It is refracted as MP' in the other medium, $\angle i$ and $\angle r$ are the angles of incidence and refraction. Let us assume that the ray follows path *PM'P'* instead of *PMP'*. It is evident that

$$PM' + M'P' > PM + MP'$$

Therefore, path *PM'P'* is not possible. If you shift *M'* towards *M,* the path from *P* to *P'* through M shortens. It is shortest when *M'* is coincident with *M* which is in accordance with Fermat's principle and proves the first law.

Second Law:

The ratio of the sine of angle of incidence to the sine of angle of refraction is a constant for a given pair of media.

You can further prove that the ratio of the sin *i* to sin *r* is equal to the refractive index of second medium with respect to the first medium which is also known as Snell's law.

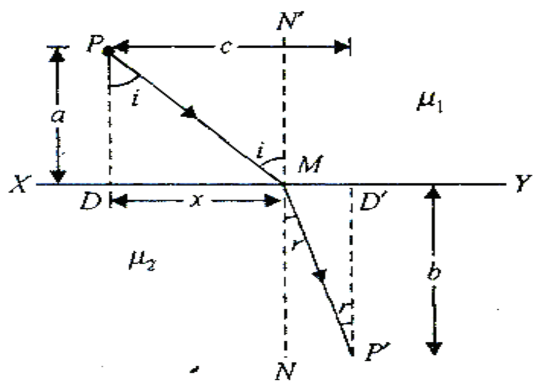

Fig. 1.4

In figure 1.4, XY is a plane surface dividing two media of refractive indices μ_1 and μ_2. Consider a point object *P* in the first medium, *PM* and *MP'* are the incident and refracted rays, *i* and *r* are the angle of incidence and refraction.

$$PD = a,\ DM = x,\ DD' = c,\ D'P' = b$$

If a ray of light travels a distance S in a medium of refractive index μ, than product μS is called the optical path in the medium. The optical path from P to P' is given by

$$S = PMP' = \mu_1 PM + \mu_2\ MP'$$

$$= \mu_1 \sqrt{(PD^2 + DM^2)} + \mu_2 \sqrt{(D'M^2 + D'P^2)}$$

$$= \mu_1 \sqrt{(a^2 + x^2)} + \mu_2 \sqrt{\{(c - x)^2 + b^2\}}$$

Now, for S to be minimum dS/dx must be zero and d^2S/dx^2 positive. Differentiating S with respect to x, we get

$$\frac{dS}{dx} = \frac{\mu_1}{2} \cdot \frac{2x}{\sqrt{(a^2 + x^2)}} - \frac{\mu_2 (c-x)}{\sqrt{\{(c-x)^2 + b^2\}}} = 0$$

or
$$\frac{\mu_1 x}{\sqrt{(a^2 + x^2)}} = \frac{\mu_2 (c-x)}{\sqrt{\{(c-x)^2 + b^2\}}}$$

Using triangles PMD and $P'MD'$, you can write the above relation as

$$\mu_1 \sin i = \mu_2 \sin r$$

or
$$\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1} = {}_1\mu_2 \qquad \ldots\ldots\ (1.10)$$

Where ${}_1\mu_2$ is the refractive index of the second medium with respect to the first medium. This is the **Snell's law** of refraction.

You can show that the second differential coefficient of S, i.e., $\left(\frac{d^2s}{dx^2}\right)$ for the plane surface comes out to be positive. It proves that the second law of refraction is in accordance with Fermat's principle, i.e., the actual path is minimum or path of least time. But you will see that, this condition is satisfied in the case of plane surfaces only and not in the case of curved surfaces. For curved surfaces, the path may be a maximum or minimum.

Case I:

When the reflecting surface is more curved than the ellipse passing through the point of reflection O and having points A and B as foci (figure 1.5)

Let us consider two fixed points A and B and a curved mirror MON (figure 1.5). Let AOB be the actual path of a ray of light traveling from A to B.

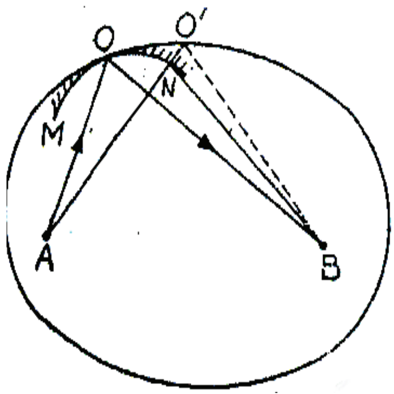

Fig. 1.5

Let us draw an ellipse having A and B as foci and passing through O. By the property of ellipse, (AO + OB) is a constant for all positions of O on the ellipse. For example,

$$AO + OB = AO' + O'B$$

An alternative neighboring path for the ray of light by reflection at the mirror is ANB. The difference between the actual path AOB and the neighboring path ANB is,

$$\Delta = AOB - ANB = (AO + OB) - (AN + NB)$$

$$= (AO + OB) - (AO' - NO' + NB)$$

$$= (AO' + O'B) - (AO' - NO' + NB)$$

$$= O'B + NO' - NB$$

This is positive because O'B and NO' are the two sides of a triangle of which NB is the third side. Thus, in this case, any neighboring path ANB is smaller than the actual path AOB. Thus from above discussion, we can conclude that the actual path is maximum compared with all neighboring paths. Hence Fermat's law is a law of extremum path.

Case II:

When the reflecting surface is less curved than the ellipse passing through the point of reflection O and having points A and B as foci (figure 1.6).Let us consider a reflecting surface *MON* (figure 1.6),which is less curved than the ellipse passing through *O* and having points *A, B* as foci. In this case, the difference between the actual path *AOB* and aneighboring path *ANB* is given by,

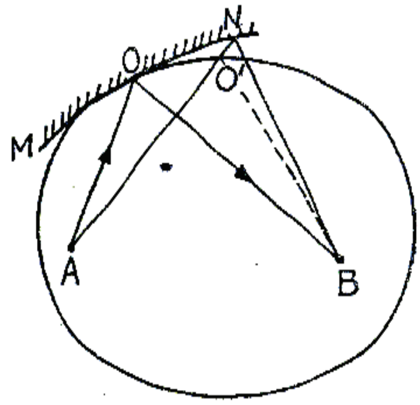

Fig. 1.6

$$\Delta = AOB - ANB = (AO + OB) - (AN + NB)$$

$$= (AO + OB) - (AO' + O'N + NB)$$

$$= (AO' + O'B) - (AO' + O'N + NB) \text{ (by property of ellipse)}$$

$$= -(O'N + NB - O'B).$$

This is negative because $O'N$ and NB are the two sides of a triangle of which $O'B$ is the third side. Thus, in this case, any neighboring path ANB is longer than the actual path AOB. You can say that, the actual path is minimum among all neighboring paths.

Hence the curvature of the reflecting surface relative to that of the ellipse through the point of reflection decides whether the path of light from the focus of the ellipse to the other focus through reflection at the surface will be a maximum or a minimum.

1.5.3. Refractive Index

The refractive index is a relative property of two media. The refractive index of any medium with respect to free space (or air) is called the absolute refractive index. The absolute refractive index of any medium depends on its nature, wavelength of incident light and the temperature. The frequency of refracted ray remains the same as that of incident light, but its velocity and wavelength change. Let us now consider the following cases.

Case I:

If μ_1 and μ_2 are the absolute refractive indices of first and second media respectively and light ray enters from rarer medium to the denser medium then, we have, $\mu_2 > \mu_1$

$$\therefore \qquad \frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1} > 1$$

or $$\sin\ i > \sin r$$

or $$i > r$$

This simply shows that refracted ray is deviated towards the normal.

Case II:

If light ray enters from denser medium to rarer medium, then $\mu_2 < \mu_1$

$$\therefore \qquad \frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1} < 1$$

or $$\sin i < \sin r$$

or $$i < r$$

Thus the refracted ray is deviated away from the normal

.

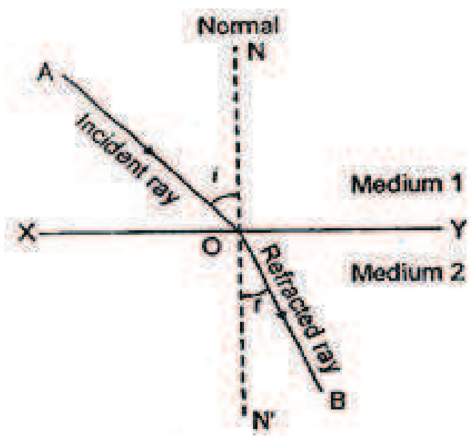

Fig.1.7

Case III:

If i = 0, then r = 0, i.e., the incident light ray is along normal then the refracted ray passes undeviated, but its velocity gets changed.

The absolute refractive index can be defined in other ways also. The absolute refractive index of a medium is defined as the ratio of speed of light in the free space to the speed of light in medium. Accordingly, if c is speed of light in free space and v the speed of the light in the medium, then the refractive index

$$\mu = \frac{c}{v} \qquad .... \quad (1.11)$$

In refraction the frequency of wave (υ) remains unchanged, therefore, c = $\upsilon\lambda$ and v = $\upsilon\lambda_m$. Where, λ_m is wavelength of light in medium. Thus refractive index of a medium may also be given by the expression,

$$\mu = \frac{\lambda}{\lambda_m} \qquad .... \quad (1.12)$$

1.5.4. Total Internal Reflection

When a ray of light moves from denser to rarer medium, then the refracted ray is deviated away from normal (figure 1.8 (a)). With increase in angle of incidence, the angle of refraction increases. For a certain angle of incidence in denser medium, the corresponding angle of refraction in rarer medium is 90^0 (figure 1.8(b)).

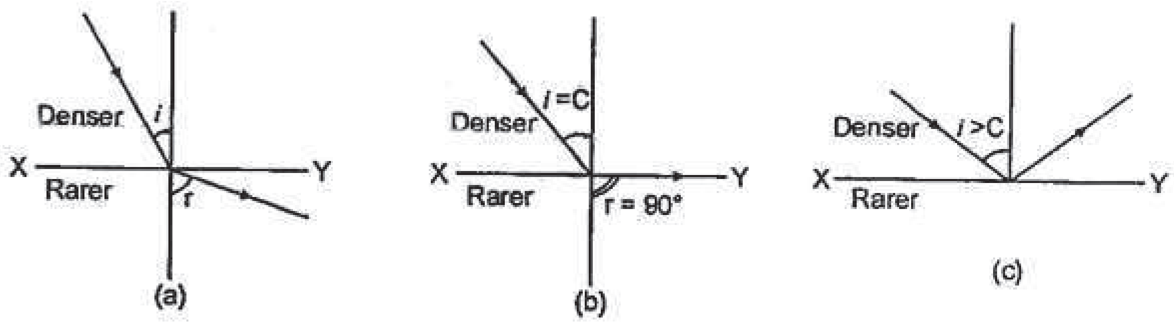

Fig. 1.8

This particular angle of incidence for which the corresponding angle of refraction is 90^0 is called the critical angle and is denoted by C. The value of the critical angle depends on the nature of the two media.

If angle of incidence is increased beyond its critical value, the incident ray is not refracted but returns back to denser medium [figure 1.8(c)]. This phenomenon is called **total internal reflection**.

Conditions of Total Internal Reflection

(i) The ray must pass from denser medium to rarer medium.
(ii) The angle of incidence in the denser medium must be greater than the critical angle for the given pair of media.

Relation between Refractive Indices of Media and Critical Angle

If μ_d and μ_r are the refractive indices of denser and rarer media respectively then from Snell's law, we have,

$$\frac{\sin i}{\sin r} = \frac{\mu_r}{\mu_d} \qquad \ldots\ldots. \quad (1.13)$$

For critical angle of incidence, $i = C$ and $r = 90^0$

$$\therefore \qquad \frac{\sin C}{\sin 90^0} = \frac{\mu_r}{\mu_d} \quad \Rightarrow \sin C = \frac{1}{{}_r\mu_d} \qquad \ldots\ldots. \quad (1.14)$$

Where ${}_r\mu_d$ = refractive index of denser medium with respect to rarer medium. In most of the problems the rarer medium is chosen to be air with refractive index 1.

Example 1: The absolute refractive indices of glass and water are 4/3 and 3/2 respectively. If the speed of light in glass is 2×10^8 m/s, calculate the speed of light in (i) vacuum and (ii) water.

Solution: Refractive index of glass, $\mu_g = \frac{4}{3}$

$$\therefore \qquad \mu_g = \frac{Speed\ of\ light\ in\ vaccum}{Speed\ of\ light\ in\ glass}$$

or

$$\frac{4}{3} = \frac{Speed\ of\ light\ in\ vaccum}{2\times10^8}$$

Thus, speed of light in vacuum = $\frac{4\times2\times10^8}{3} = 2.67 \times 10^8\ m/s$

Refractive index of water (given), $\mu_w = \frac{3}{2}$

But we know that $\mu_w = \frac{Speed\ of\ light\ in\ vaccum}{Speed\ of\ light\ in\ water}$

$$\therefore \qquad \frac{3}{2} = \frac{2.6\times10^8}{Speed\ of\ light\ in\ water}$$

Therefore, speed of light in water = 1.73×10^8 m/s

Example 2: Refractive index of water with respect to air is 4/3 and glass is 3/2. What is the refractive index of glass with respect to water?

Solution: For three media air, water and glass, we have

$${}^a\mu_w \times {}^w\mu_g \times {}^g\mu_a = 1$$

$$\therefore \qquad {}^w\mu_g = \frac{1}{{}^a\mu_w \times {}^g\mu_a} = \frac{{}^a\mu_g}{{}^a\mu_w} = \frac{3/2}{4/3} = \frac{9}{8}$$

Thus, refractive index of glass with respect to water is 9/8.

Example 3: If the angle of incidence (i) for a light ray in air be 45^0 and the angle of refraction (r) in glass be 30^0. Find refractive index of glass.

Solution: Refractive index of glass, $n = \frac{\sin 45^0}{\sin 30^0} = \frac{1}{\sqrt{2}} \times 2 = \sqrt{2}$

Self Assessment Question (SAQ)

1. What is total internal reflection?
2. What is critical angle for a medium of refractive index $\sqrt{2}$?
3. Using Fermat principle, establish condition of total internal reflection.

1.6. GAUSS'S GENERAL THEORY OF IMAGE FORMATION

In a coaxial symmetric system, Gauss's theory deals with ideal image formation. This includes the cases where there is a point-for-point, line-for-line and plane-for-plane relationship between the object and its image. With the various reflecting or refracting coaxial surfaces, the common axis is taken as *X*-axis. Due to symmetry about this axis, it is enough to deal with any one plane through the *X*-axis, and the transverse distances is given along *Y* – axis.

With arbitrary choice of origins for the object and image space, let (x, y) be the co-ordinates of an object point and (x', y') those for its image. Then the most general linear relationships of x', y' and x, y would be

$$x' = \frac{a_1x+b_1y+d_1}{ax+by+d} \qquad \text{and} \qquad y' = \frac{a_2x+b_2y+d_2}{ax+by+d}$$

We state this without a rigorous mathematical proof. We can see that out of the nine constants involved, five are redundant.

Firstly, if y changes sign, x' should remain unaffected. This condition makes

$$b = b_1 = 0$$

Again if y changes sign, y' should only change in sign, not in magnitude. This makes $a_2=d_2=0$. Finally dividing all coefficients by a, and expressing the new values by the corresponding capital letters, we get

$$x' = \frac{A_1x+D_1}{x+D} \qquad \text{and} \qquad y' = \frac{B_2y}{x+D}$$

Thus, we reach a very important conclusion that in any co-axial symmetrical system forming ideal images the co-ordinates x', y' of an image point corresponding to the object point x, y are uniquely determined by four and only four independent constants.

In actual practice, two of these constants are used to specify the origins for object space and image space on the x-axis for measurement of distances. Then only two other constants specify the object to image conjugate relations uniquely. There are several alternative ways of choosing the origins.

1.7. SUMMARY

In this unit you have studied the following

1. Fermat's principle.
2. Before explaining Fermat's principle it is necessary to know about optical path. It has been defined as, when a ray of light travels a distance d in a medium of refractive index μ, the product μd is called the optical path.
3. In 1960 Fermat's principle was stated as "Fermat's principle of least time, later it was stated in a more general form, based on the number of cases observed practically. This is known as "Fermat's principle of stationary time" or Fermat's principle of extreme path (a maximum or a minimum or stationary).
4. The laws of reflection and refraction have been deduced with the help of Fermat's Principle.
5. Condition of total internal reflection is also discussed. You have seen, this condition occurs when a ray of light moves from denser medium to rarer medium. One of the examples of total internal reflection is the formation of rainbow, which we see usually in our day to day life.
6. Many solved examples are given in this unit to make concepts clear. In the last "Gauss General theory of image formation" is explained. You will use this theory in the next unit.
7. To check your progress Self Assessment Questions (SAQs) are also given.

1.8 GLOSSARY

Beam – group of rays
Extreme – maximum or minimum
Angle of Incidence – angle between a beam striking a surface and the normal to surface
Angle of Reflection – angle formed between the normal to a surface and reflected ray
Angle of Refraction – angle formed between a refracted ray and the normal to the surface
Homogeneous – of the same kind, alike
Iterative – Frequentative

1.9 REFERENCES

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1.10 SUGGESTED READINGS

1. Milton Katz (1994), *Introduction to Geometrical Optics*.
2. B.K. Mathur (2015), *Optics*, Edition 2, Digital Library of India Item.462758.
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1.11. TERMINAL QUESTIONS

1.11.1. Short Answer Type

1. Write a short note on optical path.

2. Write a note on Fermat's principle.

3. Using Fermat's principle, establish the condition of total internal reflection.

4. Define critical angle. State the relation between refractive index of medium and critical angle.

1.11.2. Long Answer Type

1. Discuss Fermat's principle in brief and prove laws of reflection and refraction with its help.

2. Give examples to show that the path of reflected ray is

i. Maximum in some case and

ii. Minimum in other

3. What is Fermat's principle? Prove that Snell's law follows the Fermat's principle.

1.11.3. Numerical Questions

1. A man walks on the hard ground with a speed of 5 ft/s, but he has speed of 3 ft/s on the sand ground. Suppose he is standing at the border of sandy and hard ground and wishes to reach a tree situated on the sandy ground. The man can reach the tree by walking 100ft. along the border and 120 ft. on the sandy ground normal to the border. Find out the value of path which requires minimum time to reach the tree.

2. In fig. 1.9, two stations A and B in different territories are separated by a border line CD. A messenger can travel in the upper territory with a speed V_a and in the lower territory with speed V_b. Several messengers start from A and follow different paths like APB having different position of P specified by a distance x from M. It is found that the messenger who chooses x as 4.0 km reaches B in the minimum time.

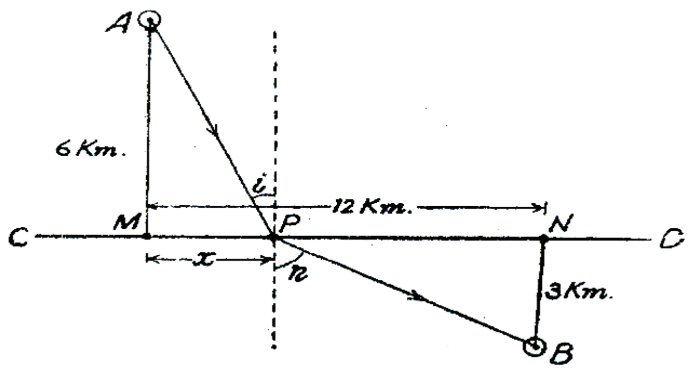

Fig. 1.9

Answer the following

(a) What is the relation between V_a and V_b?

(b) If the speeds V_a and V_b are interchanged, what will be the new value of x to give the fastest path?

(c) Explain the formation of a ray of light on the basis of Fermat's Principle of extremum path.

3. In fig. 1.10, light starts from point A and after reflection from the inner surface of the sphere reaches the diametrically opposite point B. Calculate the length of the hypothetical

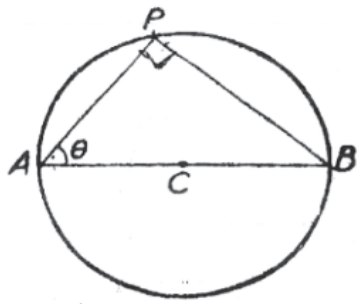

Fig.1.10

path APB and using Fermat's Principle find the actual path of light. Is the path minimum? **4.** In fig. 1.14, *P* is a point source of light. If the distance of *P* from the centre *O* of the spherical reflecting surface is 0.8*r* and if the light ray starting from *P* and after being reflected

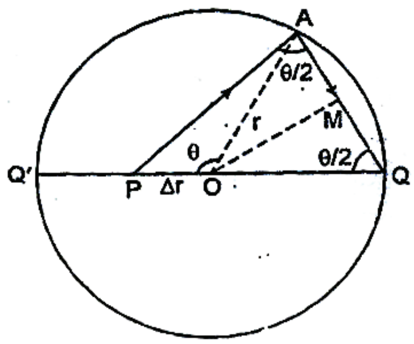

Fig.1.11

at *A* reaches at a point *Q*. Using Fermat's principle show That $\cos\frac{\theta}{2} = \frac{3}{4}$

5. A man walks with a speed of 1.8 m/s when he walks on the hard ground, but has a speed of only 1 m/s when he walks on sandy area. Suppose he is standing at the border of sandy and hard ground and he wishes to reach a tree deep inside the sandy area. He can reach the tree by walking 30m along the border and followed by 36m walk on the sandy area normal to the border. Find the suitable path which requires minimum time to reach.

1.11.4. Objective Answer Type

1. The product of μ of the medium and the distance travelled by the light in medium is known as

   a. Ray path b. Fermat's path

   c. Optical path d. Actual path

2. A ray of light traveling from one point to another by any number of reflections and refractions follows that particular path for which time taken is

   a. highest b. least

   c. maximum d. depends upon path

3. The incident, reflected and normal ray at the point of incidence are in same plane, is the statement for

   a. First law of reflection b. Second law of reflection

   c. First law of refraction d. All above

4. The optical path of monochromatic light is same as it goes through 200cm of glass or 2.25cm of water. If the refractive index of water is 1.33, the refractive index of glass is

   a. 1.00 b. 1.23

   c. 1.33 d. 1.50

5. Fermat Principle is the principle of:-

   a. Maximum path only b. Extreme path only

   c. Minimum path only d. None above

6. Refractive indices of water, sulphuric acid, glass and carbon disulphide are 1.33, 1.43, 1.53 and 1.63respectively. The light travels slowest in:

   a. Sulphuric acid b. Glass

   c. Water d. Carbon Disulphide

7. The refractive index of glass with respect to air is $\frac{3}{2}$ and the refractive index of water with respect to air is $\frac{4}{3}$. The refractive index of glass with respect to water will be:

   a. 1.525 b. 1.225

   c. 1.425 d. 1.125

1.12 ANSWERS

1.12.1 Self Assessment Questions (SAQs):

1. Refer Article 1.5.4

2. $Sin\ C = \frac{1}{{}_1\mu_2}$ or $Sin\ C = \frac{1}{\sqrt{2}} \Rightarrow \angle C = 45^0$

3. In the last question, we have proved, using Fermat's principle that

$$\frac{\sin i}{\sin r} = {}_1\mu_2.$$

If medium 2 is rarer (${}_1\mu_2 < 1$), then,

$$sin\ i < sin\ r \quad \text{or} \quad i < r$$

For $r = 90^0$, $sin\ r = 1$, and we have

$$sin\ i = {}_1\mu_2$$

At angles of incidence larger than this limiting angle, the light is reflected back into the denser medium.

1.12.2 Terminal Questions

Numerical Questions

If the man follows the path followed by a light ray he will take minimum time to reach the tree. This is in accordance with Fermat's principle. Let M be the starting position of the man on the border line AB (Fig. 1.12). Distance MM" = 100 ft. on the border. *T* is the tree on the sandy ground so that the distance M"T = 120 ft. Let the man follow the path from *M* to *M'* and *M'* to *T* in order to reach the tree in minimum time. If distance *MM'* = x then *M'M"* = (100 – x) ft. From fig. 1, we have

1. $M'T = \sqrt{[(M'M")^2 + (M"T)^2]} = \sqrt{\{(100 - x)^2 + (120)^2\}}$

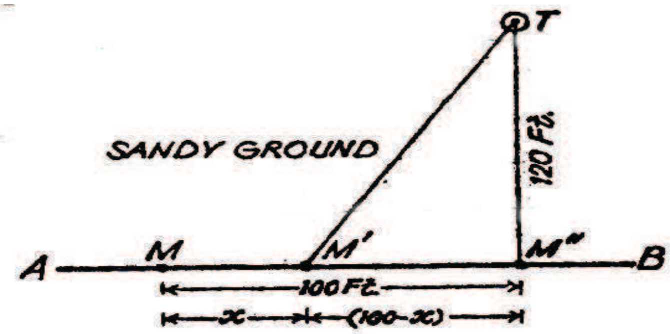

Fig. 1.12: Hard Ground

Total time taken by the man to reach from *M* to *M'* and *M'* to *T* is given by,

$$t = \frac{distance\ MM'}{velocity\ on\ border} + \frac{distance\ M'T}{velocity\ on\ sandy\ ground}$$

$$= \frac{MM'}{5} + \frac{M'T}{3} = \frac{x}{5} + \frac{\sqrt{[(100-x)^2 + (120)^2]}}{3}$$

Since the value of *t* depends upon the distance x, hence for *t* to be minimum, we have

$$\frac{dt}{dx} = 0 = \frac{1}{5} + \frac{1}{3} \cdot \frac{1}{2}\ [(100 - x)^2 + (120)^2]^{-1/2} .\ 2\ (100 - x)\ (-1)$$

or
$$\frac{1}{3} \cdot \frac{(100-x)}{\sqrt{[(100-x)^2 + (120)^2}} = \frac{1}{5}$$

Squaring it, we get, $\frac{1}{9} \cdot \frac{(100-x)^2}{(100-x)^2 + (120)^2} = \frac{1}{25}$

or $(100 - x)^2 . 25 = 9 [(100 - x)^2 + (120)^2]$

or $(100 - x)^2 [25 - 9] = 9. (120)^2$

∴ $x = 100 \pm \frac{360}{4} = 100 \pm 90 = 190$ or 10

As per data, the value 190 is out of question; hence the man should walk 10 ft. along the border and then make a head-way towards the tree from M'.

2. **(a).** See fig. 2 The path AP transverse with a speed V_a and the path PB with a speed V_b. The path PA is given by :

$$AP = \sqrt{\{(AM)^2 + (MP)^2} = \sqrt{\{(6)^2 + x^2\}}$$

The time taken to traverse this path

$$t_a = \frac{AP}{V_a} = \frac{\sqrt{\{(6)^2 + x^2\}}}{V_a}$$

Similarly path PB and time taken to traverse it are given by

$$PB = \sqrt{\{(BN)^2 + (PN)^2\}} = \sqrt{\{(3)^2 + (12 - x)^2\}}$$

$$t_b = \frac{PB}{V_b} = \frac{\sqrt{\{(3)^2 + (12-x)^2\}}}{V_b}$$

Hence total time taken to travel down the path APB is given by:

$$t = t_a + t_b = \frac{\sqrt{\{(6)^2 + x^2\}}}{V_a} + \frac{\sqrt{\{(3)^2 + (12-x)^2\}}}{V_b}$$

For *t* to be minimum, we have,

$$\frac{dt}{dx} = 0 = \frac{1}{V_a} \cdot \frac{1}{2} [6^2 + x^2]^{-1/2} . 2x + \frac{1}{V_b} \cdot \frac{1}{2} [(12 - x)^2 + 3^2]^{-1/2} \times 2 (12 - x) (-1)$$

or $$\frac{x}{V_a \sqrt{(6^2 + x^2)}} = \frac{12-x}{V_b \sqrt{(12-x^2 + 3^2)}}$$

But we are given that x = 4 km, for *t* to be minimum, hence

$$\frac{4}{V_a \sqrt{(36+16)}} = \frac{12-4}{V_a \sqrt{[(12-4)^2+9]}}$$

or $$\frac{4}{V_a \sqrt{52}} = \frac{8}{V_a \sqrt{73}},$$

or $$\frac{V_a}{V_b} = \frac{4}{8} \sqrt{\left(\frac{73}{52}\right)} = \frac{1}{2} \sqrt{\left(\frac{73}{52}\right)} = \sqrt{\left(\frac{73}{208}\right)}$$

(b) If the speeds are interchanged, then we have

$$t' = \frac{AP}{V_b} + \frac{PB}{V_a} = \frac{\sqrt{(6^2 + x^2)}}{V_b} + \sqrt{\frac{\{3^2 + (12-x)^2\}}{V_a}}$$

For the path to be the fastest, t' should be minimum, hence

$$\frac{dt'}{dx} = 0$$

$\therefore \quad \frac{1}{V_b}\frac{1}{2}(6^2 + x^2)^{-\frac{1}{2}} . 2x + \frac{1}{V_a}\frac{1}{2}[3^2 + (12-x)^2]^{-1/2}$ (12 – x) (-1) = 0

or $$\frac{x}{V_b\sqrt{(6^2+x^2)}} = \frac{12-x}{V_a\sqrt{\{3^2+(12-x)^2\}}}$$

or $$\frac{V_a}{V_b} = \frac{(12-x)\sqrt{(36+x^2)}}{\sqrt{\{9+(12-x)^2\}}}$$

Squaring and solving, we get

x = 10.2km.

(c) See figure 2 for the solution of this part.

3. See figure 3. Let r be the radius of the sphere. If L be the hypothetical length of the path followed by light, that is,

$$L = AP + PB$$

If angle PAB be θ, then the length L is given by :

$$L = AB\cos\theta + AB\sin\theta, \qquad [\because \angle APB = 90^0]$$

or $$L = 2r(\cos\theta + \sin\theta), \qquad [\because AB = 2r]$$

Following Fermat's principle, the path length L should be maximum or minimum. But in both cases, we have

$$\frac{dL}{d\theta} = 0 = 2r(-\sin\theta + \cos\theta) \qquad \ldots\ldots(1)$$

or $$\sin\theta = \cos\theta,$$

$\therefore$ $$\theta = \frac{\pi}{4}$$

Substituting the value of θ, the actual path length which light shall follow :

$$L = 2r(\cos 45^0 + \sin 45^0) = 2r\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)$$

$$= 2r \times \sqrt{2} = \sqrt{2} \times \text{diameter of the sphere.}$$

Differentiating equation (1) again to see whether the path is a maximum, or minimum, we get

$$\frac{d^2L}{d\theta^2} = 2r(-cos\theta - sin\theta) = -2r(\cos\theta + \sin\theta)$$

$$= -2r\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) = -\sqrt{2} \times 2r.$$

Which is negative quantity; hence the actual path is maximum in this case.

4. Let PQ = ar, we have

$$(PA)^2 = (PO)^2 + (OA)^2 - 2(PO)(OA)\cos\theta$$

$$= (ar)^2 + r^2 - 2(ar)r\cos\theta$$

$$= r^2[a^2 + 1 - 2a\cos\theta];$$

$$PA = r\,(1 + a^2 - 2a\cos\theta]^{1/2}$$

Draw a perpendicular OM on AQ from point O. In $\Delta AOQ,$

$$OA = OQ = r$$

$$\angle OAQ = \angle OQA = \frac{\theta}{2}.$$

$$AQ = AM + MQ = r\cos\frac{\theta}{2} + r\cos\frac{\theta}{2} = 2r\cos\frac{\theta}{2}.$$

If the length of the total path is *l*, then

$$L = PA + AQ = r\,[1 + a^2 - 2a\cos\theta]^{1/2} + 2r\cos\frac{\theta}{2}. \qquad \text{......(1)}$$

But according to Fermat's Principle, $\frac{dl}{d\theta} = 0$

∴ Differentiating equation (1) w.r.t. θ and equating to zero, we get

$$\frac{dl}{d\theta} = r.\frac{1}{2}\,[1 + a^2 - 2a\,cos\theta]^{-1/2}\,(2a\sin\theta) - 2r\sin\frac{\theta}{2}\,.\left(\frac{1}{2}\right) = 0$$

or
$$\frac{ar\sin\theta}{[1+\alpha^2 - 2\alpha\cos\theta]^{1/2}} - r\sin\frac{\theta}{2} = 0$$

or
$$\frac{ar\,.2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{[1+\alpha^2 - 2\alpha\cos\theta]^{\frac{1}{2}}} - r\sin\frac{\theta}{2} = 0$$

or
$$r\sin\frac{\theta}{2}\left[\frac{2\alpha\cos\frac{\theta}{2}}{[1+\alpha^2 - 2\alpha\cos\theta]^{1/2}} - 1\right] = 0 \qquad \text{.....(2)}$$

∴ Either $\sin\frac{\theta}{2} = 0$ or $\frac{2\alpha\cos\frac{\theta}{2}}{\{1+\alpha^2 - 2\alpha\cos\theta\}^{1/2}} - 1 = 0$ .....(3)

Equation (3) represents that the light ray PQ' starting from P, after being reflected at Q' reaches at Q. But we are given that the light ray is PA which after reflection at A reaches at Q. Therefore, equation (3) does not give satisfactory solution or equation (2). Equation (4) gives

$$\frac{2\alpha\cos\frac{\theta}{2}}{\{1+\alpha^2 - 2\alpha\cos\theta\}^{1/2}} = 1$$

or
$$4a^2\cos^2\frac{\theta}{2} = 1 + a^2 - 2a\cos\theta = 1 + a^2 - 2a\,(2\cos^2\frac{\theta}{2} - 1)$$

or
$$4a^2\cos^2\frac{\theta}{2}\,(1 + a) = (1 + a)^2$$

$$\cos^2\frac{\theta}{2} = \frac{1+a}{4a}$$

In this example, $a = 0.8$, therefore, $\cos\frac{\theta}{2} = \frac{3}{4}$

5. Let the positions of the man and tree be A and T respectively. Given AC = 30 m and CT = 36m. Let us suppose that the man walks on the hard ground along the border from *A* to

B and then from *B* to *T* on the sandy area to reach the tree *T* in minimum time.

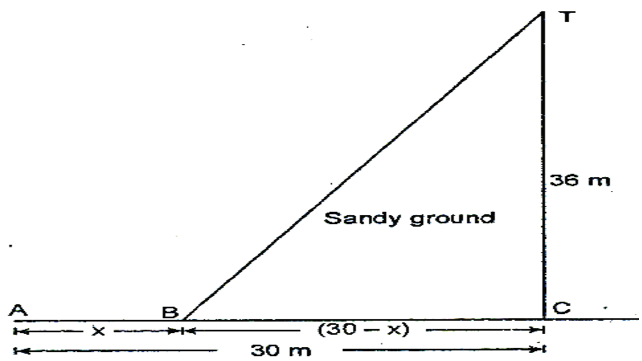

Fig. 1.13

Let *AB* = x m, then

BC = (100 – x) m.

∴ BT = $\sqrt{[(BC)^2 + (CT)^2]} = \sqrt{[(30-x)^2 + (36)^2]}$

The man traverse the distance AB with speed of 1.8 m/s and the distance BT with a speed of 1m/s. Therefore, time taken by the man to reach the tree

$$t = \frac{BT}{1.8} + \frac{BT}{1} = \frac{x}{1.8} + \frac{\sqrt{(30-x)^2 + (36)^2}}{1} \qquad \text{.......(1)}$$

If t is minimum, then $\frac{dt}{dx} = 0$, so differentiating equation (1) w.r.t. time *t*, we get,

$$\frac{dt}{dx} = \frac{1}{1.8} + \frac{1}{2}\,[30-x)^2 + (36)^2]^{-\frac{1}{2}} \times 2(30-x)(-1) = 0$$

$$\frac{1}{1.8} = \frac{30-x}{[(30-x)^2 + (36)^2]^{\frac{1}{2}}}$$

Solving we get

$$x \approx 30 \pm 24 = 54\text{m or } 6\text{m}$$

From figure it is obvious that for minimum time x cannot be 54m;

Therefore, x = 6m

Thus, to reach the tree in minimum time the man should walk 6m along the border on the hard ground and then he should walk on the sandy area along the line *BT*.

1.12.3 Objective Type Questions

1. (b), 2. (b), 3. (a), 4. (d), 5. (b) 6. (d) 7. (d)

UNIT 2: REFRACTION BY SPHERICAL SURFACES

CONTENTS

2.1 INTRODUCTION

In the previous unit you have studied about the Gauss's theory of image formation for a coaxial system. In the present unit we will apply this theory to understand the image formation by a lens or combination of lenses. Gauss has shown that if in an optical system the positions of certain specific points are known, the system may be treated as a single unit. The position and size of the image of an object may then directly be obtained by same relations as used for thin lenses or single surface, however, complicated the system may be. These points are called Cardinal points or Gauss points of an optical system.

This simplifies the understanding and processing of image formation to a great deal. The cardinal points and their use in the image formation have formed the basis of the present unit. Before dealing with the cardinal points, it is imperative to know the sign convention used in the optical system. Hence you are made accustomed to it, as well, in the initial part of this unit. An important aspect of image magnification in terms of lateral, axial and angular magnifications is discussed and a relation between different types of magnifications is established in the later part of the unit. Finally an important relation in the form of Newton's formula is derived, which will enhance your learning about the optical system of thin lenses.

2.2 OBJECTIVES

After studying this unit you will be able to

- Draw the ray diagrams for the image formation in a coaxial lens system
- Locate the six cardinal points in a lens system
- Apply the sign convention to solve various problems
- Determine the different types of magnifications produced in an optical system
- Derive Newton's formula

2.3 SIGN CONVENTION: OBJECT AND IMAGE SPACES

Before studying the different cases of image formation in an optical system, you must know the sign convention. In figure 2.1, XX' represents the optical axis and the dotted line represents the normal to it. The region on the left of optical system in which the object is placed is known as **object Space** and the region on the right of the optical system where the image is formed is called as **image space**. You can divide the sign convention into following three parts.

2.3.1 Axial or Longitudinal Distances

The distances measured along optical axis XX' or parallel to it are taken positive (+ve) or negative (-ve) with respect to the direction of incident ray. Conventionally, all distances measured in the direction of incidence are taken as positive while distances opposite in direction to the direction of incidence are taken as negative.

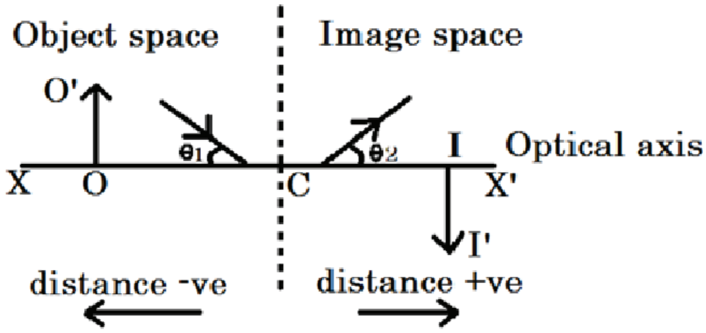

Fig. 2.1

This is clearly depicted in the figure 2.1. All distances are measured from the point C i.e. the centre of the reflecting or refracting surface known as **optical centre**. Thus distance OC is –ve while distance CI is +ve.

2.3.2 Lateral or Transverse Distance

The distances at right angle to the optical axis XX' are known as **lateral or transverse distances**. In figure 2.1, OO' and II' are the lateral distances of object and image respectively. Conventionally, lateral distance above XX' is taken as +ve while below it is taken as –ve. Hence OO' is +ve and II' is –ve.

2.3.3 Angles

Conventionally, anticlockwise angles with respect to XX' axis are taken as +ve and clockwise angles with respect to XX' axis are taken as –ve. In figure 2.1, the light ray in object space forms an angle θ_1 in clockwise direction (-ve) while light ray in the image space forms an angle θ_2 in anticlockwise direction (+ve) with respect to XX' axis respectively.

2.4 COAXIAL OPTICAL SYSTEM

A lens system having common optical axis is known as co-axial lens system. Generally, it consists of a number of lenses placed apart with a common optical axis known as **Principal axis**. We can determine the position and size of the image of an object formed by such a system by considering refraction at each lens separately. This process is however very tedious. Gauss showed that, if in an optical system the positions of certain specific points be known, the system may be treated as a single unit. The position and size of the image of an object may then directly be obtained by same relations as used for thin lenses or single surfaces.

2.5. CARDINAL POINTS

An optical system consists of six cardinal points which are as follows

(i) Two focal point

(ii) Two principal point
(iii) Two nodal point

2.5.1 Focal points and Focal Planes

These points lie on the principal axis of the optical system. Figure 2.2 (a) shows a convergent coaxial optical system having its axis XX'. An incident ray parallel to the axis, after refraction through the system, passes through a point F_2 on the axis, whatever is the path inside the system.

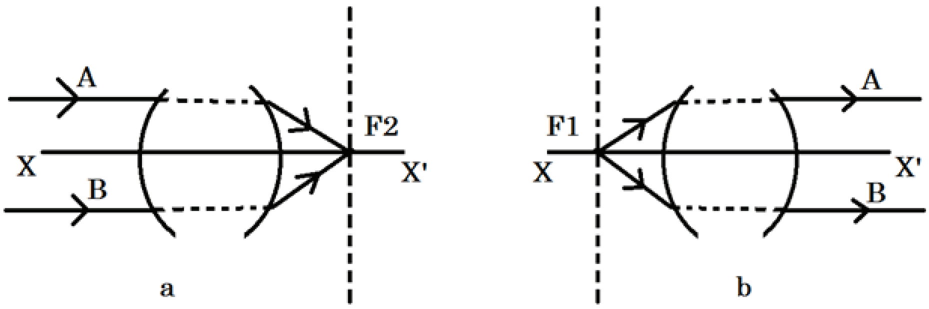

Fig. 2.2

This would be true for all the incident rays parallel to the axis (For divergent system, the emergent rays would appear to diverge from a point on the axis of the system). The point F_2 is called the **second focal point** of the system. You can define it as the image-point on the axis for which the object-point lies at infinity.

Similarly, an incident ray *b* passing through a point F_1 on the axis, after refraction through the system, becomes parallel to the axis (figure 2.2 (b)). The point F_1 is called the **first focal point** of the system. You can define it as the object-point on the axis for which the image-point lies at infinity.

The planes through F_1 and F_2 and perpendicular to the axis are called **first focal plane** and **second focal plane** of the system respectively. The first focal plane of an optical system is also called principal focal plane of the object space and second focal plane is called as principal focal plane of the image space.

2.5.2 Principal Points and Principal Planes

The Principal points are a pair of conjugate points on the principal axis of the optical system having unit positive linear magnification.

In figure 2.3, an incident ray A parallel to the principal axis, after refraction through the optical system, passes through the second focal point F_2. Produce the incident ray A onwards and emergent ray A backwards such that they intersect at A_2. The plane through A_2 and perpendicular to the axis XX' is called the **second principal plane**, and its point of intersection H_2 with the axis is called the **second principal point**.

Similarly, the incident ray B passing through the first focal point F_1, after refraction through optical system, emerges parallel to the axis. Produce the emergent ray B backward and incident ray B forward such that they intersect at A_1. The plane through A_1 and perpendicular to the XX' axis is called the **first principal plane**, and its point of intersection H_1 with the axis is called the **first principal point**.

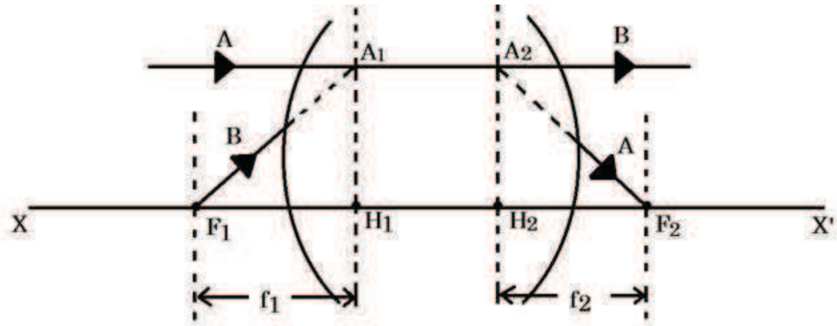

Fig. 2.3

The incident ray B has been so chosen that the corresponding emergent ray lies at the same distance above the axis as the incident ray A. It is thus seen that the incident rays A and B are converging towards the point A_1 and the corresponding emergent rays appear to diverge from the point A_2. Hence A_2 is the image of A_1, where $H_1A_1 = H_2A_2$. Thus, A_1 and A_2 are conjugate points. This is true for all such pairs on the principal planes. Hence if an object be placed in the first principal plane, an erect image of the same size would be formed in the second principal plane, that is, the linear magnification is +1.

Finally, it follows that if an incident ray passes through the first principal plane at a certain height from the principal axis, the corresponding emergent ray will pass through the second principal plane at the same height and on the same side of the axis.

Focal Lengths

The distance H_1F_1 is called the **first focal length** f_1 and the distance H_2F_2 the **second focal length** f_2 of the system. f_1 and f_2 are also known as focal lengths in object space and image space respectively. If the medium be same on the two sides of the system then $f_1 = f_2$ (numerically).

2.5.3 Nodal Points and Nodal Planes

The nodal points are a pair of conjugate points on the principal axis of the system, having unit positive angular magnification. They are such that an incident ray directed towards the one nodal point emerges parallel to its original direction through the other nodal point. In figure 2.4, N_1 and N_2 are the nodal points, and the emergent ray N_2B is parallel to the incident ray AN_1. The nodal points therefore do for angles what the principal planes do for transverse distance.

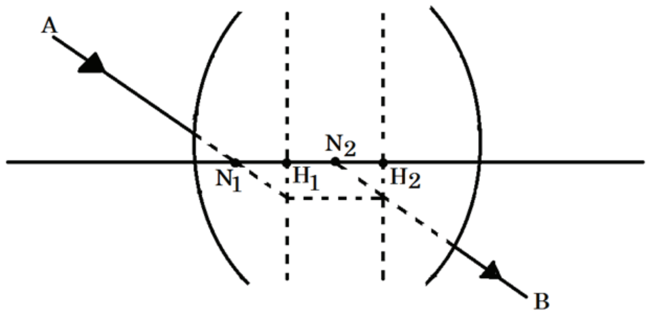

Fig. 2.4

The planes through N_1 and N_2 and perpendicular to the axis are called **nodal planes**. The distances of these nodal points are measured from the focal points. Also the distance between two nodal points is equal to the distance between Principal Points.

2.6 RULES OF IMAGE FORMATION

When the positions of cardinal points are known, you can draw the image of an object formed by a lens system using the following rules which obeys the properties of the cardinal points:

(i) An incident ray parallel to the principal axis, after refraction, passes or appears to pass through the second focal point F_2.

(ii) An incident ray passing through the first focal point F_1 or directed towards F_1 becomes parallel to the axis after refraction.

(iii) An incident ray passing through the first principal plane at a certain height from the principal axis, emerges through the second principal plane at the same height and on the same side of the axis.

(iv) An incident ray directed towards the first nodal point N_1, emerges parallel to its original direction through the second nodal point N_2.

Usually, the principal points H_1, H_2 and the focal points F_1, F_2 are enough to trace the image formed by a system. Hence, we can replace an actual optical system by a skeleton consisting simply of the axis and these four points.

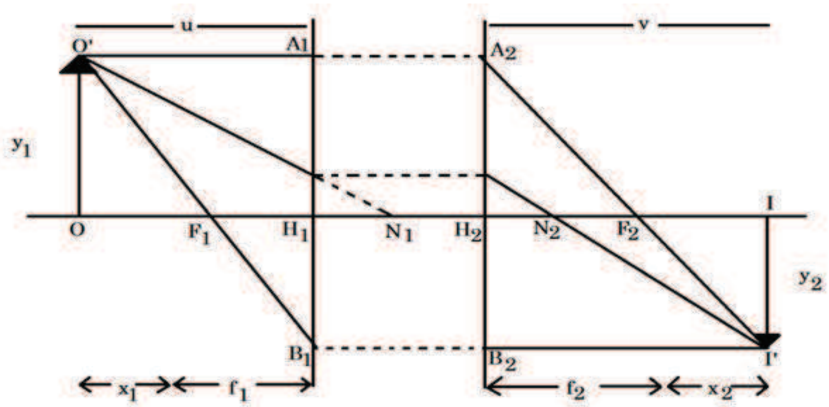

Fig. 2.5

Figure 2.5 shows the construction of the image of an object OO' by a convergent system. $O'A_1$ is ray parallel to the axis meeting the first principal plane in A_1. It must emerge from the system through A_2 on the second principal plane such that $H_1A_1 = H_2A_2$, and must also pass through F_2. $O'F_1$ is another ray through the first focal point F_1 and meeting the first principal plane in B_1. It must emerge parallel to the axis through B_2 such that $H_1B_1 = H_2B_2$. The two emergent rays meet at I', which is the image of O'. A third ray through N_1 and its conjugate parallel ray through N_2 may also be drawn, but it is not necessary. The perpendicular II', drawn from I' on the axis, is the image of the object OO'.

2.7 MAGNIFICATION OF A LENS SYSTEM

All optical systems are used to form a magnified image of an object. The magnification produced by an optical system is defined as its ability to enlarge the size of image with respect to the size of the object. It is denoted by m and expressed as:

$$m = \frac{size\ of\ image}{size\ of\ object}$$

There are three types of magnifications associated with coaxial lens systems:

(i) Lateral or Transverse Magnification: The lateral magnification of a lens-system is defined as the ratio of the length of the image to the length of the object, both being measured perpendicular to the axis of the system. As the distances above the axis are taken as positive and those below the axis negative, the lateral magnification is positive for an erect image and negative for an inverted image.

$$m = \frac{height\ of\ image}{height\ of\ object}$$

(ii) Axial or Longitudinal Magnification: If the object and the image have small extensions dx_1 and dx_2 respectively along the axis of the system, the ratio of dx_2 and dx_1 is called the longitudinal magnification of the system. It is given by

$$m_x = \frac{dx_2}{dx_1}$$

(iii) Angular Magnification: If θ_1 and θ_2 are the angles which the incident and the corresponding emergent rays make with the principal axis of the system, then the ratio of $\tan\theta_2$ to $\tan\theta_1$ is called the angular magnification of the system

$$m_\theta = \frac{\tan\theta_2}{\tan\theta_1}$$

2.7.1 Relation among Three Magnifications

Consider a convergent coaxial lens-system as shown in figure 2.6, where XX' represents the principal axis. H_1, H_2 be the principal points, and F_1, F_2 the focal points of the system. Then $H_1F_1 = f_1$ and $H_2F_2 = f_2$, where f_1 and f_2 are the focal lengths of the system.

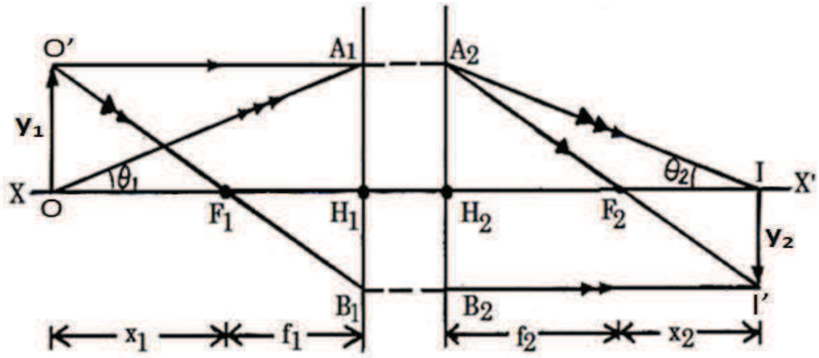

Fig. 2.6

Let OO' be a small object of size y_1 placed perpendicular to the axis. An incident ray $O'A_1$, parallel to the axis, emerges through A_2 (where $H_1A_1 = H_2A_2$) and passes through F_2. Another ray $O'B_1$ passing through F_1 emerges through B_2 (where $H_1B_1 = H_2B_2$) and becomes parallel to the axis. The two emergent rays meet at I', which is the image of O'. The perpendicular II' is the complete image of OO'.

Let x_1 and x_2 be the distances of the object OO' and the image II' respectively from F_1 and F_2. According to the sign convention, (Section 2.3) y_1, f_2 and x_2 are positive while y_2, f_1 and x_1 are negative. In similar triangles OO' F_1 and $H_1B_1F_1$, we have,

$$\frac{H_1B_1}{OO'} = \frac{H_1F_1}{F_1O}$$

or $$-\frac{y_2}{y_1} = \frac{-f_1}{-x_1} = \frac{f_1}{x_1}. \qquad [\because H_1B_1 = II' = -y_2]$$

Hence, the lateral magnification, m_y is,

$$m_y = \frac{y_2}{y_1} = -\frac{f_1}{x_1} \qquad \text{.......} \quad (2.1)$$

Also, in similar triangles $A_2H_2F_2$ and II' F_2, we have,

$$\frac{II'}{H_2A_2} = \frac{F_2I}{H_2F_2}$$

or $$-\frac{y_2}{y_1} = \frac{x_1}{f_2} \qquad [\because H_2A_2 = OO' = y_1]$$

Hence, the lateral magnification, m_y, is given by

$$m_y = \frac{y_2}{y_1} = -\frac{x_2}{f_2} \qquad \text{........} \quad (2.2)$$

Comparing equations (2.1) and (2.2), we have,

$$\frac{-f_1}{x_1} = -\frac{x_2}{f_2}$$

or $$x_1 x_2 = f_1 f_2 \qquad \text{........} \quad (2.3)$$

This is known as Newton's formula. Differentiating above equation, we have,

$$x_1 (dx_2) + x_2 (dx_1) = 0.$$

Hence the longitudinal magnification m_x is given by

$$m_x = \frac{dx_2}{dx_1} = -\frac{x_2}{x_1} \qquad \text{.......} \quad (2.4)$$

Let us now consider an incident ray OA_1 and its corresponding emergent ray A_2I. Let θ_1 and θ_2 be the angles which these rays make with the principal axis. According to sign convention, θ_1 is positive and θ_2 negative. Now, from the figure 2.7, we can write

$$\tan\theta_1 = \frac{A_1H_1}{H_1O} = \frac{y_1}{-f_1 - x_1} = \frac{-y_1}{f_1 + x_1}$$

and $$\tan(-\theta_2) = \frac{A_2H_2}{H_2I} = \frac{y_1}{f_2 + x_2}$$

or $$\tan\theta_2 = \frac{-y_1}{f_2 + x_2}$$

Hence the angular magnification m_θ can be written as

$$m_\theta = \frac{\tan\theta_2}{\tan\theta_1} = \frac{-y_1/(f_2 + x_2)}{-y_1/(f_1 + x_1)} = \frac{f_1 + x_1}{f_2 + x_2}$$

Using Newton's formula $x_1x_2 = f_1f_2$

$$m_\theta = \frac{f_1 + x_1}{f_2 + \left(\frac{f_1f_2}{x_1}\right)} = \frac{f_1 + x_1}{\frac{f_2}{x_1}(x_1 + f_1)} = \frac{x_1}{f_2}$$

∴ $$m_\theta = \frac{x_1}{f_2} = \frac{f_1}{x_2} \qquad \text{.......} \quad (2.5)$$

We can write various magnifications by using the expressions given by equations 2.1, 2.2, 2.3, 2.4 and 2.5 as

Transverse magnification (Lateral Magnification) $m_y = -\frac{f_1}{x_1} = -\frac{x_2}{f_2}$

Longitudinal magnification $m_x = -\frac{x_2}{x_1}$

Angular magnification $m_\theta = \frac{x_1}{f_2} = \frac{f_1}{x_2}$

From these, we get the following relations:

Relation I:

$$m_x \times m_\theta = -\frac{x_2}{x_1} \times \frac{x_1}{f_2} = -\frac{x_2}{f_2} = m_y$$

That is, longitudinal magnification × angular magnification = lateral magnification.

Relation II:

$$m_y \times m_\theta = -\frac{f_1}{x_1} \times \frac{x_1}{f_2} = -\frac{f_1}{f_2},$$

That is, the product of lateral and angular magnifications is a constant and equal to the ratio of the focal lengths of the system.

Relation III:

$$m_x = -\frac{x_2}{x_1}$$

Substituting values of x_2 and x_1 from equation 2.1 and 2.2, we get,

$$m_x = -\frac{-m_y f_2}{-f_1/m_y} = -m_y^2 \left(\frac{f_2}{f_1}\right)$$

But f_2/f_1 is a constant

∴ $m_x \propto m_y^2$,

That is, the longitudinal magnification is proportional to the square of the lateral magnification.

2.8 HELMHOLTZ - LAGRANGE'S MAGNIFICATION FORMULA

This is also known as Helmholtz's magnification formula. This formula gives the inter relationship between angular magnification, lateral magnification and axial magnification. Let SPS' (figure 2.7) be the convex spherical refracting surface separating media of refractive indices μ_1 and μ_2 respectively, μ_2 being denser. Let *P* be the pole and *C* the centre of curvature of the surface.

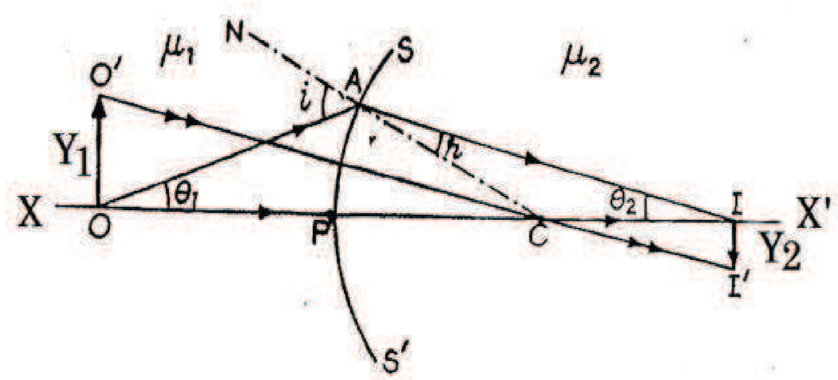

Fig. 2.7

Let OO' be an object placed on the axis perpendicular to it. An incident ray OA, after refraction at A, bends towards the normal CN (drawn on spherical surface) and goes in the direction *AI*. Another ray OP meets the surface normally and passes undeviated. The two refracted rays meet at *I,* which is the image of *O*. To find the image of *O'*, let us take a ray *O'C*. As it passes through the centre of curvature C, it strikes the surface normally and goes undeviated. The image of O' lies somewhere on this line. Also it lies on the perpendicular to the axis at *I.* Hence I', the point of intersection of the two, is the image of O' and II' is the complete image of OO'.

Now if y_1 and y_2 be the sizes of the object and the image respectively then by sign convention y_1 is positive and y_2 is negative. Let θ_1 and θ_2 be the angles made by the conjugate rays OA and IA respectively with the axis. By sign convention, θ_1 is positive and θ_2 is negative. Let *i* and *r* be the angles of incidence and refraction respectively at A.

Since triangle COO' is similar to CII', therefore,

$$\frac{II'}{OO'} = \frac{CI}{CO}$$

or $$\frac{-y_2}{y_1} = \frac{CI}{CA}\left(\frac{CA}{CO}\right) \qquad \ldots\ldots. \ (2.6)$$

Now, in ΔCIA, we have

$$\frac{CI}{CA} = \frac{\angle CAI}{\angle CIA} = \frac{\sin r}{\sin\ (-\theta_2)} = -\frac{\sin r}{\sin \theta_2}$$

and in ΔCOA, we have

$$\frac{CA}{CO} = \frac{\angle COA}{\angle CAO} = \frac{\sin \theta_1}{\sin\ (180^0 - i)} = \frac{\sin \theta_1}{\sin i}$$

Substituting these values in equation (2.6), we get,

$$\frac{y_2}{y_1} = \frac{\sin r}{\sin \theta_2}\left(\frac{\sin \theta_1}{\sin i}\right)$$

According to Snell's law, $\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1}$

∴ $$\frac{y_2}{y_1} = \frac{\mu_1 \sin \theta_1}{\mu_2 \sin \theta_2}$$

or $$\mu_1 y_1 \sin \theta_1 = \mu_2 y_2 \sin \theta_2 .$$

For paraxial rays, (the rays which make small angles with the principal axis) θ_1 and θ_2 are small, so that we may put,

$$\sin \theta_1 = \theta_1 = \tan \theta_1$$

and $$\sin \theta_2 = \theta_2 = \tan \theta_2.$$

Then, $$\mu_1 y_1 \tan \theta_1 = \mu_2 y_2 \tan \theta_2 \qquad \ldots\ldots. \ (2.7)$$

This relation can also be written as

$$\mu_1 y_1\ \theta_1 = \mu_2 y_2\ \theta_2 \qquad \ldots\ldots. \ (2.8)$$

This formula, which was first given by Lagrange, relates the linear transverse magnification (y_2/y_1) and the angular magnification ($\tan \theta_2 / \tan \theta_1$ or θ_2/θ_1) conjugate planes OO' and II'.

Let us now consider a coaxial system having (n–1) refracting surfaces separating n media of refractive indices $\mu_1, \mu_2, \mu_3, \ldots\ldots. \mu_n$ respectively. Let a small object of linear size y_1 perpendicular to the axis be placed in the first medium and a ray from it make an angle θ_1 with the axis. After refraction at the first surface this ray makes an angle θ_2 with the axis and appears to form an image of linear size y_2. After refraction at the second surface, it makes an angle θ_3 with the axis and appears to form an image of linear size y_3, and so on. Hence applying equation 2.7 to each surface in turn, we obtain,

$$\mu_1\ y_1 \tan \theta_1 = \mu_2\ y_2 \tan \theta_2 = \mu_3\ y_3 \tan \theta_3 = \ldots\ldots\ldots = \mu_n\ y_n \tan \theta_n$$

This is the Helmholtz Lagrange's equation of magnification for refraction at a system of coaxial surfaces. If the angles $\theta_1, \theta_2, \theta_3 \ldots\ldots\ldots \theta_n$ are small, then it can also be written as

$$\mu_1\ y_1\ \theta_1 = \mu_2\ y_2\ \theta_2 = \ldots\ldots\ldots\ldots\ldots\ldots = \mu_n\ y_n\ \theta_n$$

Example 1: A convex lens of focal length 25cm and made of glass (${}_a\mu_g = 1.5$) is immersed in water (${}_a\mu_w = 4/3$). Calculate the change in the focal length of the lens.

Solution: Refractive index of glass with respect to water is

$$ {}_w\mu_g = \frac{{}_a\mu_g}{{}_a\mu_w} = \frac{1.5}{4/3} = \frac{9}{8}. $$

∴ Focal length of glass lens in water is

$$ {}_wf_g = \frac{({}_a\mu_g - 1)}{({}_w\mu_g - 1)} \times \ {}_af_g = \frac{(1.5-1)}{(9/8\ \ -1)} \times 25 = 100cm. $$

2.9 SOME IMPORTANT RELATIONS

Consider a convergent coaxial lens-system (figure 2.8), having principal axis XX', separating two media of refractive indices μ_1 and μ_2. Let H_1, H_2 be the principal points and F_1, F_2 the focal points of the system.

Suppose OO' is a small object placed perpendicular to the axis. An incident ray $O'A_1$ parallel to the principal axis meets the first principal plane at A_1. It emerges through A_2 in the second principal plane such that $H_1A_1 = H_2A_2$ and passes through F_2. Another incident ray $O'B_1$, passing through F_1, meets the first principal plane at B_1. It emerges through B_2 in the second principal plane such that $H_1B_1 = H_2B_2$ and becomes parallel to the axis. The two emergent rays meet at I' which is the image of O'. II', which is the perpendicular from I' on the axis, is the complete image of OO'.

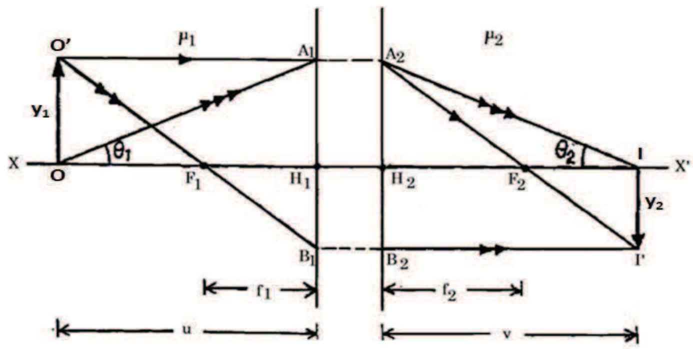

Fig. 2.8

Let y_1 and y_2 be the sizes of the object and the image and u and v their distances from the first and the second principal points respectively. Let f_1 and f_2 be the focal lengths of the lens-system. Thus, according to the sign convention, we have

$$ OO' = y_1,\ II' = -\ y_2,\ H_1O = -\ u,\ H_2I = +v,\ H_1F_1 = -\ f_1 \text{ and } H_2F_2 = +f_2 $$

Now proceeding in four steps:

Step-1: In similar triangles $F_1H_1B_1$ and $O'A_1B_1$, we have

$$ \frac{H_1F_1}{A_1O'} = \frac{H_1B_1}{A_1B_1} = \frac{H_1B_1}{H_1A_1 + H_1B_1} $$

or $$\frac{H_1F_1}{H_1O} = \frac{II'}{OO' + II'}$$

Putting the values according to the sign convention we get

$$\frac{-f_1}{-u} = \frac{-y_2}{y_1 - y_2} \quad \text{.......} \quad (2.9)$$

Again, in similar triangles $A_2H_2F_2$ and A_2B_2I', we have

$$\frac{H_2F_2}{B_2I'} = \frac{H_2A_2}{B_2A_2} = \frac{H_2A_2}{H_2A_2 + H_2B_2}$$

or $$\frac{H_2F_2}{H_2I} = \frac{OO'}{OO' + II'}$$

Putting the values according to sign convention, we get

$$\frac{f_2}{v} = \frac{y_1}{y_1 - y_2} \quad \text{.......} \quad (2.10)$$

On adding equations (2.9) and (2.10), we get,

$$\frac{-f_1}{-u} + \frac{f_2}{v} = \frac{-y_2 + y_1}{y_1 - y_2}$$

or $$\frac{f_1}{u} + \frac{f_2}{v} = 1 \quad \text{.......} \quad (2.11)$$

Step-2: Dividing equation (2.10) by (2.11), we obtain,

$$\frac{f_1/u}{f_2/v} = -\frac{y_2}{y_1}$$

∴ The linear transverse magnification is

$$m = \frac{y_2}{y_1} = -\frac{v}{u}\left(\frac{f_1}{f_2}\right) \quad \text{.......} \quad (2.12)$$

Step-3: Let us now consider a ray OA_1. The corresponding emergent ray meet the second principal plane at A_2, where $H_1A_1 = H_2A_2$. It also passes through I. Since I is the image of O, A_2I is the emergent ray. If θ_1 and θ_2 be the angles which the rays OA_1 and A_2I make with the principal axis then by sign convention, θ_1 is positive and θ_2 is negative. Now, from figure 2.8, we have,

$$\tan\theta_1 = \frac{A_1H_1}{H_1O} = \frac{y_1}{-u}$$

and $$\tan(-\theta_2) = \frac{A_2H_2}{H_2I} = \frac{y_1}{v}$$

∴ $$\frac{\tan\theta_1}{\tan\theta_2} = \frac{v}{u}$$

But, by Helmholtz's law of magnification ($\mu_1 y_1 \tan\theta_1 = \mu_2 y_2 \tan\theta_2$), we have

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\mu_2 y_2}{\mu_1 y_1}$$

∴ $$\frac{\mu_2 y_2}{\mu_1 y_1} = \frac{v}{u}$$

Hence the linear transverse magnification is

$$\frac{y_2}{y_1} = \frac{\mu_1}{\mu_2}\left(\frac{v}{u}\right)$$

Comparing this with equation 2.12, we get

$$-\frac{v}{u}\left(\frac{f_1}{f_2}\right) = \frac{\mu_1}{\mu_2}\left(\frac{v}{u}\right)$$

or $$\left(\frac{f_1}{f_2}\right) = -\frac{\mu_1}{\mu_2} \quad .... (2.13)$$

Step-4: If the medium on the two sides of the system is the same, that is, $\mu_1 = \mu_2$, then equation 2.13 can be written

$$f_2 = -f_1 \quad .... (2.14)$$

Now, putting $f_2 = -f_1 = f$ (say) in equation (2.10), we get,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \quad .... (2.15)$$

Similarly, putting $f_2 = -f_1$ in equation (2.11), we get,

$$m = \frac{y_2}{y_1} = \frac{v}{u} \quad .... (2.16)$$

These formulae are similar to those for a thin lens.

Thus, when the medium on both sides of lens-system is the same and, u and v are measured from the first and the second principal planes respectively then the formula for conjugate distances is exactly similar to that for a thin lens.

Self Assessment Questions

1. Define Cardinal points of a lens system.
2. Show that the distance between Principal points is the same as distance between nodal points.
3. Show that principal points coincide with nodal points if the medium is same on both side of the system.
4. Define lateral, axial and angular magnifications of a lens system. Establish relations between them.

2.10 NEWTON'S FORMULA

If the distances of two conjugate points (object and image) on the principal axis from the respective focal points be x_1 and x_2 then from Newton's formula (equation 2.3), we have,

$$x_1 x_2 = f_1 f_2$$

This Newton's formula can be easily derived by making use of cardinal points of the lens system. Figure 2.10 shows a convergent coaxial lens-system in which XX' is the principal axis, H_1 and H_2 are the principal points, and F_1, F_2 are the focal points of the system. Then $H_1F_1 = f_1$ and $H_2F_2 = f_2$, where f_1 and f_2 are the focal lengths of the system.

Let OO' be a small object placed perpendicular to the principal axis. An incident ray $O'A_1$ parallel to the axis meets the first principal plane at A_1. It emerges through a point A_2 in the second principal plane such that $H_1A_1 = H_2A_2$, and passes through F_2. Another ray $O'B_1$ passing through F_1 meets the first principal plane at B_1. It emerges through B_2 in the second principal plane such that $H_1B_1 = H_2B_2$ and becomes parallel to the axis. The two emergent rays meet at I' which is the image of O'. The perpendicular II', drawn from I' on the axis, is the complete image of OO'.

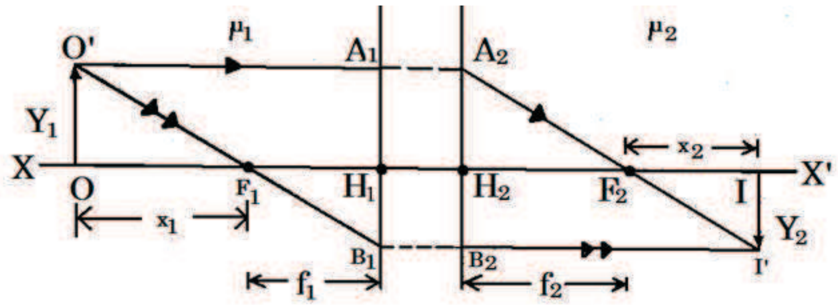

Fig. 2.9

Let $OO' = y_1$, $II' = y_2$, $F_1O = x_1$ and $F_2I = x_2$. According to the sign convention y_1, x_2 and f_2 are positive while y_2, x_1 and f_1 are negative.

Using similar triangles $H_1B_1F_1$ and $OO'F_1$, we have

$$\frac{H_1B_1}{OO'} = \frac{H_1F_1}{F_1O}$$

Putting the values with proper signs, we get,

$$-\frac{y_2}{y_1} = \frac{-f_1}{-x_1} = \frac{f_1}{x_1} \qquad \text{.......} \quad (2.17)$$

Again from similar triangles triangles $A_2H_2F_2$ and $II'F_2$, we have,

$$\frac{II'}{H_2A_2} = \frac{F_2I}{H_2A_2}$$

or

$$\frac{-y_2}{y_1} = \frac{x_2}{f_2} \qquad \text{.......} \quad (2.18)$$

Comparing equations (2.17) and (2.18), we obtain,

$$\frac{f_1}{x_1} = \frac{x_2}{f_2}$$

$$x_1x_2 = f_1f_2 \qquad \text{.......} \quad (2.19)$$

This is Newton's formula. If the medium on both sides of the system is the same, then we have, $f_2 = -f_1 = f$. Hence the Newton's formula (equation 2.19) becomes

$$x_1\,x_2 = -f^2 \qquad \text{.......} \quad (2.20)$$

2.11 SUMMARY

1. The distances are measured from optical center of the lens. The distances measured along the optical axis in the direction of incident ray are positive while those in opposite direction are negative.
2. The distances measured above optical axis are positive while those below are negative.
3. The angles measured anticlockwise with respect to optical axis are positive while those measured clockwise are negative.
4. A lens has six cardinal points: a pair of focal points, a pair of principal points and a pair of nodal points. The planes passing through these points and normal to optical axis are known as focal-, principal- and nodal- planes respectively.
5. The ray diagram for the image formation in lens follows the rules given below-
   a. An incident ray parallel to optical axis passes or appears to pass through second focal point after refraction.
   b. An incident ray through first focal point or directed towards it emerges parallel to optical axis after refraction.
   c. An incident ray passing through the first principal plane at a certain height from the principal axis emerges through the second principal plane at the same height and on the same side of the axis.
   d. An incident ray directed towards the first nodal point emerges parallel to its original direction through the second nodal point.
6. Magnification is the ability to enlarge the size of image relative to the size of the object. It is expressed as the ratio of size of image to that of object. It is of three kinds:
   a) Lateral Magnification: It is the ratio of the length of the image to the length of object, both being measured perpendicular to the axis of the system
   b) Axial Magnification: It is the ratio of dx_2 and dx_1, where dx_1 and dx_2 are small extensions of the object and the image respectively along the axis of the system.
   c) Angular Magnification: It is the ratio of $\tan\theta_2$ and $\tan\theta_1$, where θ_1 and θ_2 are the angles made by incident ray and the corresponding emergent ray with the optical axis.
7. The three types of magnification can be related as-

   a) $m_x \times m_\theta = m_y$, (b) $m_y \times m_\theta = -\frac{f_1}{f_2}$, (c) $m_x = -m_y^2\left(\frac{f_2}{f_1}\right)$

   Where, m_x = axial magnification, m_θ = angular magnification, m_y = lateral magnification, f_1= first focal length and f_2 = second focal length
8. Helmholtz Lagrange's magnification equation gives the interrelationship between angular magnification, lateral magnification and axial magnification as

   $\mu_1 y_1 \tan\theta_1 = \mu_2 y_2 \tan\theta_2 = \mu_3 y_3 \tan\theta_3 = \ldots\ldots\ldots = \mu_n y_n \tan\theta_n$
9. Some important relations in coaxial lens system are as follows-

   a) $\left(\frac{f_1}{f_2}\right) = -\frac{\mu_1}{\mu_2}$, (b) $m = -\frac{v}{u}\left(\frac{f_1}{f_2}\right)$, (c) $\frac{f_1}{u} + \frac{f_2}{v} = 1$
10. Newton's Formula states that, if the distances of two conjugate points (object and image) on the principal axis from the respective focal points be x_1 and x_2 then $x_1x_2 = f_1f_2$, where, f_1 and f_2 are the first and second principal focal lengths of the system respectively.

2.12 GLOSSARY

Pole: The midpoint of a spherical mirror.

Radius of curvature: The linear distance between the pole and the centre of curvature.

Centre of curvature: The centre of the sphere of which the spherical mirror is a part.

Optical Centre: The point on the optical axis of a lens where all rays passing through it remain unrefracted.

Principal axis: The imaginary line passing through the pole and the centre of curvature of a spherical mirror.

Paraxial rays: The rays that makes small angle (θ) to the optical axis of the system, and lies close to the axis throughout the system.

Geometrical Centre: Physical centre of a lens as determined by measurement

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2.15 TERMINAL QUESTIONS

2.15.1 Short Answer Type

1. What are cardinal points of a coaxial optical system?
2. What are paraxial rays?
3. What are object space and image space?
4. Write down the rules of formation of image in a coaxial lens system.

2.15.2 Long Answer Type

1. Define cardinal points of a coaxial optical system and give their characteristics. Draw a ray diagram to clarify each.
2. Define Axial, Lateral and Angular magnifications. How they are interrelated with each other?
3. Deduce Helmholtz Lagrange equation for image formation for paraxial rays.

4. Deduce Newton's formula $x_1x_2 = f_1f_2$ for a coaxial optical system of two thin lenses. Where x_1 and x_2 are the distances of the object and the image from the first and second focal points respectively.

5. Show that the distance between two principal points is same as the distance between two nodal points. Also, show that the principal points coincide with nodal points if the medium is same on both sides of the system.

6. Prove the following relations for a coaxial lens-system separating two media of refractive indices μ_1 and μ_2 :

(i) $\frac{f_1}{u} + \frac{f_2}{v} = 1$, (ii) $m = -\frac{v}{u}\left(\frac{f_1}{f_2}\right)$ and (iii) $\frac{f_1}{f_2} = -\frac{\mu_1}{\mu_2}$,

where u and v are the distances of the object and the image from the first and the second principal points respectively, f_1 and f_2 the focal lengths of the system and m is linear transverse magnification.

7. What form do the three expressions, given in question number 6, take when the medium on the two sides of the system is the same?

2.15.3 Numerical Questions

1. Calculate the axial and angular magnification for a coaxial optical system in which the initial media is air and final media is water. The lateral magnification of the object is -3.0. (Take refractive index of water = 4/3)

2.15.4 Objective Type Questions

1. The points lying on the principal axis of the optical system and conjugate to points at infinity are

(a) Principal points (b) Focal points
(c) Nodal points (d) Cardinal points

2. Conjugate points on the principal axis of the optical system having unit positive linear magnification are called

(a) Principal Points (b) Focal points
(c) Nodal points (d) Cardinal point

3. Two Principal planes on a thin lens coincide and pass through

(a) First Principal Point (b) First focal point
(c) First Nodal Point (d) The optical centre of the lens

4. If h be the transverse distance of the point from the axis at which the ray meets the lens and f the focal length of the lens, then the angular deviation of the ray will be

(a) h / f (b) f / h
(c) h^2 / f^2 (d) f^2 / h^2

5. L_1 and L_2 be two thin convergent lenses of focal lengths f_1 and f_2 placed coaxially at a distance d apart. If H_1, H_2 be two principal points, then

(a) $L_1H_1 = + \frac{Fd}{f_1}$ (b) $L_2H_2 = - \frac{Fd}{f_1}$

(c) $L_1H_1 = - \frac{Fd}{f_1}$ (d) $L_2H_2 = \frac{Fd}{f_1}$

6. If a thin lens is placed such that refractive indices on the two sides are n_1 and n_2. The ratio of focal length f_1 / f_2 is

(a) $- n_1 / n_2$ (b) $- n_2/n_1$

(c) n_1 / n_2 (d) n_2 / n_1

7. An incident ray directed towards the first nodal point emerges

(a) Perpendicular to its original direction through first principal point

(b) Parallel to its original direction through second nodal point

(c) Opposite to its original direction through first focal point

(d) None of these

2.16 ANSWERS

Numerical Questions

Given, $\mu_1 = 1,\ \mu_2 = \frac{4}{3}$ and $m_y = 3.0$

Axial magnification, $m_x = - m_y^2 \left(\frac{f_2}{f_1}\right)$

Also, $\left(\frac{f_1}{f_2}\right) = - \frac{\mu_1}{\mu_2}$

$m_x = - m_y^2 \left(- \frac{\mu_2}{\mu_1}\right) = 12$

Angular magnification, $m_\theta = m_y / m_x = - 0.25$

Objective Type Questions

1. (b), **2.** (a), **3.** (d), **4.** (a), **5.** (b), **6.** (a), **7.** (c)

UNIT 3: THICK LENS, LENSES COMBINATIONS AND TELESCOPE

3.1 INTRODUCTION

So far we have discussed the image formation of an optical system consisting of thin lenses. However, most practical applications require the use of thick lenses. In order to produce sufficient illumination of the image for virtual observation the optical instruments such as telescopes and photographic objectives require wide apertures. Practically, the lenses thus produced are thick in nature.

In this unit, we will discuss the image formation by thick lens. The thickness of a lens is defined as the separation between the poles of its spherical refracting surfaces. When this thickness is comparable to its focal length then the lens is said to be thick. The distances in case of thick lens cannot be measured from a single optical centre as in the case of thin lens; however the distances can be referred to from the poles on the two surfaces. It was shown by C.F. Gauss that the formulae of thin lenses are applicable to thick lenses too if the position of certain specific points for a lens called cardinal points are known. In this way, a thick lens is treated as a combination of thin lenses. We have already studied that there are six cardinal points of a lens namely two principal points, two focal points and two nodal points. We will also discuss the variation of focal length of different types of lenses with its thickness by considering different examples.

3.2 OBJECTIVES

After studying this unit, you will be able to,

- Know the difference between thin and thick lenses
- Locate the cardinal points of a thick lens
- Classify the lens on the basis of their shape
- Calculate power of a lens
- Know about the optical devices like different types of telescopes
- Understand the advantages of reflecting telescope over refracting telescope

3.3 CARDINAL POINTS OF A THICK LENS

There are six cardinal points of a thick lens; two focal points F_1, F_2; two principal points H_1, H_2 and two nodal points N_1, N_2. You can see the cardinal points and the corresponding cardinal planes for a thick convex lens in figure 3.1. When the lens is placed in air, the nodal points N_1, N_2 coincide with the principal points H_1, H_2 respectively. The formation of the image II' of an object OO' is also shown in this figure. Although, the cardinal points are explained in the previous unit but we are again discussing them briefly, in this unit, with reference to thick lenses.

3.3.1 Focal Points

The focal points F_1, F_2 are a pair of points lying on the principal axis and conjugate to points at infinity. An incident ray $O'A_1$ parallel to the principal axis, after refraction through the lens, passes through the second focal point F_2, while an incident ray $O'B_1$ through the first point F_1, after refraction, emerges parallel to the principal axis XX'.

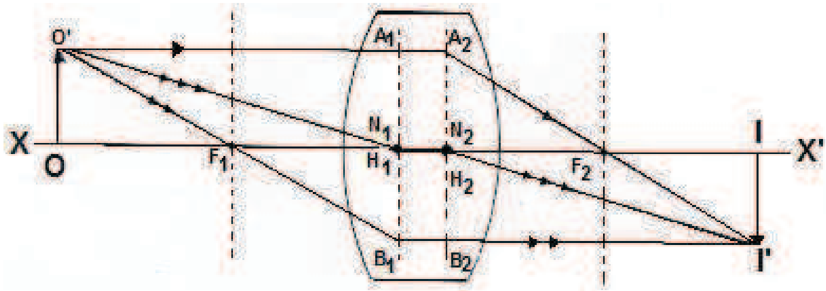

Figure 3.1

3.3.2 Principal Points

The principal points H_1, H_2 are a pair of conjugate points on the principal axis having unit positive linear transverse magnification. An incident ray meeting the first principal plane at a certain height from the principal axis emerges through the second principal plane at the same height and on the same side of the axis.

You can see $O'A_1$ is a ray parallel to the axis meeting the first principal plane at A_1. It will emerge from the lens through A_2 on the second principal plane such that $H_1A_1 = H_2A_2$, and also pass through F_2. $O'F_1$ is another ray through the first focal point F_1 and meeting the first principal plane in B_1. It will emerge parallel to the axis through B_2 such that $H_1B_1 = H_2B_2$.

3.3.3 Nodal Points

The nodal points N_1, N_2 are a pair of conjugate points on the principal axis having unit positive angular magnification. They are such that an incident ray directed towards N_1 emerges through N_2 parallel to itself. An incident ray $O'N_1$ and its conjugate parallel emergent ray N_2I' are shown in figure 3.1.

3.4 FOCAL LENGTH OF A THICK LENS

Let us consider a convex lens of thickness t and refractive index μ placed in air. Let R_1 and R_2 be the radii of curvature of the faces of the lens. The lens is a combination of two refracting surfaces with poles P_1 and P_2 (Figure 3.2). Draw a ray *PQ* parallel to the principal axis, incident on the first surface at a height h_1 above the axis. After refraction at the first surface, it follows the path QR in the lens and meets the second surface of the lens at a height h_2 above the axis. This ray, if produced forward, will meet the axis at point S, which acts as virtual object for the second surface. After refraction at the second surface, the emergent ray intersects the principal axis at F_2 which is the second focal point of the lens.

Let us produce the incident ray PQ forward and the emergent ray RF_2 backward such that they meet at A_2. The plane through A_2 and perpendicular to the axis is the second principal plane, and its point of intersection with the principal axis, H_2 is the second principal point. H_2F_2 is the focal length *f* of the lens.

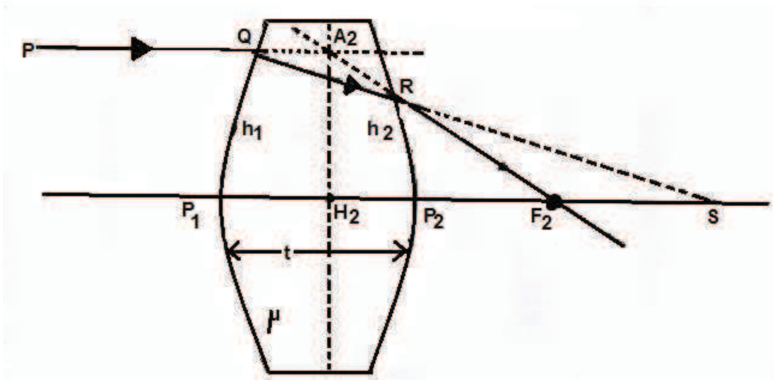

Figure 3.2

The refraction formula for a single surface is

$$\frac{\mu}{v} - \frac{1}{u} = \frac{\mu - 1}{R}$$

For refraction at the first surface (from air to lens) we can write

$$\mu = \infty, v = P_1S \text{ and } R = R_1$$

$\therefore$

$$\frac{\mu}{P_1S} - \frac{1}{\infty} = \frac{\mu - 1}{R_1}$$

or

$$\frac{1}{P_1S} = \frac{\mu - 1}{\mu R_1} \qquad \text{....... (3.1)}$$

For refraction at the second surface (from lens to air), we can write

$$u = P_2S,\ v = P_2F_2 \text{ and } R = R_2.$$

Also in this case, from lens to air (i.e., from denser to rarer), μ will be replaced by $1/\mu$. Hence

$$\frac{1/\mu}{P_2F_2} - \frac{1}{P_2S} = \frac{\left(\frac{1}{\mu}\right) - 1}{R_2}$$

or

$$\frac{1}{P_2F_2} = \frac{\mu}{P_2S} + \frac{1 - \mu}{R_2} \qquad \text{....... (3.2)}$$

Now, from similar triangles $A_2F_2H_2$, RF_2P_2 and from similar triangles QSP_1 and RSP_2, we have,

$$\frac{H_2F_2}{P_2F_2} = \frac{h_1}{h_2} = \frac{P_1S}{P_2S}$$

or

$$\frac{1}{P_2F_2} = \frac{1}{H_2F_2}\frac{P_1S}{P_2S} = \frac{1}{f}\frac{P_1S}{P_2S} \qquad \text{....... (3.3)}$$

Substituting the value of $\frac{1}{P_2F_2}$ in equation (3.2), we get,

$$\frac{1}{f}\frac{P_1S}{P_2S} = \frac{\mu}{P_2S} + \frac{1-\mu}{R_2}$$

or $$\frac{1}{f} = \frac{\mu}{P_1S} + \frac{P_2S}{P_1S}\frac{1-\mu}{R_2}$$

You can see from figure 3.2 that, $P_2S = P_1S - P_1P_2 = P_1S - t$.

∴ $$\frac{1}{f} = \frac{\mu}{P_1S} + \frac{P_1S-t}{P_1S}\frac{1-\mu}{R_2}$$

$$= \frac{\mu}{P_1S} + \left(1 - \frac{t}{P_1S}\right)\frac{1-\mu}{R_2}$$

Putting the value of $\frac{1}{P_1S}$ from equation 3.1 in this expression, we obtain,

$$\frac{1}{f} = \frac{\mu-1}{R_1} + \left\{1 - \frac{t(\mu-1)}{\mu R_1}\right\}\frac{1-\mu}{R_2}$$

$$= \frac{\mu-1}{R_1} - \frac{\mu-1}{R_2} + \frac{t(\mu-1)^2}{\mu R_1R_2}$$

or $$\frac{1}{f} = (\mu-1)\left\{\frac{1}{R_1} - \frac{1}{R_2} + \frac{(\mu-1)t}{\mu R_1R_2}\right\} \qquad \text{.......} \quad (3.4)$$

This is the expression for compute focal length of a thick lens.

3.4.1 Position of Cardinal Points

Let us now compute the positions of cardinal points.

Second Focal Point ($\boldsymbol{\beta_2}$): The distance of the second focal point F_2 from the second surface of the lens is P_2F_2. Using equation 3.3, we can write,

$$P_2F_2 = f\frac{P_2S}{P_1S} = f\,\frac{P_1S-t}{P_1S} = f\left(1 - \frac{t}{P_1S}\right)$$

Substituting the value of $\frac{1}{P_1S}$ from equation 3.1 we get

$$P_2F_2 = +f\left[1 - \frac{(\mu-1)\,t}{\mu R_1}\right]$$

or $$\beta_2 = +f\left[1 - \frac{(\mu-1)\,t}{\mu R_1}\right] \qquad \text{.......} \quad (3.5)$$

Second Principal Point ($\boldsymbol{\propto_2}$): The distance of the second principal point H_2 from the second surface P_2 is

$$P_2H_2 = F_2H_2 - F_2P_2$$

$$= -H_2F_2 + P_2F_2$$

Substituting the value of P_2F_2 from equation 3.5 we get

$$P_2H_2 = -f + f\left[1 - \frac{(\mu-1)\,t}{\mu R_1}\right]$$

$$= -f\,\frac{(\mu-1)\,t}{\mu R_1}$$

or
$$\propto_2 = -f\,\frac{(\mu-1)\,t}{\mu R_1} \qquad \text{.......} \quad (3.6)$$

First Focal Point (β_1): If we consider the incident ray PQ as shown in figure 3.2, coming from right, then R_1 and R_2 will interchange and the signs of f, R_1 and R_2 will become opposite. Now, the distance of the first focal point F_1 from the first surface P_1 is (from equation 3.5)

$$P_1F_1 = -f\left[1 - \frac{(\mu-1)\,t}{\mu(-R_2)}\right]$$

$$= -f\left[1 + \frac{(\mu-1)\,t}{\mu R_2}\right]$$

$$\beta_1 = -f\left[1 + \frac{(\mu-1)\,t}{\mu R_2}\right] \qquad \text{.......} \quad (3.7)$$

First Principal Point ($\propto_1$): The distance of the first principal point H_1 from the first surface P_1 is (from equation 3.6)

$$P_1H_1 = +f\,\frac{(\mu-1)\,t}{\mu(-R_2)}$$

$$= -f\,\frac{(\mu-1)\,t}{\mu R_2}$$

$$\propto_1 = -f\,\frac{(\mu-1)\,t}{\mu R_2} \qquad \text{.......} \quad (3.8)$$

Nodal Points: Since the medium on both sides of the lens is same (air), the nodal points N_1 and N_2 are the same as the principal points H_1 and H_2 respectively.

3.5 VARIATION OF FOCAL LENGTH OF A THICK BI-CONVEX LENS

We have calculated the focal length of a thick lens in a previous section as

$$\frac{1}{f} = (\mu-1)\left\{\frac{1}{R_1} - \frac{1}{R_2} + \frac{(\mu-1)t}{\mu\,R_1R_2}\right\} \qquad \text{.......} \quad (3.9)$$

Let us consider a biconvex lens having $R_1 = +r_1$ and $R_2 = -r_2$ where $r_1, r_2 > 0$, we have,

$$\frac{1}{f} = (\mu-1)\left\{\frac{1}{r_1} + \frac{1}{r_2} - \frac{(\mu-1)t}{\mu\,r_1r_2}\right\} \qquad \text{.......} \quad (3.10)$$

For a thin lens i.e. t = 0 the focal length f_o can be obtained as

$$\frac{1}{f_0} = (\mu-1)\left\{\frac{1}{r_1} + \frac{1}{r_2}\right\} \qquad \text{.......} \quad (3.11)$$

Substituting equation 3.11 in equation 3.10, we get,

$$\frac{1}{f} = \frac{1}{f_0} - \frac{(\mu-1)^2 t}{\mu\,r_1r_2} \qquad \text{.......} \quad (3.12)$$

From this equation we can infer that if the value of t increases from zero to upwards, the value of $\frac{1}{f}$ decreases which means the focal length of a lens increases with the increase in thickness. We know that the power of a lens is inversely proportional to the focal length; therefore the power of a thick lens is less than the power of a thin lens for the same refracting surfaces. When we increase the thickness of a biconvex lens continuously a situation arises when $\frac{1}{f}$ becomes zero and on further increasing thickness, the lens becomes a concave lens (diverging). This is the critical value of thickness, denoted by t_c and can be obtained by equation 3.10. Therefore, we can write

$$\frac{1}{f} = (\mu - 1)\left\{\frac{1}{r_1} + \frac{1}{r_2} - \frac{(\mu-1)t_c}{\mu\, r_1 r_2}\right\} = 0$$

or $$\frac{1}{r_1} + \frac{1}{r_2} - \frac{(\mu-1)t_c}{\mu\, r_1 r_2} = 0$$

or $$\frac{(\mu-1)t_c}{\mu\, r_1 r_2} = \frac{1}{r_1} + \frac{1}{r_2}$$

or $$t_c = \frac{\mu}{(\mu-1)}\,(r_1 + r_2) \qquad \text{....... (3.13)}$$

If the radii of curvature of both the surfaces of biconvex lens is same then we can write $r_1 = r_2 = r$, so

$$t_c = \frac{\mu}{(\mu-1)}\,.\,2r \qquad \text{....... (3.14)}$$

3.6 POWER OF A THICK LENS

The focal length of a thick lens is given by

$$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2} + \frac{(\mu-1)}{R_1 R_2}\,.\frac{t}{\mu}\right]$$

$$= \frac{\mu-1}{R_1} - \frac{\mu-1}{R_2} + \frac{(\mu-1)^2}{R_1 R_2}\,.\frac{t}{\mu}$$

If P is the power of the lens then we have,

$$P = \frac{1}{f} = \frac{\mu-1}{R_1} - \frac{\mu-1}{R_2} + \frac{(\mu-1)^2}{R_1 R_2}\,.\frac{t}{\mu}$$

The power of the first refracting surface is given by

$$P_1 = \frac{\mu-1}{R_1}$$

Similarly, the power of second refracting surface is given by

$$P_2 = \frac{1-\mu}{R_2} = -\frac{\mu-1}{R_2}.$$

∴ $$P = P_1 + P_2 - P_1 P_2\,.\,\frac{t}{\mu} \qquad \text{....... (3.15)}$$

Self Assessment Questions

1. What is a thick lens?
2. How many cardinal points a thick lens has? Name them.
3. Obtain an expression to show the variation of focal length of thick lens with thickness.
4. What is critical thickness of a lens?

Example 3.1: A convergent thick lens has radii of curvature 10.0cm and – 6.0cm, μ = 1.60 and thickness t = 5.0cm. Deduce its focal length. At what value of t will the lens become divergent?

Solution: The focal length of a lens of thickness t is given by

$$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2} + \frac{(\mu-1)t}{\mu R_1 R_2}\right].$$

Here, $\mu = 1.60, \mathrm{R}_1 = +10.0 \text{ cm}, \mathrm{R}_2 = -6.0 \text{ cm and t} = 5.0 \text{ cm}.$

$$\therefore \qquad \frac{1}{f} = (1.60 - 1)\left[\frac{1}{10.0} + \frac{1}{6.0} + \frac{(1.60-1)\times 5.0}{1.60\times 10.0\times(-6.0)}\right]$$

or

$$f = \frac{480}{67.8} = +7.08 \text{ cm}.$$

At critical value of thickness t_c the lens will become convergent. Using equation 3.13 we can write

$$t_c = \frac{\mu(r_1+r_2)}{\mu-1}$$

Putting $\mu = 1.60$, r_1 = 10.0 cm and r_2 = 6.0 cm, we get

$$t_c = \frac{1.60\ (10.0+6.0)}{(1.60-1)} = 42.7 \text{ cm}$$

Beyond a thickness of 42.7cm, the lens will become divergent.

Example 3.2: Show that for a thin lens the two principal planes coincide.

Solution: The positions of first and second principal planes of a thick lens are

$$\alpha_1 = P_1H_1 = \frac{-f\ (n-1)t}{nR_2}$$

$$\alpha_2 = P_2H_2 = \frac{-f\ (n-1)t}{nR_1}$$

For a thin lens t = 0 and P_1 and P_2 coincide, so $P_1H_1 = P_2H_2 = 0$, i.e., H_1 and H_2 coincide and lie at the optical centre of the lens.

3.7 TELESCOPES

A telescope is an optical device which enables us to see the distant objects clearly. It provides angular magnification of the distant objects. There are two types of telescopes.

1. **Refracting Telescopes:** These types of telescopes work on refraction phenomenon of light and therefore make use of lenses to view distant objects. As its name suggests this device works on refraction phenomenon that is why lenses are used here. These are of two types :
   (a) **Astronomical Telescope:** It is used to see heavenly objects like sun, stars, planets, etc. The final image formed by this telescope is inverted but it does not make any difference in the case of heavenly bodies because of their round shape.
   (b) **Terrestrial Telescope:** It is used to see distant objects on the surface of the earth. The final image formed by this telescope is erect. This is an essential condition of viewing the objects on earth's surface correctly.
2. **Reflecting Telescopes:** These make use of converging mirrors to view the distant objects. For example, Newtonian and Cassegrain telescopes.

3.7.1 Astronomical Telescope

Astronomical telescope uses refraction phenomenon of light to see heavenly bodies like sun, stars, planets, satellites etc. It consists of two converging lenses mounted coaxially at the outer ends of two sliding tubes. One of which is used as an eyepiece while the other is used as objective.

1. **Objective:** It is a convex lens of large focal length and a much larger aperture. It faces the distant object. In order to form bright image of the distant object, the aperture of the objective is taken large so that it can gather sufficient light from the distant objects.
2. **Eyepiece:** It is also a convex lens but of short focal length. It faces the eye. The aperture of the eyepiece used is also taken small so that whole light of the telescope may enter the eye for distinct vision. It is mounted in a small tube which can slide inside the bigger tube carrying the objective.

Working

1. **When the Final Image is Formed at the Least Distance of the Distinct Vision:**

The parallel beam of light coming from the distant objects falls on the objective at some angle α as shown in figure 3.3. The objective focuses the beam in its focal plane and forms a real, inverted and diminished image A'B'. This image A'B' acts as an object for the eyepiece. The distance of the eyepiece is so adjusted that the image A'B' lies within its focal length.

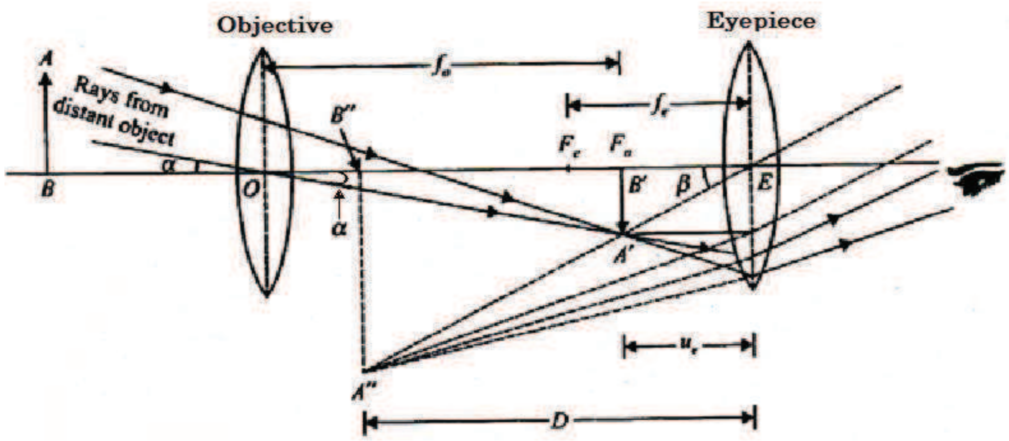

Fig. 3.3

The eyepiece magnifies this image so that final image A"B" is magnified and inverted with respect to the object. The final image is seen distinctly by the eye at the least distance of the distinct vision.

Magnifying Power: The magnifying power of a telescope is defined as the ratio of the angle subtended at the eye by the final image formed at the least distance of the distinct vision to the angle subtended at the eye by the object at infinity, when seen directly.

Since the distance of the object from the telescope is very large therefore the angle subtended by it at the eye is practically equal to the angle α subtended by it at the objective. Hence,

$$\angle A'OB' = \alpha$$

Suppose $\angle A''EB'' = \beta$

Magnifying power, $m = \frac{\tan\beta}{\tan\alpha} = \frac{\beta}{\alpha}$ ($\because$ α and β are small angles)

According to the sign convention

$OB' = +f_o$ = focal length of the objective

$B'E = -u_e$ = distance of A'B' from the eyepiece acting as an object for it

$$\therefore \quad m = -\frac{f_o}{u_e}$$

Again, for the eyepiece, $u = -u_e$ and $v = -$ D. Thus from equation $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, we have,

$$\therefore \quad \frac{1}{-D} - \frac{1}{-u_e} = \frac{1}{f_e}$$

or

$$\frac{1}{u_e} = \frac{1}{f_e} + \frac{1}{D} = \frac{1}{f_e}\left(1 + \frac{f_e}{D}\right)$$

Substituting this in above expression of m, we get,

$$m = -\frac{f_o}{u_e}\left(1 + \frac{f_e}{D}\right) \quad \text{....... (3.16)}$$

As we can see, for large magnifying power, $f_o >> f_e$. The negative sign of the magnifying power indicates that the final image is real and inverted.

2. When the Final Image is Formed at Infinity: Normal Adjustment:

When a parallel beam of light is incident on the objective as shown in figure 3.4, it forms a real, inverted and diminished image A'B' in its focal plane. The eyepiece is so adjusted that the image A'B' exactly lies at its focus. Therefore, the final image is formed at infinity and is highly magnified and inverted with respect to the object.

Magnifying Power: It is defined as the ratio of the angle subtended at the eye by the final image as seen through the telescope at the eye to the angle subtended by the object seen directly when both the image and object lie at infinity at the eye. As the distance of the object from the telescope is very large, the angle subtended by it at the objective is

$$\angle A'OB' = \alpha$$

Also, let $\angle A'EB' = \beta$

∴ Magnifying power, $m = \frac{\tan\beta}{\tan\alpha} = \frac{\beta}{\alpha}$ (∵ α and β are small angles)

$$= \frac{A'B'/B'E}{A'B'/OB'} = \frac{OB'}{B'E}$$

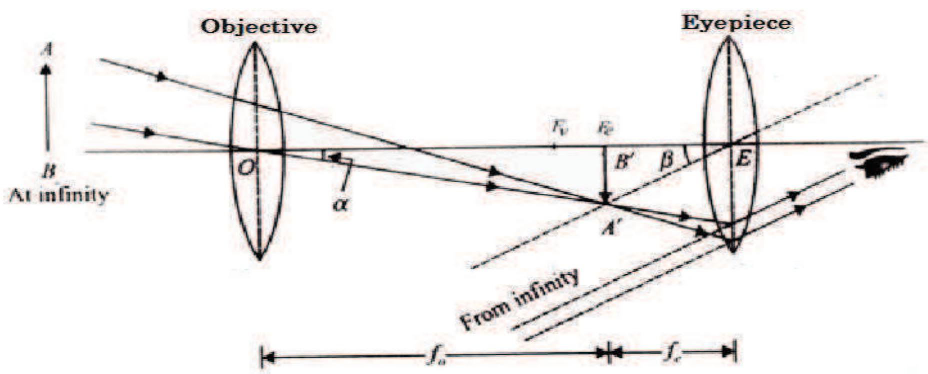

Fig. 3.4

Applying the cartesian sign convention.

$OB' = +f_o$ = Distance of A'B' from the objective along the incident light

and $B'E = -f_e$ = Distance of A'B from the eyepiece against the incident light

∴ $$m = -\frac{f_o}{f_e} \qquad .... (3.17)$$

As we can see, for large magnifying power, $f_o >> f_e$. The negative sign of m indicates that the image is real and inverted.

3.7.2 Terrestrial Telescope

It is a refracting telescope which is used to see erect images of distant earthy objects. It uses an additional convex lens between objective and eyepiece for obtaining an erect image.

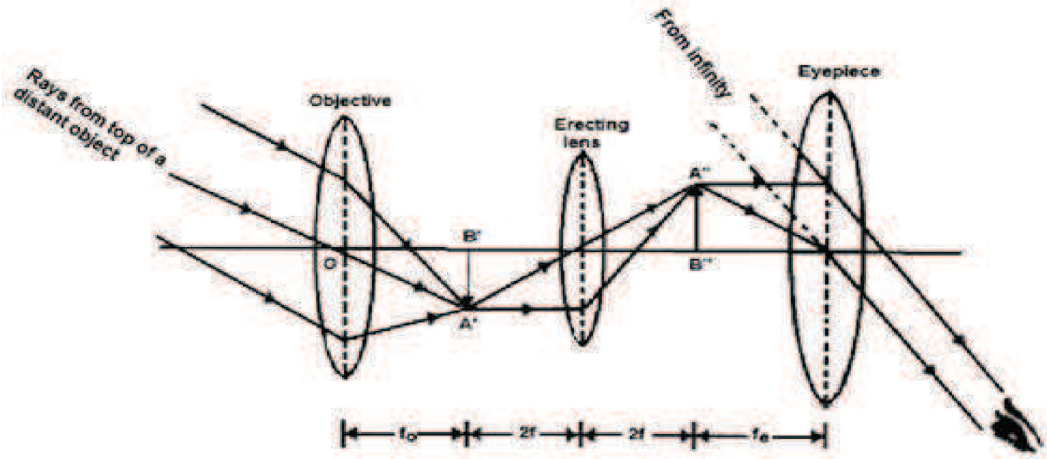

Fig. 3.5

In this telescope the objective forms a real, inverted and diminished image A'B' of the distant object in its focal length from the focal plane of the objective as shown in figure 3.5. This lens forms a real, inverted and equal size image A"B" of A'B'. This image is now erect with respect to the distant object. The eyepiece is so adjusted that the image A"B" lies at its principal focus. Hence the final image is formed at infinity and is highly magnified and erect with respect to the distance object. As the erecting lens does not cause any magnification, the angular magnification of the terrestrial telescope is the same as that of the astronomical telescope.

If the image is formed at infinity the

$$m = \frac{f_o}{f_e}$$

and if image is formed at the least distance of distinct vision then

$$m = \frac{f_o}{f_e}\left(1 + \frac{f_e}{D}\right) \qquad \text{.......} \quad (3.18)$$

Drawbacks

1. The length of the terrestrial telescope is much larger than the astronomical telescope. In normal adjustment, the length of a terrestrial telescope = $f_o + 4f + f_{e'}$ where f is the focal length of the erecting lens.
2. Due to extra reflection at the surfaces of the erecting lens, the intensity of the final image decreases.

3.7.3 Newtonian Reflecting Telescope

The first reflecting telescope was set up by Newton in 1668. It consists of a large concave mirror of large focal length as the objective, made of an alloy of copper and tin as shown in figure 3.6.

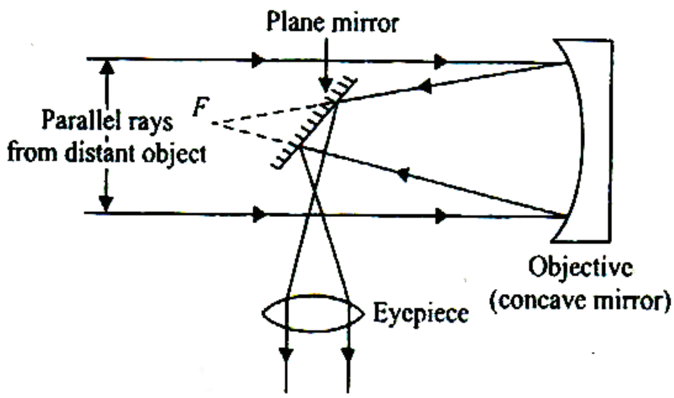

Fig. 3.6

A beam of light from the distant star is incident on the objective. Before the rays are focused at F, a plane mirror inclined at 45^0 intercepts them and turns them towards an eyepiece adjusted perpendicular to the axis of the instrument. The eyepiece forms a highly magnified, virtual and erect image of the distant object.

3.7.4 Cassegrain Reflecting Telescope

It consists of a large concave parabolic (primary) mirror having a hole of its centre. It is a small convex (secondary) mirror near the focus of the primary mirror. The eyepiece is place on the axis of the telescope near the hole of the primary mirror (figure 3.7).

The parallel rays from the distant object are reflected by the large concave mirror. Before these rays come to focus at F, these are reflected by a small convex mirror and are converged to a point I, just outside the hole. The final image formed at I is viewed through the eyepiece. As the first image at F is inverted with respect to the distant object and the second image I is erect with respect to the first image F, hence the final image is inverted with respect to the object.

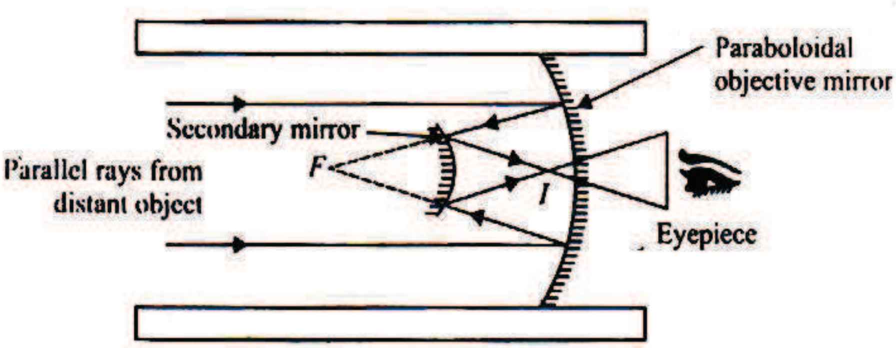

Fig. 3.7

Let f_o be the focal length of the objective and f_e that of the eyepiece then for the final image formed at the least distance of distinct vision, we have,

$$m = \frac{f_o}{f_e}\left(1 + \frac{f_e}{D}\right)$$

For the final image formed at infinity

$$m = \frac{f_o}{f_e} = \frac{R/2}{f_e} \qquad .... (3.19)$$

3.7.5 Advantages of a Reflecting Telescope over Refracting Telescope

A reflecting–type telescope has the following advantages over a refracting–type telescope.

- A concave mirror of large aperture has high gathering capacity and absorbs very less amount of light than the lenses of large apertures. The final image formed in reflecting telescope is very bright. So even very distant or faint stars can be easily viewed.
- Due to the large aperture of the mirror used, the reflecting telescopes have high resolving power.
- As the objective is a mirror and not a lens, it is free from chromatic aberration (formation of coloured image of a white object).
- The use of parabolic mirror reduces the spherical aberration (formation of non-point blurred image of a point object).
- A mirror requires grinding and polishing of one surface only. So it costs much less to construct a reflecting telescope than a refracting telescope of equivalent optical quality.
- A lens of large aperture tends to be very heavy, and therefore, difficult to make and support by its edges. On the other hand, a mirror of equivalent optical quality weighs less and can be supported over its entire back surface.

Currently, reflecting and refracting telescopes have their own roles to play. Reflecting telescopes are used more and more in astronomy due to their ability to see much farther and much clearer. On the other hand, refracting telescopes are used more in everyday items like binoculars and camera lens system due to their straightforward designs and lower construction costs.

Example 3.3: A refracting astronomical telescope uses objective lens and eyepiece of focal lengths 60 cm and 3 cm. Find the magnifying power of telescope and also distance between the objective and eye piece, if the final image formed at infinity.

Solution: Given $f_o = 60\ cm$ and $f_e = 3\ cm$

The magnifying power of telescope when the final image formed at infinity is

$$M = -\frac{f_0}{f_e} = -\frac{60}{3} = -20$$

Also the distance between objective and eye piece is given by L = $f_0 + f_e = 60 + 3 = 63\ cm.$

Thus magnifying power of telescope is 20 and distance between objective and eye piece is 63 cm.

Example 3.4: The magnifying power of a telescope in normal adjustment position is 30 and its length is 93 cm. Find the focal length of the objective and the eye piece.

Solution: The magnifying power of telescope when the final image formed at infinity is

$$M = -\frac{f_0}{f_e}$$

Also the length of telescope is given by L= $f_0 + f_e$

Given M = 30 and L = 93

Thus, $\frac{f_0}{f_e} = 30$

$and\ f_0 + f_e = 93 \quad Or\ f_0 = 93 - f_e$

Hence $\frac{f_0}{f_e} = 93$ or $\frac{(93-f_e)}{f_e} = 30 \quad or \quad f_e = 3\ cm.$

Also as $\frac{f_0}{f_e} = 30 \quad or\ \frac{f_0}{3} = 30 \quad f_0 = 90\ cm.$

Thus the focal lengths of objective lens and eye lens are 90 cm and 3 cm respectively.

3.8 SUMMARY

1. Thickness of a lens is a separation between the poles of its spherical refracting surfaces.
2. When the thickness of a lens is comparable to its focal length, the lens is said to be thick.
3. A thick lens is considered as a combination of thin lenses.
4. A thick lens has six cardinal points: two focal points F_1, F_2; two principal points H_1, H_2 and two nodal points N_1, N_2.
5. The focal length increases with the increase in a thickness of a lens.
6. A telescope is an optical device which enables us to see the distant objects clearly. It provides angular magnification of the distant objects.
7. Reflecting telescopes use converging mirrors to show distant objects. Examples are: Newtonian and Cassegrain telescope.
8. Refracting telescopes use lenses to see heavenly objects like Sun, Stars, Planets etc. from the surface of the earth. Example: Astronomical and Terrestrial telescopes.
9. Generally all telescopes consist of two converging lenses mounted coaxially at the outer ends of two sliding tubes. One lens is called objective, facing the distant object and another lens is eyepiece, facing the eye. Both lenses are convex in nature.

3.9 GLOSSARY

Aperture – Opening which allows light to reach the lens.

Erect – upright or strait

Inverted – opposite position

Distance of distinct vision – a minimum comfortable distance between the naked human eye and a visible object

Infinity – endless, limitlessness

Terrestrial – something relating to the earth

Intercept – obstruct

Parabolic mirror – it is a reflective surface used to collect or project light rays (energy)

Principal axis – the straight line joining the centres of curvature of two bounding surfaces is called the principal axis of the lens.

3.10 REFERENCES

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3. Introductory University Optics, PHI Learning, New Delhi

3.11 SUGGESTED READINGS

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2. Principles of Optics- B.K. Mathur, Digital Library of India
3. Introduction to Optics, The Indian Press, Allahabad

3.12 TERMINAL QUESTIONS

SHORT ANSWER TYPE QUESTIONS

1. Write formula for the focal length of a thick lens.
2. What is meant by chromatic aberration of a lens?
3. What is achromatism?
4. What is the condition of achromatism of two thin lenses of same material placed at distance d apart?
5. What is the condition for minimum spherical aberration for two lens placed at a distance d apart?

OBJECTIVE–TYPE QUESTIONS

1. When the medium on the two sides of a lens-system is same, the principal points coincide with:

   (a) Focal points (b) nodal points (c) centres of curvature (d) none of these

2. If m_x, m_y and m_θ be the longitudinal, lateral and angular magnifications respectively, then choose the correct relation (s):

   (a) $m_x \times m_y = m_\theta$ (b) $m_x \times m_\theta = m_y$

   (c) $m_x \propto m_y$ (d) $m_x \propto m_\theta^2$

3. A convex lens of focal length f_1 and a concave lens of focal length f_2 are placed at a distance d apart. The focal length of the combination is:

   (a) $\frac{f_1\, f_2}{f_1 + f_2 - d}$ (b) $\frac{f_1\, f_2}{f_1 + f_2 + d}$

(c) $\frac{f_1 f_2}{d+ f_1- f_2}$ (d) $\frac{f_1 f_2}{d- f_1+ f_2}$

4. When an extended white object is placed before a convex lens, coloured images are formed. The image of the least size will be of the colour:

(a) red (b) yellow (c) green (d) violet

5. For a parallel incident white beam, the longitudinal chromatic aberration of a lens of unit focal length is numerically equal to:

(a) its focal length (b) its dispersive power

(c) 1 (d) ∞

6. Two lenses of focal lengths f_1 and f_2, made of glasses of dispersive powers ω and $2\,\omega$ respectively, form an achromatic combination when placed in contact. Then:

(a) $f_2 = f_1/2$ (b) $f_2 = -f_1/2$

(c) $f_2 = -2f_1$ (d) $f_2 = 2f_1$

7. The condition of achromatism $\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0$ holds for:

(a) longitudinal and lateral chromatic aberrations

(b) longitudinal chromatic aberration only

(c) lateral chromatic aberration only

(d) neither longitudinal nor lateral chromatic aberration

8. Two lenses in contact form an achromatic doublet. Their focal lengths are in the ratio 2: 3. The dispersive powers of their material must be in the ratio:

(a) 2 : 3 (b) 1 : 3 (c) 3 : 1 (d) 3 : 2

9. A convex crown-glass lens of focal length 20cm and dispersive power 0.018 forms an achromatic doublet when placed in contact with a flint-glass lens of dispersive power 0.036. The focal length of the combination is:

(a) – 20cm (b) – 40 cm (c) +20cm (d) +40cm

10. Two lenses of same material and focal lengths f_1 and f_2 show achromatism when the distance between them is:

(a) zero (b) $f_1 \sim f_2$ (c) $\frac{f_1+ f_2}{2}$ (d) $f_1 + f_2$

11. Parallel paraxial rays incident on a convex lens are converged by the lens at an axial points *F*. The marginal rays incident on the same lens will be converged at :

(a) a points further away than *F* (b) a point nearer the lens than *F*

(c) *F* only

(d) a point further or nearer than *F* depending on the focal length of the lens.

12. The variation in focal length of a lens when we pass from the central portion to the periphery is called:

(a) spherical aberration (b) astigmatism

(c) comma (d) chromatic aberration

13. Spherical aberration of a lens may be reduced by designing the lens so that the deviation of a ray is:

(a) maximum at the first surface (b) minimum at the first surface

(c) equally shared by the two surfaces (d) reduced to a minimum

NUMERICAL QUESTIONS

1. The focal length of objective lens and eyepiece of a telescope is 72 cm and 1.2 cm. Find its angular magnification and length for relaxed eye. (Ans. angular magnification = 60, length for relaxed eye = 73.2 cm.)
2. The magnifying power of a telescope for relaxed eye is 24 and its length is 75 cm. Find the focal length of the objective and the eye piece. (Ans. focal length of the objective = 72 cm and the focal length of eye piece = 3 cm)

3.13 ANSWERS

SHORT ANSWER TYPE QUESTIONS

1. **Ans.** $\frac{1}{f} = (\mu - 1)\left\{\frac{1}{R_1} - \frac{1}{R_2} + \frac{(\mu-1)t}{\mu\, R_1 R_2}\right\}$
2. The image of a white object formed by a lens is usually coloured and blurred due to different refractive indices of the lens material for different wavelengths of light. This defect of image is called ‘chromatic aberration’.
3. The process of elimination of chromatic aberration by combining two or more lenses is called **achromatism**.
4. $d = \frac{f_1 + f_2}{2}$, where f_1 and f_2 are the focal lengths of the two lenses.
5. $d = f_1 - f_2$.

OBJECTIVE TYPE QUESTIONS:

1	(b)	5	(b)	9	(d)	13	(c)
2	(b) and (c)	6	(c)	10	(c)		
3	(d)	7	(a)	11	(b)		
4	(d)	8	(a)	12	(a)		

UNIT 4: INTERFERENCE OF LIGHT WAVES

Structure

4.1 INTRODUCTION

In 1680 Huygens proposed the wave theory of light. But at that time, it was not clear about the nature of light wave, its speed and way of propagation. In 1801 Thomas Young performed an experiment called Young's double slit experiment and noticed that bright and dork fringes are formed which is called inference pattern. At that time it was a surprising phenomenon and is to be explained.

After the Maxwell's electromagnetic theory it was cleared that light is an electromagnetic wave. In physics, interference is a phenomenon in which two waves superimpose on each other to form a resultant wave of greater or lower or of equal amplitude. When such two waves travel in space under certain conditions the intensity or energy of waves are redistributed at certain points which is called interference of light and we observe bright and dark fringes.

4.2 OBJECTIVES

After reading this unit you will able to understand

- The wave nature of light
- Phase and phase changes in light wave
- Coherence and coherent source of light
- Principle of superposition
- Young's double slit experiment and explanation
- Interference
- Interference phenomena in biprism and thin sheets

4.3 WAVE NATURE OF LIGHT

Light wave is basically an electromagnetic wave. Electromagnetic wave consists of electric and magnetic field vectors. The directions of electric and magnetic vectors are perpendicular to direction of propagation as shown in the figure 4.1. The electric and magnetic vectors are denoted by E and H and vary with time.

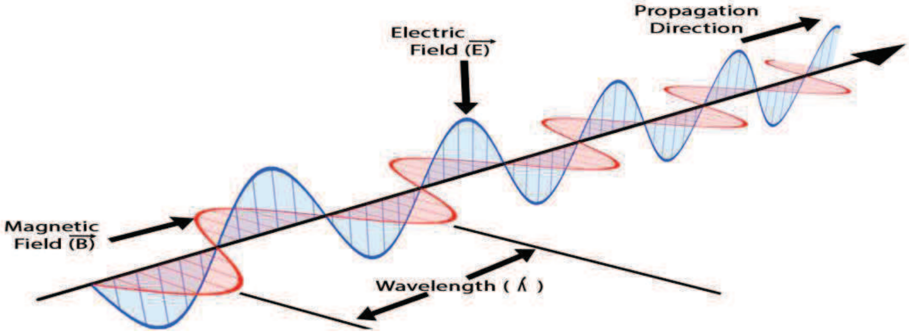

Figure 4.1

In light, electric vectors (or magnetic vectors) vary in sinusoidal manner as shown in figure 4.1. Therefore the electric vectors can be given as

$$E = E_0 \sin(kz - \omega t)$$

Where E = Electric field vector, E_0 = maximum amplitude of field vector, k = wave number (= $2\pi/\lambda$), z = displacement along the direction of propagation (say z axis), ω = angular velocity and t = time.

Before understanding the interference we should understand some terms and properties of light which are related to interference.

4.3.1 Monochromatic Light

The visible light is a continuous spectrum which consist a large number of wavelengths (approximately 3500Å to 7800Å). Every single wavelength (or frequency) of this continuous spectrum is called monochromatic light. However, the individual wavelengths are sufficiently close and indistinguishable. Some time we consider very narrow band of wave lengths as monochromatic light.

Ordinary light or white light, coming from sun, electric bulb, CFL, LED etc. consists a large number of wave lengths and hence non-monochromatic. But some specific sources like sodium lamp and helium neon laser emit monochromatic lights with wave lengths 589.3 nm and 632.8 nm respectively. It should be noted that sodium lamp, actually emits two spectral lines of wavelengths 589.0 nm and 589.6 nm which are very close together, and source is to be consider monochromatic.

4.3.2 Plane Wave

A plane wave is a wave whose wave front remains in a plane during the propagation of wave. In light wave, the maximum amplitude of electric vector E_0 remains constant and confined in a plane perpendicular to direction of propagation. Such type of wave called plane wave.

4.3.3 Polarized and Unpolarized Light

Light coming from many sources like sun, flame, incandescent lamp produce unpolarized light in which electric vector are oriented in all possible directions perpendicular to direction of propagation. But in polarized light electric vector are confined to only a single direction. The detail about polarized light will be discussed in the next block.

4.3.4 Phase Difference and Coherence

Wave is basically transportation of energy by mean of propagation of disturbance or vibrations. In wave motion through a medium, the particles of medium vibrate but in case of electromagnetic wave the electric or magnetic vectors vibrate form its equilibrium position.

The term phase describes the position and motion of vibration at any time. For example if y= a sin (ωt +θ) represents a wave, then the term (ωt +θ) represents the phase of wave. The unit of phase is degree or radium. After completion of 360^0 or 2π, the cycle of wave or phase repeats.

Phase difference

If there are two waves have some frequency then the phase difference is the angle (or time) after which the one wave achieves the same position and phase as of first wave. In the figure 4.2, two waves with phase different θ are shown.

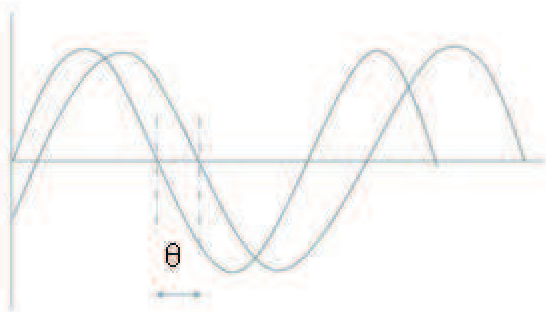

Figure 4.2

Coherence

If two or more waves of same frequencies are in same phase or have constant phase difference, those waves are called coherent wave. Figure 4.3 shows coherent wave with same phase (zero phase difference) and with constant phase difference.

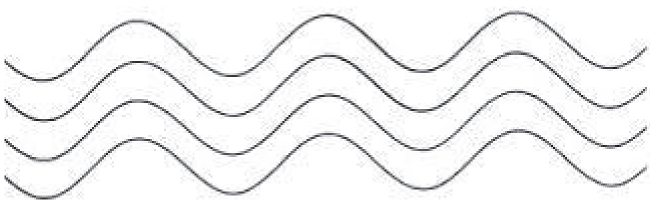

Figure 4.3

4.3.5 Optical path and Geometric Path

Optical path length (OPL) denoted by Δ is the equivalents path length in the vacuum corresponding to a path length in a medium. Path length in a medium can be considered as geometric path length (L). Suppose a light wave travels a path length L in a medium of refractive index μ and velocity of light is v in this medium, then for a time period t the geometric path length L is given by

$$L = vt$$

In the same time interval t, the light wave travel a distance Δ in vacuum which is optical path length corresponding to length L. Then

$$\Delta = ct = c\frac{L}{v}$$

Where, c is the velocity of light in vacuum.

or $$\Delta = \mu L$$

or The Optical path length = μ × (Geometrical path length in a medium).

In case of interference we always calculate optical path for simplification of understanding and mathematical calculations.

4.4 PRINCIPLE OF SUPERPOSITION

According to Young's principle of superposition, if two or more waves are travelling and overlap on each other at any point then the resultant displacement of wave is the sum of the displacement of individual waves (figure 4.4). If two waves are represented by $y_1 = a_1 \sin \omega t$ and $y_2 = a_2 \sin (\omega t+\delta)$. Then according to principle of superposition, the resultant wave is represented by $y = y1 + y2$

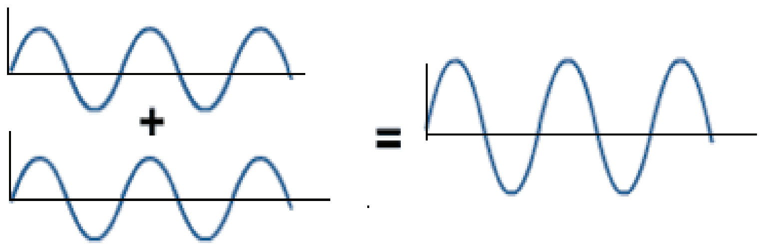

Figure 4.4

4.5 INTERFERENCE

When two light waves of some frequency, nearly same amplitude and having constant phase difference travel and overlap on each other, there is a modification in the intensity of light in the region of overlapping. This phenomenon is called interference.

The resultant wave depends on the phases or phase difference of waves. The modification in intensity or change in amplitude occurs due to principle of superposition. In certain points the two waves may be in same phase and at such point the amplitude of resultant wave will be sum of amplitude of individual waves. Thus, if the amplitudes of individual waves are a_1 and a_2 then the resultant amplitude will be $a = a_1 + a_2$. In this case, the intensity of resultant wave increases ($I \propto a^2$) and this phenomena is called constructive interference. Corresponding to constructive interference we observe bright fringes.

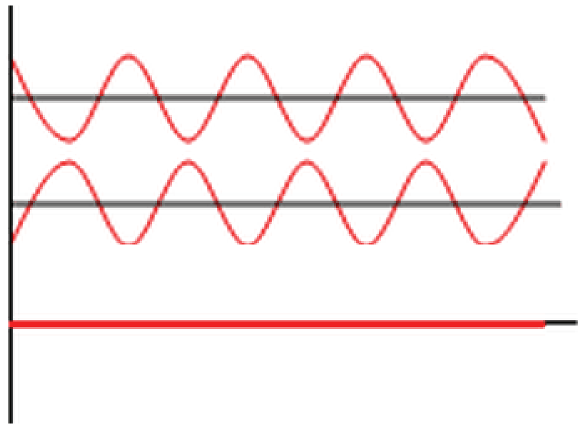

Figure 4.5

On the other hand, at certain points the two waves may be in opposite phase as shown in figure 4.4. In these points the resultant amplitude of waves will be sum of amplitude of individual waves with opposite directions. If the amplitudes of individual waves are a_1 and a_2 then the resultant amplitude will be $a = a_1 - a_2$ and the intensity of resultant wave will be minimum. This case is called destructive interference. Corresponding to such points we observe dark fringes. Figure 4.5 depicts two waves of opposite phase and their resultant.

4.5.1 Theory of Superposition

Let us consider two waves represented by $y_1 = a_1 \sin \omega t$ and $y_2 = a_2 \sin (\omega t + \delta)$. According to Young's principle of superposition the resultant wave can be represented by

$$y = y_1 + y_2$$

$$= a_1 \sin \omega t + a_2 \sin(\omega t + \delta)$$

$$= a_1 \sin \omega t + a_2 (\sin \omega t \cos \delta + \cos \omega t \sin \delta)$$

$$= (a_1 + a_2 \cos \delta) \operatorname{Sin} \omega t + (a_2 \sin \delta) \cos \omega t \qquad \text{....... (4.1)}$$

Let $$a_1 + a_2 \cos \delta = A \cos \emptyset \qquad \text{....... (4.2)}$$

and $$a_2 \sin \delta = A \sin \emptyset \qquad \text{....... (4.3)}$$

Where A and Ø are new constants, then above equation becomes

$$y = A \cos \emptyset \sin \omega t + A \sin \emptyset \cos \omega t$$

or $$y = A \sin (\omega t + \emptyset) \qquad \text{....... (4.4)}$$

This is the equation of the resultant wave. In this equation y represents displacement, A represents resultant amplitude, $\emptyset$ is the phase difference.

From equation (4.2) and (4.3) we can determine the constant A and Ø. Squaring and adding the two equations, we get,

$$A^2 = a_1^2 + a_2^2 \cos^2 \delta + 2\, a_1 a_2 \cos \delta + a_2^2 \sin^2 \delta$$

or $$A^2 = a_1^2 + a_2^2 + 2\, a_1 a_2 \cos\delta \qquad \text{....... (4.5)}$$

On dividing equation (4.3) by eq (4.2), we obtain,

$$\frac{\sin\emptyset}{\cos\emptyset} = \tan\emptyset = \frac{a_2 \sin\delta}{a_1 + a_2 \cos\delta} \qquad \text{....... (4.6)}$$

4.5.2 Condition for Maxima or Bright Fringes

If $\cos\delta = +1$ then $\delta = 2n\pi$ where $n = 0, 1, 2, 3\ldots\ldots$(positive integer numbers).

Then, $$A^2 = a_1^2 + a_2^2 + 2a_1a_2 = (a_1+a_2)^2$$

Intensity, $$I = A^2 = (a_1+a_2)^2 \qquad \text{....... (4.7)}$$

Therefore, for $\delta = 2n\pi = 0, 2\pi, 4\pi\ldots\ldots$, we observe bright fringes.

In term of path difference Δ

$$\Delta = \frac{\lambda}{2\pi л} \times \text{phase difference} = \frac{\lambda}{2\pi л}\, 2n\pi$$

or $$\Delta = n\lambda = \lambda, 2\lambda, 3\lambda\ldots \text{ etc.} \qquad \text{....... (4.8)}$$

4.5.3 Condition for Minima or Dark Fringes

If $\cos\delta = -1$ or $\delta = (2n-1)\pi = \pi, 3\pi, 5\pi \ldots\ldots$

Then $$A^2 = a_1^2 + a_2^2 - 2\, a_1a_2 = (a_1 - a_2)^2$$

Intensity, $$I = A^2 = (a_1 - a_2)^2 \qquad \text{....... (4.9)}$$

Therefore if phase difference between two waves is $\delta = (2n-1)\pi = 0, 3\,\pi, 5\,\pi\ldots$ etc. is the condition of minima or dark fringes.

Now path difference, $\Delta = \frac{\lambda}{2\pi} \times Phase\ difference$

or $$\Delta = \frac{\lambda}{2\pi} \times (2n-1)\pi = \frac{(2n-1)}{2}\lambda = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \ldots\ldots \qquad \text{....... (4.10)}$$

Example 4.1. Two coherent resources whose intensity ratio is 81:1 produce interference fringes. Calculate the ratio of maximum intensity and minimum intensity.

Solution: If I_1 and I_2 are intensities and a_1 and a_2 are the amplitudes of two waves then

$$\frac{I_1}{I_2} = \frac{81}{1} \quad or \quad \frac{a_1^2}{a_2^2} = \frac{81}{1} \quad or \quad \frac{a_1}{a_2} = \frac{9}{1}$$

Maximum intensity = $a_1 + a_2 = 9\, a_2 + a_2 = 10\, a_2$

Minimum intensity = $a_1 - a_2 = 9\, a_2 - a_2 = 8\, a_2$

The ratio of maximum intensity to minimum intensity

$I_{max}/I_{min} = (a_1+a_2)^2 / (a_1-a_2)^2 = 10^2/8^2 = 100/64 = 25/16$

4.5.4 Intensity Distribution

The intensity (I) of a wave can be given as $I = (½)\ \epsilon_o\ a^2$ where a is the amplitude of wave, and ϵ_0 is the permittivity of free space. If we consider two waves of amplitudes a_1 and a_2 then at the point of maxima

$$I_{max} = (a_1+a_2)^2 = a_1^2+a_2^2+2a_1a_2$$

If $a_1 = a_2 = a$ then $I = 4a^2$. Therefore, at maxima points the resultant intensity is more than the sum of intensities of individual waves.

Similarly the intensity at points of minima

$$I_{min} = a_1^2+a_2^2 - 2a_1a_2 = (a_1 \sim a_2)^2$$

If $a_1 = a_2 = a$ then $I_{min}=0$. Thus the intensity at minima points is less than the intensity of any wave.

The average intensity I_{av} is given as

$$I_{av} = \frac{\int_0^{2\pi} I d\delta}{\int_0^{2\pi} d\delta} = \frac{\int_0^{2\pi}\ (a_1^2+a_2^2+2a_1\ a_2\ Cos\ \delta)d\delta}{\int_0^{2\pi} d\delta} = \frac{(a_1^2+a_2^2)2\pi л}{2\pi л} = a_1^2 + a_2^2$$

If $a_1 = a_2 = a$ then $I_{av} = 2a^2 = 2I$

Therefore, in interference pattern energy (intensity) $2a_1a_2$ is simply transferred from minima to maxima points. The net intensity (or average intensity) remains constant or conserved.

4.6 CLASSIFICATION OF INTERFERENCE

The interference can be divided into two categories.

4.6.1 Division of Wavefront

In this class of interference, the wave front originating from a common source is divided into two parts by employing mirror, prisms or lenses on the path. The two wave front thus separated traverse unequal paths and are finally brought together to produce interference pattern. Examples are biprism, Lloyd's mirror, Laser etc.

4.6.2 Division of Amplitude

In this class of interference the amplitude or intensity of incoming beam divided into two or more parts by partial reflection and refraction. Examples are thin films, Newton's rings, Michelson interferometer etc.

4.7 YOUNG'S DOUBLE SLIT EXPERIMENT

In 1801, Thomas Young performed double slit experiment in which a light first entered through a pin holes, then again divided into two pinholes and finally brought to superimpose on each other and obtained interferences. Young's performed experiment with sum light. Now the experiments are modified with monochromatic light and efficient slits.

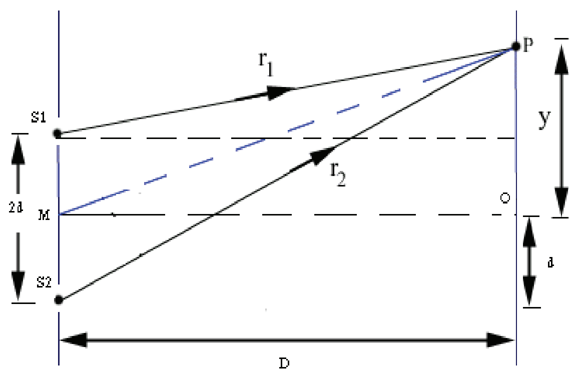

Fig. 4.6

Figure 4.6 shows the experimental setup of double slit experiment. S_1 and S_2 are two narrow slits illuminated by a monochromatic light source. The distance between two slits S_1 and S_2 is 2d. The two waves superimposed on each other and fringes are formed on the screen placed at a distance D from the centre of slits M. Let us consider a point P on the screen which is y distant from O. The two rays S_1P and S_2P meet at point P and produce interference pattern on screen.

Mathematically, path difference between rays S_1 P and S_2P is given as

$$\Delta = S_2P - S_1P \qquad \text{.......} \quad (4.11)$$

$$S_2P^2 = D^2 + (y+d)^2 = D^2[1 + (y+d)^2 / D^2]$$

$$S_2P = D[1 + (y+d)^2 / D^2]^{1/2}$$

$$= D[1 + \frac{1}{2}(y+d)^2 / D^2] \quad [\because (1+x)^n = 1 + nx + \cdots \ldots]$$

or $$S_2P = D + (y+d)^2 / 2D \qquad \text{.......} \quad (4.12)$$

Similarly

$$S_1P^2 = D^2 + (y-d)^2$$

$$S_1P = D\,[1+ (y\text{-}d)^2 / D^2]^{1/2}$$

$$= D\,[1+ \tfrac{1}{2}(y\text{-}d)^2 / D^2]$$

$$= D + (y\text{-}d)^2 /2D \qquad \ldots\ldots\ (4.13)$$

Using equation (4.12) and (4.13), the path difference becomes

$$\Delta = D + \frac{(y+d)^2}{2D} - D - \frac{(y-d)^2}{2D} = \frac{2yd}{D} \qquad \ldots\ldots\ (4.14)$$

For the position of bright fringes path difference

$$\Delta = n\lambda \qquad \text{(where n=1, 2, 3........)}$$

or $$\frac{2yd}{D} = n\lambda$$

or $$y = \frac{nD\lambda}{2D}$$

Since the expression consists of integer *n*, i.e., *y* is a function of *n*. Thus it is better to use y_n in place of *y* and we can write,

$$y_n = \frac{nD\lambda}{2D} \qquad \ldots\ldots\ (4.15)$$

Where n = 1, 2 … etc. represents the order of fringe

On putting the value of n=1, n=2 etc. we get the bright fringes at positions $y_1 = \frac{D\lambda}{2D}$, $y_2 = \frac{2D\lambda}{2D}$ etc. Similarly for the position of dark fringes, the path difference should be

$$\Delta = \frac{(2n-1)\lambda}{2}$$

or $$\frac{2yd}{D} = \frac{(2n-1)\lambda}{2}$$

or $$y_n = \frac{(2n-1)}{2}\frac{D\lambda}{2D} \qquad \ldots\ldots\ (4.16)$$

If we place the value of n = 1, 2, 3 … we get the positions of dark fringes at $y_1 = \frac{1}{2}\frac{D\lambda}{2D}$, $y_2 = \frac{3}{2}\frac{D\lambda}{2D}$, $y_3 = \frac{5}{2}\frac{D\lambda}{2D}$…… etc.

Fringe Width: Distance between two consecutive bright or dark fringes is called fringe width denoted by ω (sometimes β). In case of bright fringes, fringe width

$$\omega = y_{n+1} - y_n = (n+1)\frac{D\lambda}{2D} - n\frac{D\lambda}{2D} = \frac{D\lambda}{2D}$$

Similarly, in case of dark fringes

$$\omega = y_{n+1} - y_n = \frac{2(n+1)-1}{2}\frac{D\lambda}{2D} - \frac{(2n-1)-1}{2}\frac{D\lambda}{2D} = \frac{D\lambda}{2D}$$

4.8 COHERENCE LENGTH AND COHERENCE TIME

In case of ordinary light source, light emission takes place when an atom leaves it excited state and come to ground state or lower energy state. The time period for the process of transition from an upper state to lower state is about 10^{-8} s only. Therefore an excited atom emits light wave for only 10^{-8} s and wave remains continuously harmonic for this period. After this period, the phase changes abruptly. But in a light source, there are innumerous numbers of atoms which participate in the emission of light. The emission of light by a single atom is shown in figure 4.7. After the contribution of a large number of atoms emitting light photon, a succession of wave trains emits from the light source.

Figure 4.7

4.8.1 Coherence Length

Coherence length is propagation distance over which a coherent wave maintains coherence. If the path of the interfering waves or path different is smaller than coherent length, the interference is sustainable and we observe distinct interference pattern.

4.8.2 Coherence Time

Coherent time τ_c is defined as the average time period during which the wave remains sinusoidal and after which the phase change abruptly.

4.8.3 Spatial Coherence

Spatial coherence describes the correlation between waves at different points on a plane perpendicular to the direction of propagation. More precisely, the spatial coherence is the cross-correlation between two points in a wave for all times. If a wave has only 1 value of amplitude over an infinite length, it is perfectly spatially coherent.

4.8.4 Temporal Coherence

Temporal coherent describes the correlation between two points in the direction of propagation. In other words, it characterizes how well a wave can interfere with itself at a different time as direction of propagation indicates time line. The delay over which the phase or amplitude wanders by a significant amount (and hence the correlation decreases by significant amount) is nothing but coherence time τ_c

4.9 CONDITIONS FOR SUSTAINABLE INTERFERENCE

As we studied the different aspects of interference it is clear that under which conditions interference can take place. But for strong interference or sustained interference some more condition may be summarized. The conditions are:

1. The interfering waves must have same frequencies. For this purpose we can select a single source.
2. The interfering waves must be coherent. To maintain the coherence, the path difference of two interfering waves must be less than coherence length.
3. As fringe width is given by $\omega = \frac{D\lambda}{2d}$. Thus to obtain reasonable fringe width the distance between source and screen D should be large and distance 2d between two sources should be small.
4. For good contrast we can prefer the interfering wave of same amplitude. If amplitude of two waves, a_1 and a_2 are same or nearly same than we observe distinct maxima and minima.
5. The back ground of screen should be dark.

4.10 INTERFERENCE DUE TO THIN SHEET

When a thin transparent sheet of mica of thickness t and refractive index μ is introduced in the path of one of the interfering beam of light, then entire fringe system is displaced. Suppose a thin sheet of mica of thickness t is place in the path of a light beam as shown in figure 4.8 then suppose the fringe system is displaced by a distance x.

If *t* is the time taken by light to travel distance S_1P, then

$$t = \frac{S_1P - t}{c} + \frac{t}{v}$$

where v is velocity of light in the thin sheet and c is the velocity of light in air.

$$t = \frac{S_1P - t}{c} + \frac{t}{c}\mu \qquad \because \mu = \frac{c}{v}$$

$$t = \frac{S_1P - t + \mu t}{c}$$

For light ray reaching to P from slit S_1, the path travelled in air is S_1P-t while in thin sheet is t, the optical path can be written as

$$= S_1P - t + \mu t = S_1P + (\mu - 1)\, t$$

Now path difference between two interfering says S_1P and S_2P at P is given as

$$\Delta = S_2P - S_1P = S_2P - [S_1P + (\mu - 1)t]$$

$$= S_2P - S_1P - (\mu - 1)t$$

$$= \frac{2yd}{D} - (\mu - 1)\, t \qquad \text{(Using equation 4.14)}$$

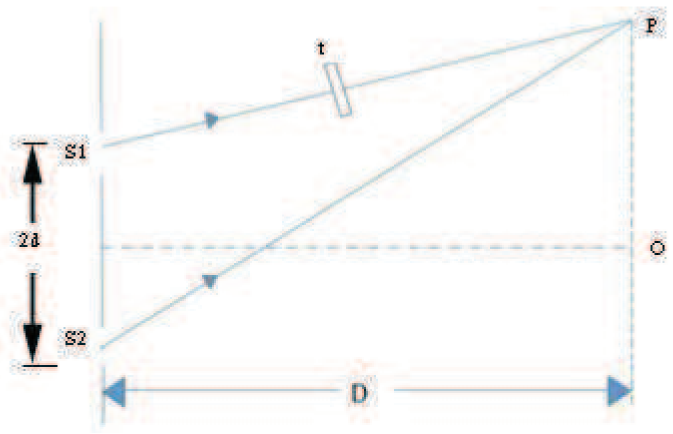

Figure 4.8

For nth maxima (bright fringe) path difference should be of the order of nλ, i.e.,

$$\frac{2yd}{\Delta} - (\mu - 1)\, t = n\lambda$$

Taking *y* as y_n we get, $\quad y_n = \frac{D}{2d}[n\lambda + (\mu - 1)\, t] \qquad \text{....... (4.17)}$

In the absence of thin sheet (*t =0*)

$$y_n = \frac{nD\lambda}{2d}$$

Therefore, net displacement in the presence and absence of sheet is given by equations 4.18 and 4.19 respectively

$$x = \frac{D}{2d}[n\lambda + (\mu - 1)\, t] - \frac{nD\lambda}{2d} \qquad \text{....... (4.18)}$$

$$x = \frac{D}{2d}(\mu - 1)\, t \qquad \text{....... (4.19)}$$

Therefore, on introducing a thin transparent sheet in the path of any interfering ray, the entire fringe system will disposed by distance of x. By measuring the value of x we can calculate the thickness of sheet.

$$t = \frac{x\,.2d}{D(\mu - 1)} \qquad \text{....... (4.20)}$$

4.11 FRESNEL'S BIPRISM

Fresnel biprism consists of two acute angle prisms with their bases in contact. Generally the angles are 179^0, $30'$ and $30'$ as shown in figure 4.9. The light coming from a source is allowed to fall symmetrically on a biprism as shown in figure 4.9. As we know, when a light beam is incident on a prism, the light is deviated from its original path through an angle called angle of deviations. Similarly in case of biprism, the light beam coming from source S, is appeared to be coming from S_1 and S_2 as shown in figure 4.10. Thus we can say for prism S_1 and S_2 behave as virtual sources for the biprism.

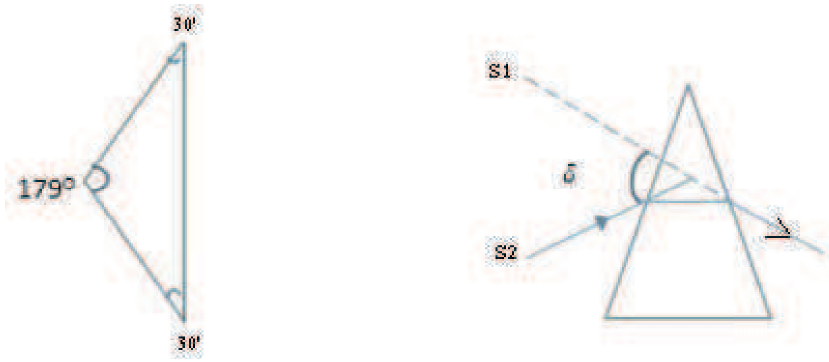

Fig. 4.9

In case of biprism, it can be considered that two cones of lights AS_1Q and BS_2P are coming from S_1 and S_2 and superimposed on each other and produce interference fringes in the region of superposition (between AB). The formation of interference fringes due to Fresnel's biprism is the same as due to Young's double slit experiment.

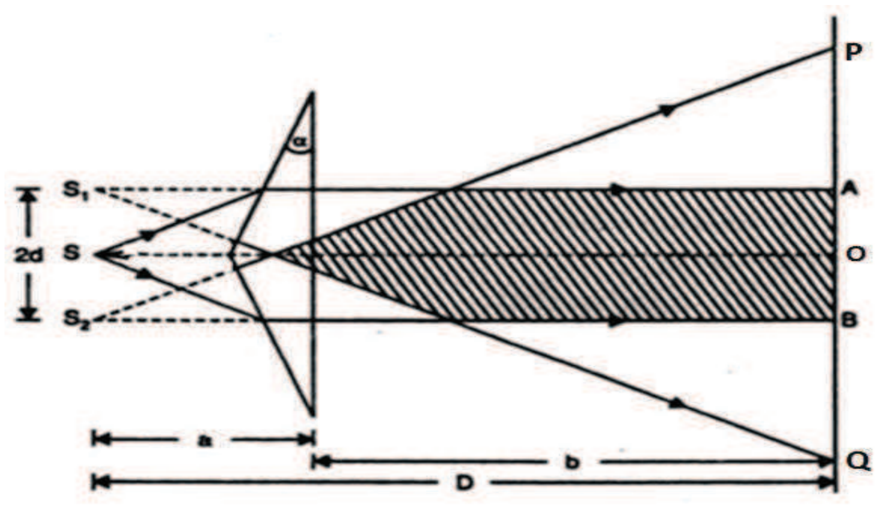

Fig. 4.10

In this experiment point O is equidistance from both slits S_1 and S_2. If we consider distance between source and screen is D and separation between two slits S_1 and S_2 is 2d the fringe width can be given as

$$\omega = \frac{D\lambda}{2d}$$

The position of n^{th} bright fringe is given by $y_n = n\frac{D\lambda}{2d}$

Similarly the position of n^{th} dark fringe is given by $y_n = \frac{2n-1}{2}.\frac{D\lambda}{2d}$

The wave length of the light source used in biprism experiment can be obtained by using above relation as

$$\lambda = \omega\frac{2d}{D} \qquad .... (4.21)$$

4.11.1 Experimental Arrangement of Biprism Apparatus

The experiment is performed on an optical bench as shown in figure 4.11. In this experiment we have an optical bench, which is an arrangement of two parallel metallic rods which are horizontal at same label. The rods or optical bench carry upright on which optical instruments are mounted. These upright are movable on the rods. In the first uprights, we have a slit illuminated by a monochromatic light source S. The slit provides a linear monochromatic light to the biprism which is mounted on the second upright. The biprism is placed in such a way that its refracting edges parallel to the slit so that light falls symmetrically on the biprism. In third upright there is a concave lens for conversing the light coming from biprism. Finally on forth upright a micrometer eyepiece is mounted in which interference fringes are observed.

For obtaining fringes, following adjustments are to be made.

(i) The optical bench is leveled with the help of spirit level.
(ii) Axis of slit is made parallel to edge of biprism.
(iii) The heights of all four uprights should be same so that line joining slit, biprism and micrometer should be parallel to optical bench.

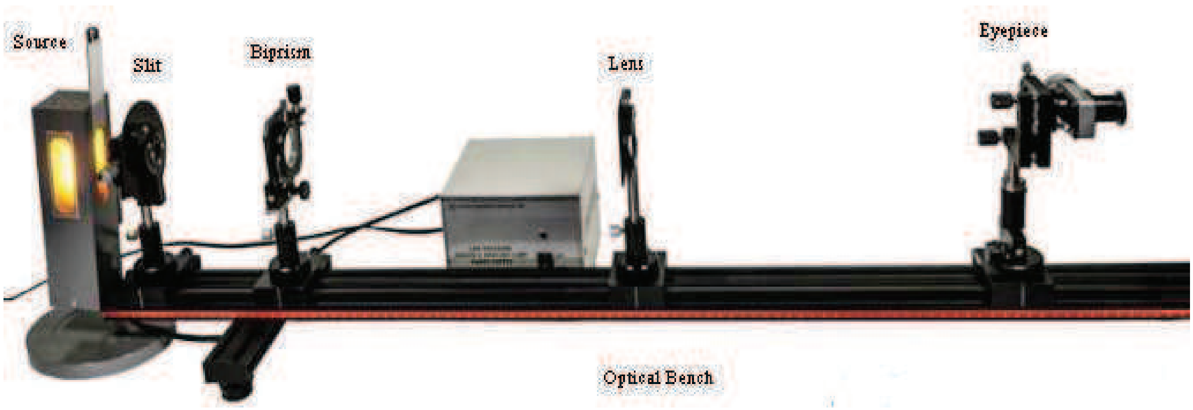

Figure 4.11

4.11.2 Lateral Shift

If the eyepiece of micrometer is moved away from the biprism, and fringes shift either left or right of bench then it is called lateral shift. Simply, we can say the shift of fringes across the bench is called lateral shift. It indicates that the line joining the slit biprism and eyepiece is not parallel to the optical bench.

To remove the lateral shift we put the eyepiece near the birprism and fix the vertical crosswire on any fringe. Now micrometer eyepiece is moved some distance away from biprism and direction of fringe shift is observed. Now biprism is moved in the direction opposite to the fringe shift so that vertical crosswise again reached on same fringe. We repeat this process again and again so that lateral shift removes compatibly.

4.11.3 Measurement of Wavelength of Light (λ) by Fresnel Biprism

By using the Fresnel biprism we can determine the wavelength of given source of light. For this purpose we use the given light source in experimental arrangement. We adjust the apparatus for fringes are to be observed on the eyepiece. We measure the fringe width on apparatus and apply the formula for fringe width as

$$\omega = \frac{D\lambda}{2d} \qquad \text{or} \qquad \lambda = \omega.\frac{2d}{D}$$

Fringe width ω can be measured with the help of micrometer on eyepiece. D is the distance between eyepiece and slit, and can be measured with the help of optical bend. The 2d is the distance between two virtual sources (S_1 and S_2) and cannot be measured directly with the help of any scale. We apply two methods for the measurement of distance 2d.

Magnification Method

To determine the distance 2d, we placed a convex lens of short focal length between biprism and screen. We find out a position L_1, of lens very near to biprism so that two sharp real images are obtained in the field of view of eyepiece. In figure 4.12 the position of Lens L_1 is denoted by bold lines. In this position, we measure distance between two images d_1, with the help of micrometer of eyepiece.

For this position the magnification is given by

$$\frac{v}{u} = \frac{d_1}{2d}$$

Now we move the lens some distance away from the biprism and obtain another position L_2 so that two sharp images are seen again in the field of view. We again measure the distance between two images, say d_2 with the help of micrometer of eyepiece.

In this case of position L_2 the magnification is given as

$$\frac{u}{v} = \frac{d_2}{2d}$$

By using above two equations (10) and (11) we get:

$$1 = \frac{d_1}{2d}.\frac{d_2}{2d}$$

or $$2d = \sqrt{d_1 d_2} \qquad .... (4.22)$$

By putting the value of d_1 and d_2 we can determine the value of 2d.

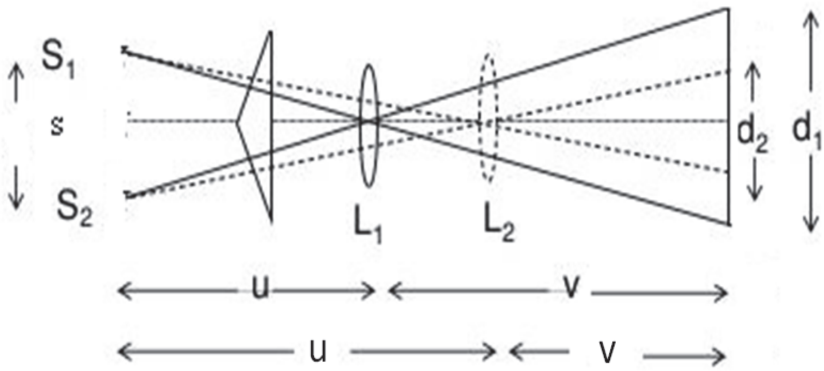

Figure 4.12

Refractive Index Method

In this method, we use the formula of angle of deviation for a prism. As shown in figure 4.13 the angle of deviation can be given as

$$\delta = (\mu - 1)\,\alpha \qquad .... (4.23)$$

Where μ is refractive index and α is angle of prism as shown in figure 4.13. Again the angle of deviation can be given as.

$$\delta = \frac{d}{a} \text{ or } d = a\,\delta \qquad .... (4.24)$$

Using equations (4.23) and (4.24), we obtain, 2d = 2a δ

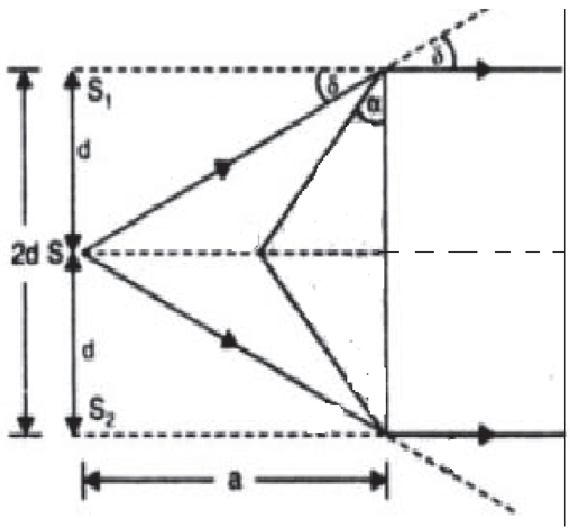

Figure 4.13

or $$2d=2a\,(\mu-1)\,\alpha \qquad \text{....... (4.25)}$$

By using any of the above mentioned methods, we can determine the value of 2d and then putting this value is equation 4.21, we can determine the wavelength of given light source.

4.12 INTERFERENCE WITH WHITE LIGHT

Now let us discuss what happen when the monochromatic light source in a Young's double slit experiment is replaced by a white light. Since the white light consists innumerable wavelengths from red to violet, when white light is used, all wavelengths have their own fringe pattern and finally superimposed on each other. Since the path different for all colours at center point is same then the waves of all colours reach at mid point without any path difference and we observed a white fringe at Center point. This central fringe is called zero order fringes. After central fringe, we observed few coloured fringes with poor contrast. These fringes are due to superposition of different fringes of different colours. Thus the interference pattern is not clear but the superposition of many colours.

Self Assessment Questions

1. What is difference between coherence and non coherence light?
2. Why non-coherent sources do not produce interference pattern?
3. What are the conditions for sustainable interference?
4. Young's double slit experiment, why the central fringe is bright?
5. How can we arrange coherence sources in practical?
6. What is meant by interference of light?
7. Explain the principle of superposition of light wave?
8. How is the shape of fringes formed by biprism?

4.13 SOLVED EXAMPLES

Example 4.2: A monochromatic light of wave length 5100 Å from a slit is incident on a double slit. If the overall separation of 30 fringes on a screen 200 cm always is 3cm, find the distance between slits.

Solution: The fringe width $\omega = \frac{D\lambda}{2D}$

Where ω = fringe width, D = distance between slit and screen, 2d= distance between slits.

It is given that D =200 cm, $\omega = \frac{3}{30} = 0.1cm$

Therefore, $2d = D\lambda/\omega = \frac{5100 \times 10^{-8} \times 200}{0.1} = 0.025$ cm

Example 4.3: In Young's double slit experiment the two slits are 0.05 mm apart and screen is located 2m away from the slit. The third bright fringe from the slit is displaced 8.3 cm apart from the central fringe. Determine the wavelength of incident light.

Solution: For the third bright fringe n= 3

$x_n = \frac{nD\lambda}{2d}$ or $\lambda = \frac{x_n .2d}{nD} = \frac{8.3 \times 10^{-2} \times 0.05 \times 10^{-3}}{3 \times 2} = 6.91 \times 10^{-7}$ m = 6910 Å

Example 4.4: In Fresnel's biprism experiment, a light of wavelength 6000 Å falls on biprism. The distance between source and screen is 1m and distance between source and birprism is 10 cm. The angle of biprism is 1^0. If the fringe width is 0.03cm, find out the refractive index of the material of biprism.

Solution: The fringe width $\omega = \frac{D\lambda}{2d}$

If the refractive index of material is μ and angle of prism is α then

$2d = 2a(\mu-1)\alpha$. Then $\omega = \frac{D\lambda}{2a\ \omega(\mu-1)\ \alpha}$

Here, D = 1m = a+b and a= 10 cm, b= 90cm, λ= 6000× 10^{-8} cm, $\alpha = 1^0 = \frac{\pi}{180}$ radian and ω = 0.03 cm

Thus, $\mu - 1 = \frac{D\lambda}{2a\ \omega\alpha} = \frac{100 \times 6000 \times 10^{-8}}{2 \times 10 \times 0.03 \times \frac{\pi}{180}} = 0.57$

∴ $\mu = 1 + 0.57 = 1.57$

Example 4.5: A light of wavelength 6900 Å is incident on a biprism of refracting angle 1^0 and refractive index 1.5. Interference fringes are observed on a screen 80 cm away from the biprism. If the distance between source and the biprism is 20 cm, calculated the fringe width.

Solution : The fringe width is given by $\omega = \frac{D\lambda}{2d}$ and $2d = 2(\mu-1)a\alpha$

Here λ=6900 Å =6900 ×10^{-8} cm, $\alpha = 1^0 = \frac{\pi}{180}$ redius, μ = 1.5, D = a+b = (20+80) cm =100cm

$$\omega = \frac{D\lambda}{2a(\mu-1)\alpha} = \frac{100 \times 6900 \times 10^{-8}}{2 \times 20 \times (1.5-1) \times \frac{\pi}{180}} = 0.02 \text{ cm.}$$

Example 4.6: A thin sheet of a transparent material of refractive index μ =1.60 is placed in the path of one of the interfering beam in a biprism experiment. The wave length of the light used is 5890Å. After placing the sheet, the central fringe shifted to a position originally occupied by 12^{th} bright fringe. Calculate the thickness of the sheet.

Solution: On introducing a thin transparent sheet in the path of one interfering say, the interfering system is shifted by a distance x and

$$x = \frac{D}{2d}(\mu - 1)t$$

In this case the fringe shifted by 12th bright fringe.

$$x = y_{12} = 12\frac{D}{2d} \qquad [\because y_n = n\frac{D\lambda}{2d}]$$

Therefore, $\quad 12\frac{D\lambda}{2d} = \frac{D}{2d}(\mu - 1)t \quad$ or $\quad t = \frac{12\lambda}{(\mu-1)} = \frac{12\times 5890\times 10^{-8}}{(1.6-1)} = 1.18 \times 10^{-3}$ cm

4.14 SUMMARY

1. When two light waves of same frequency and nearly some amplitude and having constant phase difference traverse in a medium and cross each other, there is redistribution in the intensity of light which is called interference of light.
2. If $y_1 = a_1 \sin \omega t$ and $y_2 = a_2 \sin(\omega t+\delta)$ are two waves, then resultant wave is given by $y = A \sin(\omega t+\emptyset)$. Where $A=\sqrt{a_1^2 + a_2^2 + 2a_1a_2Cos\delta}$ and $\emptyset = tan^{-1}\left[\frac{a_2 sin\delta}{a_1+a_2 cos\delta}\right]$
3. For constructive interference or bright fringes, path difference $\Delta = n\lambda$ where n=1, 2, 3…
4. For destructive interference or dark fringes, path difference $\Delta = (\frac{2n-1}{2})\, n\lambda$
5. For sustainable interference the two waves should be coherent. If two or more waves of same frequency are in the same phase or have constant phase difference then there waves are called coherent.
6. In interference pattern, the component of energy (intensity) $2a_1 a_2$ is simply transfer from minima to maxima point. The net intensity or average intensity remains constant or conserved.
7. Interference is of two types, known as division of wave front and division of amplitude.
8. Division of wave front is a class of interference in which the light from original common source is divided into two parts by employing mirror, prism, lens, biprism etc.
9. In case of division of amplitude, the incoming beam is divided into two or more parts by partial reflection or refraction. Interference due to thin film, Newton's rings, Michelson interferometer are the examples of division of amplitude.
10. In Young's double slit experiment fringe width is given by $\omega = \frac{D\lambda}{2d}$. Sometimes symbol β is to be used for fringe width. Position of n^{th} bright fringe is given by $y_n = n\frac{D\lambda}{2d}$. Similarly position of nth dark fringe $y_n = \frac{2n-1}{2}\frac{D\lambda}{2d}$, where D distance between slit and screen and 2d is the separation between slits S_1 and S_2.

11. On introducing a thin transparent sheet of thickness t in the path any interfering ray, the entire fringe system will be displaced by a distance x given as x = $\frac{D}{2d}$(μ-1) t.
Where μ is refractive index of material of sheet, 2d is distance between two slits.

12. In Fresnel's biprism the fringe width is given by ω = $\frac{D\lambda}{2d}$ and 2d = 2a (μ-1)α where a is distance between source and biprism and α is the angle of biprism and μ is refractive index of material of biprism.

4.15 GLOSSARY

Interference: Redistribution of energy due to superposition of waves.

Interference fringes: Pattern of dark and bright bands due to interference.

Superposition: Combining the displacements of two or more waves to produce a resultant displacement.

Coherence: Property of two or more waves with equal frequency and constant phase difference.

Coherent light: Light in which all wave trains have same frequency and its crests and troughs aligned in same directions which have constant phase difference.

Biprism: Combination of two prisms with their bases in contact.

Slit: A narrow opening for light.

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4.18 TERMINAL QUESTION

4.18.1 Short Answer Type Questions

1. What is interference of light? Give some example of interference of light.
2. What are the necessary conditions for interference of light?
3. What are coherent sources of light?
4. Discuss why two independent sources of some frequency are not coherent?
5. State the principle of superposition of waves.
6. Explain the optical path of light in a medium.
7. What is the difference between ordinary prism and biprism? How can we distinguish?

4.18.2 Long Answer Type Questions

1. What is interference of light? Obtain the condition for constructive and distractive interference.
2. What is Young's double slit experiment? Find out the position of bright fringes, dark fringes and fringe width.
3. Derive an expression for the resultant intensity of two coherent beam of light which are superimposed.
4. Explain the construction and working of biprism.
5. Calculate the displacement of fringe system when a transparent thin film is introduced in the path of an interfering beam in the double slit experiment.

4.18.3 Numerical Questions

1. A biprism is placed 5 cm from the slit and 75cm from the screen. The biprism is illuminated by sodium light of wavelength 5890Å. The fringe width is observed 424×10^{-2} cm. Calculate the distance between two coherent sources. [Ans. 0.5mm]
2. A biprism form interference fringes with monochromatic light of wave length 5450Å. On introducing a thin glass plate of refractive index 1.5 in the path of one of the interfering beam, the central fringe shifts to the position previously occupied by 6^{th} bright fringe. Find out the thickness of the plate.
3. The inclined faces of a biprism of refractive index 1.5 make angle 2^0 with base. A slit illuminated by a monochromatic light is placed at a distance of 10cm from the biprism. If the distance between two dark fringes observed at a distance of 1cm from the biprism is 0.18 mm, find out the wavelength of light used.

4. The inclined faces of a glass biprism of refractive index 1.5 makes angle of 1^0 width base of the prism. The distance between slit and biprism is 0.1m. The biprism is illuminated by a light of wavelength 5900Å and fringes are observed at a distance 1m from the biprism. find out the fringe width.

4.18.4 Objective Type Questions

1 . Phase difference Φ and path difference δ are related by Φ=

(a) $\frac{2\pi}{\lambda}\delta$ (b) $\frac{\lambda}{2\pi}\delta$

(c) $\frac{\pi}{2\lambda}\delta$ d) $\frac{2\lambda}{\pi}\delta$

2 . The condition for constructive interference is path difference should be equal to

(a) odd integral multiple of wavelength
(b) integral multiple of wavelength
(c) odd integral multiple of half wavelength
(d) Integral multiple of half wavelength

3. The ratio of intensities of two waves that produce interference pattern is 16:1 then the ratio of maximum and minimum intensities in the pattern is

(a) 25:9 (b) 9:25 (c) 1: 4 (d) 4:1

4. Correlation between the a point in the field and the same point in the field at later time is known as

(a) Temporal coherence (b) coherence

(c) Spatial coherence (d) none of these

5. The overlapping of waves into the regions of the geometrical shadow is

(a) Dispersion (b) polarization

(c) diffraction (d) interference

6. Interference occurs due to

(a) Wave nature of light (b) particle nature of light

(c) both a and b (d) none of these

7. Two interfering beams have their amplitudes ratio 2:1 then the intensity ratio of bright and dark fringes is

(a) 2:1 (b) 1:2 (c) 9:1 (d) 4:1

8. If a_1 and a_2 are the amplitudes of light coming from two slits in Young's double slit experiment then the minimum intensity of interference fringe is

(a) $a_1 + a_2$ (b) $a_1 - a_2$ (c) $(a_1 + a_2)^2$ (d) $(a_1 - a_2)^2$

9. Young's double slit experiment is an example of division of

(a) amplitude (b) Wavelength (c) wave front (d) None

10. In Young's double slit experiment, the fringe width ω is given by

(a) $\frac{D\lambda}{2d}$ (b) $\frac{D\lambda}{d}$ (c) $\frac{2d\lambda}{D}$ (d) $\frac{d\lambda}{2D}$

4.18.5 Answers of Objective Type Questions

1. (a), 2. (b), 3(a), 4(a), 5(d), 6(a), 7(c), 8(c), 9(c), 10(a)

UNIT 5: INTERFERENCE IN THIN FILMS AND NEWTON'S RINGS

CONTENTS

5.1. INTRODUCTION

In optics any transparent material in a shape of thin sheet of order 1μm to 10 μm is simply called thin film. The material may be glass, water, air, mica and any other material of different refractive index. When a thin film is illuminated by a light, some part of incident light get refracted from the upper surface of film and some part of get transmitted into the film. Some part of transmitted light gets reflected again from the lower surface of thin film. Now the light reflected from upper and lower surface of thin may course interference.

In case of thin film, the maximum portion of incident light is transmitted and a very few part of light reflected form the thin film. Therefore the intensity of reflected light is significantly small. For example if we consider a light beam is reflected from a glass plate of refractive index 1.5 then the reflection coefficient is given by

$$r = \left(\frac{\mu_1 - \mu_2}{\mu_1 - \mu_2}\right)^2 = \left(\frac{1.5-1}{1.5+1}\right)^2 = \left(\frac{0.5}{2.5}\right) = 0.04$$

Thus only 4% of incident light is reflected by the upper surface of glass film and 96% of light is transmitted into the glass plate. Similarly nearly 4% of light is again reflected through the lower surface of glass plate. If we consider the interference due to the light reflected from upper and lower surface of glass plate, the intensity of light will be significantly small.

When white light is incident of thin film, interference pattern is appeared as colourful bands since white light consists different wavelengths, different wavelengths produce interference bands of different colours and thicknesses. Interference in thin films also occurs in nature. Thin wings of many insects and butterflies are layer of thin films. There thin films are responsible for structural colourization which produce different colours by microscopically structured surface, and suitable enough for interference of light.

5.2. OBJECTIVE

After reading this unit you will be able to understand

- Thin film
- Interference in thin film
- Interference in wedge shaped film
- Classification of fringes and its shapes
- Newton's rings experiments and its applications

5.3. INTERFERENCE DUE TO PLANE PARALLEL THIN FILM

A plane parallel thin film is transparent film of uniform thickness with two parallel reflecting surfaces. The example is a thin glass film. Light wave generally suffers multiple reflections and refractions at the two surfaces. There are two cases of interference as given below

5.3.1 Interference in Case of Reflected Light

Let us consider a thin film of thickness *t* as shown in figure 5.1. A monochromatic light ray SA is incident on a thin film with an angle of incident *i* as shown in figure. The film is made of a transparent material (say glass) of refractive index μ. Some part of light ray reflected at point A along the direction AB and some part of light transmitted into the film along AC direction. The ray AC makes an angle of refraction *r* at point A, and the angle r becomes angle of incident ACN at point C. Some part of light of ray AC again reflected in the direction CD which comes out from the film along the direction DE. The light rays AB and DE come together and they can produced interference pattern on superposition.

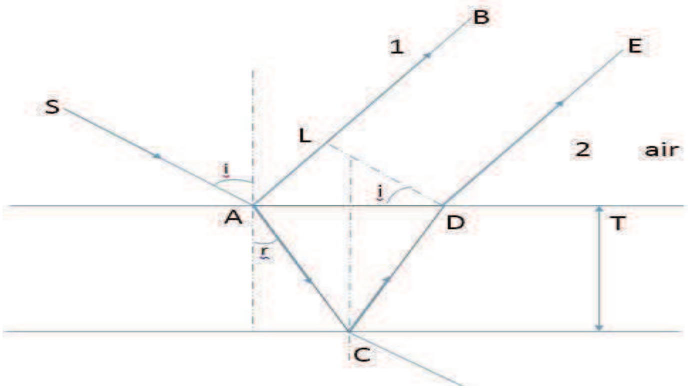


Fig 5.1

The path difference Δ between rays AB and DE is given as

$$\Delta = (AC+DC) \text{ in film- } AL \text{ in air.}$$

Since optical path in air = μ × optical path in a medium

Therefore, path difference Δ can be given as

$$\Delta = \mu\,(AC+ DC) - AL$$

From figure 5.1, we have, $\cos r = \frac{t}{AC}$ or $AC = \frac{t}{\cos r}$ and $DC = \frac{t}{\cos r}$

Again, $AL = AD \sin i = (AN+ ND) \sin i$

$$= (t \tan r + t \tan r) \sin i = 2t \tan r \,\text{Sin}\, i$$

$$\Delta = \frac{\mu\, 2t}{\cos r} - 2t.\tan r \sin i = \frac{2\mu t}{\cos r} - 2\mu t\,(\sin^2 r)$$

$$= 2\mu \frac{t}{\text{Cos}\, r}(1 - \sin^2 r) = 2\mu t \cos r$$

According to Stock's treatment, if a wave is reflected form a denser medium it involves a path difference of λ/2 or phase difference or π. Therefore, net path difference

$$\Delta = 2\mu t\, \text{Cos}\, r - \frac{\lambda}{2} \qquad \text{....... (5.1)}$$

Condition of Maxima: For maxima or bright fringes the path difference should be $n\lambda$ where n is integer number given as n= 0,1,2,3 ……..

$$\Delta = 2\mu t\, \text{Cos}\, r - \frac{\lambda}{2} = n\lambda$$

or $$2\mu t \, Cos \, r = (\frac{2n+1}{2})\lambda \qquad \text{....... (5.2)}$$

Thus maxima occur when optical path difference is $(\frac{2n+1}{2})\lambda$.

Condition for minima: Minima occur when the path difference is order of $(\frac{2n-1}{2})\lambda$. Then

$$\Delta = 2\mu t \, Cos \, r - \frac{\lambda}{2} = (\frac{2n-1}{2})\lambda$$

or $$2\mu t \, Cos \, r = n\lambda \qquad \text{....... (5.3)}$$

5.3.2 Interference in Case of Refracted Light

A light ray SA is incident at point A on a film of refractive index μ as shown in figure 5.2. Some part of light ray reflected at point A and some part of light transmitted into the film along AB. In case of interference due to refracted light we are not interested in the reflected light. At point B some part of light is again reflected along direction BC, then again reflected at point C and finally refracted at point D and comes out form the medium along DF direction. Now the light rays coming along BE and DF are coherent and can produce interference pattern in the region of superposition.

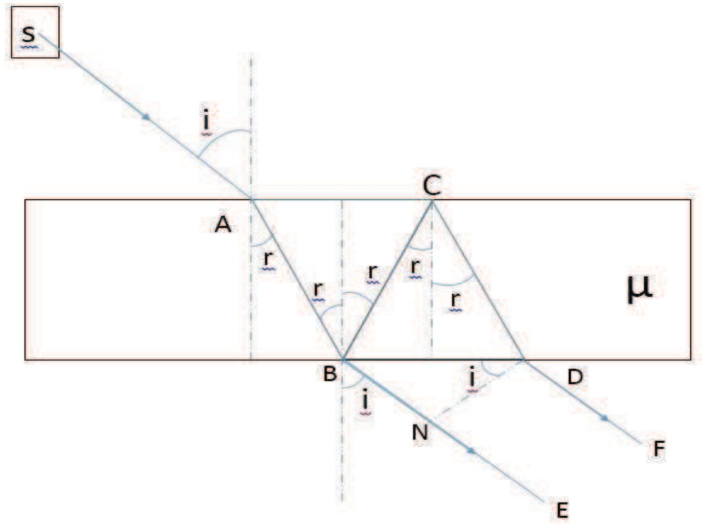


Figure 5.2

In this case path difference Δ is given as

$$\Delta = (BC + CD) \text{ in film} - BN \text{ in air}$$

As Calculated in case of reflection, the path difference comes out

$$\Delta = 2\mu t \, Cos \, r$$

In this case there is no correction according to Stoke's treatment as no wave from rarer medium is reflected back to denser medium. Therefore this is net path difference.

For maxima or bright fringes, $\Delta = 2\mu t \text{ Cos } r = n\lambda$

For minima or dark fringes, $\Delta = 2\mu t \text{ Cos } r = (\frac{2n-1}{2})\lambda$

5.4 *INTERFERENCE IN A WEDGE SHAPED FILM*

In a wedge shape film, the thickness of the film at one end is zero and it increases consistently towards another end. A glass wedge shaped film is shown in figure 5.3. Similarly a wedge shaped air film can be formed by using two glass films touch at one end and separated by a thin wire at another end.

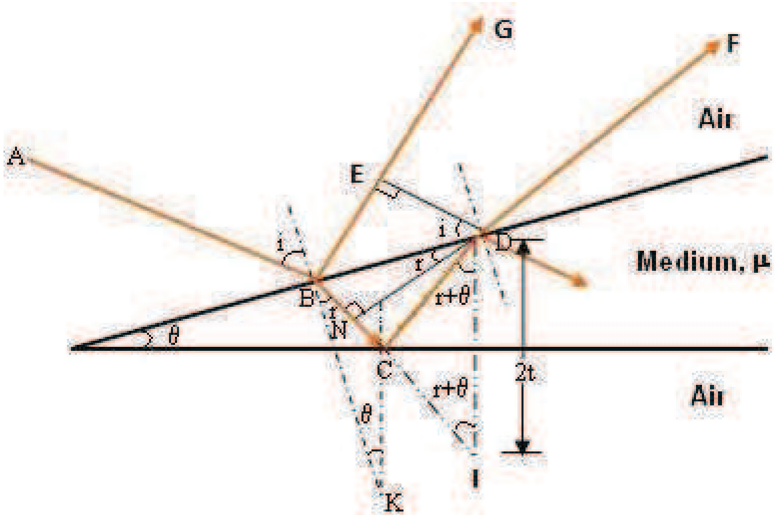


Figure 5.3

The angle made by two surfaced at touching end of wedge is called angle of wedge as shown ϴ in figure 5.3. The angle is very small in order of less than 1^0. Path difference between two reflected rays BE and DF is given by

$$\Delta = (BC+CD) \text{ in film} - BE \text{ in air}$$

$$= \mu\,(BC+CD) - BE$$

$$= \mu\,(BC+CI) - BE \qquad \because CD = CI$$

$$= \mu\,(BN+NI) - BE \qquad \text{.......}\ (5.4)$$

In right triangle Δ BED, $\sin i = \frac{BE}{BD}$

Similarly in Δ BND, $\sin r = \frac{BN}{BD}$

Refractive index μ can be given as

$$\mu = \frac{Sin\ i}{Sin\ r} = \frac{BE}{BN} \quad \text{or} \quad BE = \mu\, BN$$

Putting this value in equation (5.4) we get

$$\Delta = \mu\,(BN + NI) - \mu\, BN = \mu\, NI \qquad \text{.......}\ (5.5)$$

Now in Δ DNI, $\cos(r+\theta) = \frac{NI}{DI}$

or $cos\ (r + \Theta) = \frac{NI}{2t} \Rightarrow NI = 2t\ Cos\ (r + \Theta)$

Putting this value in equation (5.5)

Path difference, $\Delta = \mu.\ 2t\ Cos\ (r + \Theta)$ .... (5.6)

Since the light is reflecting from a denser medium therefore according to stokes treatment a path change of λ/2 occurs. Now net path difference

$$\Delta = 2t\ Cos\ (r + \Theta) - \lambda/2 \qquad \text{.......}\ (5.7)$$

For bright fringes the path difference should be in order of $\Delta = n\lambda$ where *n* is an integer (n= 0, 1, 2….....).

$$2\mu t\ Cos\ (r + \Theta) - \lambda/2 = n\lambda$$

or
$$2\mu t\ Cos\ (r + \Theta) = \left(\frac{2n+1}{2}\right)\lambda \qquad \text{where } n = 0,1,2\ldots..$$

or
$$2\mu t\ Cos\ (r + \Theta) = \left(\frac{2n-1}{2}\right)\lambda \qquad \text{.......}\ (5.8)$$

Where, $n = 1, 2, 3\ldots..$

For dark fringes path difference should be in order of $\Delta = \left(\frac{2n-1}{2}\right)\lambda$.

$$2\mu t\ Cos\ (r + \Theta) - \lambda/2 = \left(\frac{2n-1}{2}\right)\lambda$$

or
$$2\mu t\ Cos\ (r + \Theta) = n\lambda \qquad \text{.......}\ (5.9)$$

Since the focus of points of constant thickness is straight line, therefore the fringes are straight lined in shape.

According to equation (5.8), for bright fringes

$$t = \frac{(2n-1)\lambda}{4\,\mu\cos(r+\theta)} = \frac{\lambda}{4\,\mu\cos(r+\theta)} = \frac{3\lambda}{4\,\mu\cos(r+\theta)} = \cdots\ldots\ldots \qquad \text{.......}\ (5.10)$$

If x_n is the distance of fringes from the edge (position of n^{th} fringe) then,

$$\tan\theta = \frac{t}{x_n}$$

or
$$x_n = \frac{(2n-1)\lambda}{4\,\mu\cos(r+\theta)\tan\theta} \qquad \text{.......}\ (5.11)$$

Thus, $x_1 = \dfrac{\lambda}{4\,\mu\cos(r+\theta)\tan\theta}$, $x_2 = \dfrac{3\lambda}{4\,\mu\cos(r+\theta)\tan\theta}$ ....

Fringe width $\omega = x_{n+1} - x_n$

$$\omega = \frac{2\lambda}{4\,\mu\,Cos(r+\theta)\tan\theta}) \qquad \text{.......}\ (5.12)$$

If Θ is very small then tan Θ≃ Θ, and cos (r + Θ) ≅ r. Further if we consider normal incidence then $r = 0^0$ then cos 0 = 1 and equation (5.12) becomes

$$\omega = \frac{\lambda}{2\,\mu\,\theta} \qquad \text{.......}\ (5.13)$$

5.4.1 Properties of Fringes Due to Wedge Shaped Film

1. As the locus of the points of constant thickness is a straight line therefore the fringes are straight lime and parallel.

2. The fringe width ω is constant for a particular wave length or colour, therefore the fringes are of equal thickness and equidistant.

3. Fringes are localized

5.4.2. Applications of Wedge Shaped Film

By observing the interference pattern, the thickness of a spacer or wire which is placed between two films at one end can be determined. Suppose *t* is the thickness of a wire or spaces and *l* is length of wedge shaped film as shown in figure 5.4 then we can calculate the thickness of spacer as

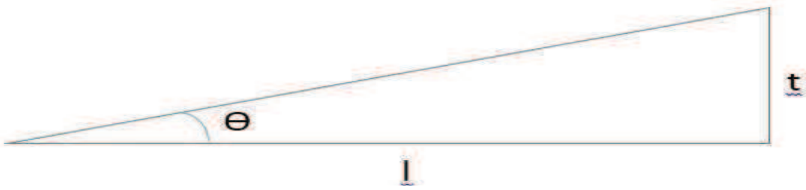


Figure 5.4

$$\tan \Theta \cong \Theta = \frac{t}{l}$$

If we know the fringe width ω then by using relation $\omega = \frac{\lambda}{2\mu\theta} = \frac{\lambda}{2\mu\frac{t}{l}}$ we get,

$$t = \frac{\lambda l}{2\mu.\omega}$$

Example 5.1: A white light is normally incident on a soap bobble film of thickness 0.40 μm and refractive index 1.4. Which are the wavelengths may cause bright fringes.

Solution: For bright fringes, due to thin films, the condition is

$$2\mu t \text{ Cos } r = (2n+1)\frac{\lambda}{2}, \text{ where } n= 0,1,2,3\ldots..$$

or $$\lambda = \frac{4\ \mu t \cos r}{(2n+1)}$$

Here r = 0, μ = 1.4 and t = 0.40 μm.

$$\lambda = \frac{4\times1.4\times0.40\times10^{-6}}{(2n+1)} = \frac{2.24\times10^{-6}}{(2n+1)}\ m$$

For n = 0; $\lambda = 2.24 \times 10^{-6}\ m$

n = 1; $\lambda = 0.74 \times 10^{-6}\ m$

n = 2; $\lambda = 0.44 \times 10^{-6}\ m$

Example 5.2: White light is incident on an oil film of thickness 0.01mm and reflected at an angle 45^0 to vertical. The refractive index of oil is 1.4 and refracted light falls on the slit of a spectrometer, calculate the number of dark bands seen between wavelengths 4000Å and 5000Å.

Solution: For the dark band, formed by interference, due to thin film

$$2\mu t \cos r = n\lambda$$

In case of wave length λ_1 =4000Å and μ=1.4 , t= 0.01 mm

$$n_1 = \frac{2\mu t \, Cos\, r}{\lambda_1}$$

Now $$\mu = \frac{Sin\, i}{Sin\, r} = Sin\, r = \frac{Sin\, i}{\mu}$$

$$Cos\, r = \sqrt{1 - Sin^2 r} = \sqrt{1 - \frac{Sin^2 r}{\mu^2}} = \sqrt{1 - \frac{1}{2\times(1.4)^2}} = 0.86$$

Thus $$n_1 = \frac{2\times1.4\times0.001\times.86}{4000\times 10^{-8}} = 60$$

Thus corresponding to λ_1=4000Å wavelength light we observe 60^{th} order band

Similarly corresponding to λ_2 wavelength

$$n_2 = \frac{2\mu t \, Cos\, r}{\lambda_2} = \frac{2\times1.4\times.001\times0.86}{5000\times10^{-8}} = 48$$

Thus corresponding to wavelength λ_2 =5000Å light we observe 48^{th} order band.

Thus the number of dark bands between λ_2 and λ_1 = $n_1 - n_2$ =60-48 =12.

Example 5.3: A parallel beam of light λ = 5890 Å is incident on a thin glass film and the angle of refraction into the film is 60^0. Calculate the smallest thickness of the film which appear dark on reflection.

Solution: The film appears dark if the destructive interference takes place in reflection.

Path difference in dark bands

$$\Delta = 2\mu t \, Cos\, r = n\lambda$$

For smallest thickness n=01 then

$$t = \frac{\lambda}{2\mu Cos\, r} = \frac{5890\times10^{-10}}{2\times1.5\times0.5} = 3927 \times 10^{-10} m = 3927 \text{ Å}$$

Example 5.4: A monochromatic light of wavelength 5890 Å is incident normally on glass plates enclosing a wedge shaped air film. The two plates touch at one end and are separated at 15cm apart from that end by a wire of 0.05 mm diameter. Calculate the fringe width of bright fringes.

Solution: In case of wedge shaped film the fringe width is given by

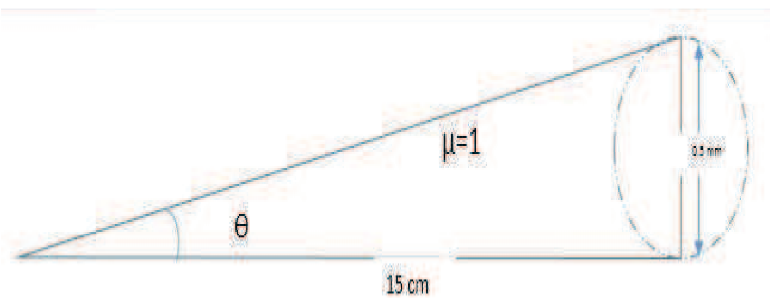


Figure 5.5

$$\omega = \frac{\lambda}{2\mu\theta}$$

Given λ = 5890 Å, μ=1, ϴ= tan ϴ = $\frac{0.05 \times 10^{-1}}{15}$ = 3.3×10^{-4}

$$\omega = \frac{5890 \times 10^{-10}}{2\times 1.0\times 3.3\times 10^{-4}} = 892.4 \times 10^{-6} m = 0.89 \text{ mm}$$

Example 5.5: Sodium light of wavelength λ = 5890Å is incident on a wedge shaped air film. When viewed normally 10 fringes are observed in a distance of 1cm. Calculate the angle of the wedge.

Solution: The fringe width ω for wedge shaped film is given us

$$\omega = \frac{\lambda}{2\mu\theta}$$

In this case, 10 fringes are observed in a distance of 1 cm. Therefore, fringe width

$$\omega = \frac{1}{10} = 0.1\ cm$$

Now $$\Theta = \frac{\lambda}{2\mu\omega} = \frac{5890 \times 10^{-8}}{2\times 2\times 0.1} = 2.94 \times\ 10^{-4}\ radians$$

$$= 2.94 \times\ 10^{-4} \times \frac{180}{\pi} \text{ degree} = 3.94 \times\ 10^{-6} \times \frac{180\times 60}{\pi} \text{ minute}$$

$$= 1.01 \text{ minute}$$

Example 5.6: A Wedge shaped film is form by using two glass plates of length 10cm touch at one end and separate at another end by introducing a thin foil of thickness 0.02mm. If the sodium light of wavelength 5890Å is indent normally on it. Find the separation between two consecutive fringes.

Solution: The separation between two consecutive fringes is the same as the fringe width.

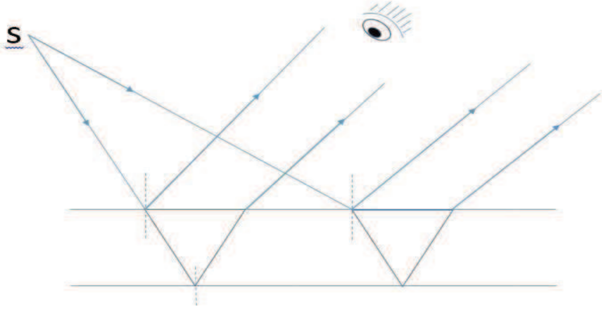
$$\omega = \frac{\lambda}{2\mu\theta} \quad \text{where } \Theta = \tan\Theta = \frac{t}{x} = \frac{0.02}{100} = 2 \times 10^{-4}$$

Given λ =5890Å, μ = 1 then

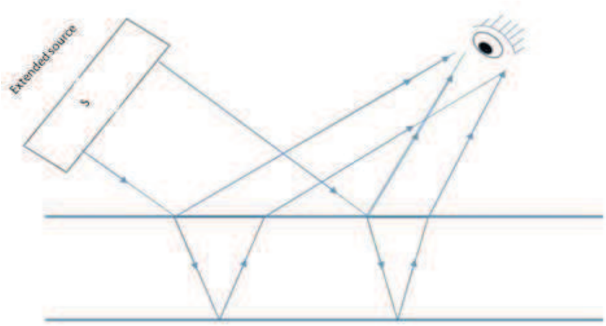
$$\omega = \frac{5890 \times 10^{-8}}{2\times 1\times 2\times 10^{-4}} \text{ cm} = 0.14 \text{ cm}$$

5.5 NECESSITY OF EXTENDED SOURCE FOR INTERFERENCE DUE TO THIN FILMS

If we use narrow source of light in case of interference due to thin film the light rays are diverged as shown in figure 5.6 (a) and we can view a limited portion of interference pattern. On the other hand, if we use an extended or broad source of light a large number of rays are available for the production of interference pattern as shown in figure 5.6 (b). A large number of rays are incident on film at different angles, and a large area of film can be viewed by our eye at the field of view. Therefore, extended source of light is beneficial to observe the good interference pattern in thin film.



(a)



(b)
Figure 5.6

5.6 COLOURS OF THIN FILMS

When light coming from extended source is reflected by thin film of oil, mica, soap or coating etc., different colours are shown due to interference of light. For interference, the optical path difference is $\Delta = 2\mu t \; Cos \; r = (2n+1) \; \lambda/2$ for bright fringes. If thickness t is constant then for different wavelengths, angle of refraction r should be different. Therefore different colours are observed at different angle of incident. Sometime different colours are over lopped on each other's and a mixed colour may be observed.

5.7 CLASSIFICATION OF FRINGES

As we know, in case of thin films, the path difference Δ is given as

$$2\mu t \; Cos \; r = \left(\frac{2n+1}{2}\right)\lambda$$

For a monochromatic light, μ and λ remain constant. Now the path difference for constructive interference arises due to variation in thickness t and angle of incident (inclination) *r*. On the basis of *t* and *r* the fringes are two types.

5.7.1 Fringes of Equal Thickness

If the thickness of film is varying and the light is coming at same angle of incident then the fringes are formed due to variation in thickness. For example in case of wedge shaped film where thickness is varying, the locus of points of constant thickness is a straight line corresponding to which fringes are formed. Such fringes are called fringes of equal thickness. Newton's rings are example of such type of fringes.

5.7.2 Fringes of Equal Inclination

If the thickness of film is constant then path difference for constrictive interference is only due to variation in angle of inclination r. In this case we consider a locus of points on film at which the angle of inclination of light is equal. Corresponding to such points of equal inclination we observed fringes which are called fringes of equal inclination. Since the light rays of equal inclinations pass through the plate is a parallel beam of light, and hence meet at infinity but by using telescope focused on such rays the fringes can be observed. In such case fringes are called the fringes localized at infinity. Such fringes are also called Haidinger's fringes. The fringes formed in Michelson interferometer is an example of fringes of equal inclination.

5.8 NEWTON'S RINGS

Newton's rings in a special case of wedge shaped film in which an air film is formed between a glass plate and a convex surface of lens. The thickness of air film is zero at the center and increases gradually towards the outside.

When a plano-convex lens of large focal length is placed on a plane glass plate, a thin air film is formed between the lower surface of plano-convex lens and upper surface of glass plate. When a monochromatic light falls on this film the light reflected from upper and lower surfaces of air film, and after interference of these rays, we get an inner dark spot surrounded by alternate bright and dark rings called Newton's rings. These rings are first observed by Newton and hence called Newton's rings.

5.8.1 Experimental Arrangement for Reflected Light

The experimental arrangement for Newton's rings experiment is shown in Figure 5.7. A beam of light from a monochromatic source S is made parallel by using a convex lens L. The parallel beam of light falls on a partially polished glass plate inclined at an angle of 45^0. The light falls on glass plate is partially reflected and partially transmitted. The reflected light normally falls on the plano-convex lens placed on plane glass plate.

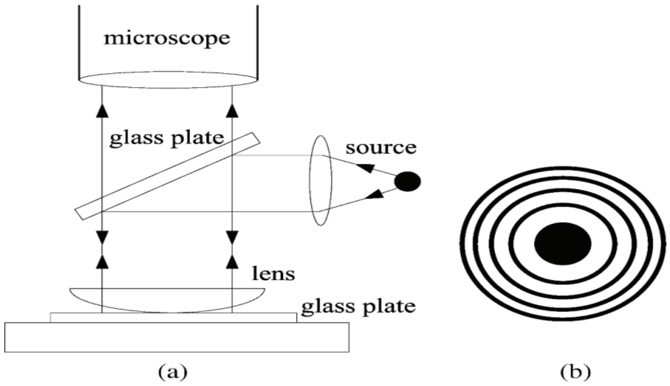


Fig. 5.7

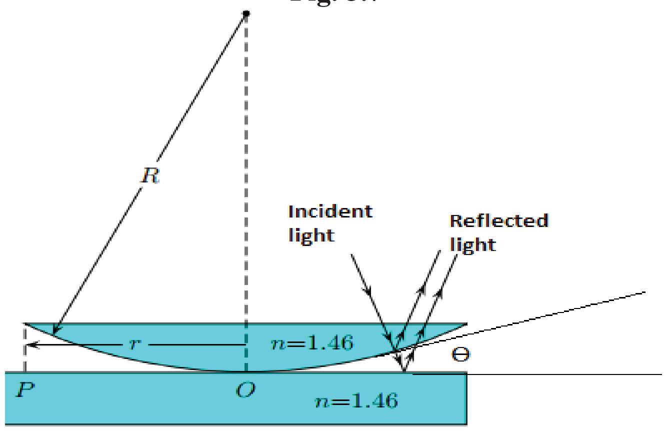


Fig 5.8

This light reflected from upper and lower surface of the air film form between plane glass plate and plno-convex lens. These rays interfere and rings are observed in the field of view. The figure 5.8 shows the reflection of light form upper and lower surfaces of air film which are responsible for interference.

5.8.2 Formation of Bright and Dark Rings

As we know the interference occurs due to light reflected from upper and lower surface of air film form between glass plate and plano-convex lens. The air film can be considered as a special case of wedge shaped film. In this case, angle wedge is the angle made between the plan glass plate and tangent from line of contact to curved surface of plano convex lens as shown in figure. 5.8.

The path difference between two interfering rays reflected by air film

$$\Delta = 2\mu t \,\mathrm{Cos}\,(r + \Theta) - \frac{\lambda}{2} \qquad \text{.......} \quad (5.14)$$

where μ is the refractive index of the air film, t is the thickness of air film at the point of reflection (say point P) r is angle of refraction and Θ is angle of wedge.

In this case the light normally falls on the plane convex lens for the angle of refraction r = 0. Further, as we use a lens of large focal length the angle of wedge Θ is very small. So Cos $(r+\Theta)$ = Cos Θ = Cos 0^0 =1 and thus the path difference

$$\Delta = 2\mu t - \frac{\lambda}{2} \qquad \text{.......} \quad (5.15)$$

At point of contact t = 0, therefore, $\Delta = \frac{\lambda}{2}$

Which is the condition of minima. Hence at centre or at point of contact there is a dark spot.

Condition of Bright Rings or Maxima

The condition for bright rings is path difference $\Delta = n\lambda$ therefore

$$\Delta = 2\mu t - \frac{\lambda}{2} = n\lambda \text{ where } n= 0, 1, 2, 3\ldots\ldots$$

or $$2\mu t = \left(\frac{2n+1}{2}\right)\lambda$$

or $$2\mu t = \left(\frac{2n-1}{2}\right)\lambda \qquad \ldots\ldots. (5.16)$$

Where n = 1, 2, 3………

Condition of Dark Ring or Minima

In case of dark rings, the path difference, $\Delta = \left(\frac{2n-1}{2}\right)\lambda$

Where n= 1, 2, 3………

Therefore $$\Delta = 2\mu t - \frac{\lambda}{2} = \left(\frac{2n-1}{2}\right)\lambda$$

or $$2\mu t = n\lambda \qquad \ldots\ldots. (5.17)$$

Thus corresponding to n = 1, 2, 3….. we observe first, second third…..etc. bright or dark rings. In Newton's rings experiment the locus of points of constant thickness is a circle therefore the fringes are circular rings.

5.8.3 Diameter of Bright and Dark Rings

In figure 5.9 the plano-convex lens BOPF is place on glass plate G and O is the point of contact. Suppose, C is the centre of the sphere OBFP from which the plano-convex lens is constructed. P is point on the air film at which the thickness of air film is t. At point P, the light is incident and reflected form the upper and lower surface of air film, and rings are formed. AP is the radius of ring passes through point P. According to property of circle

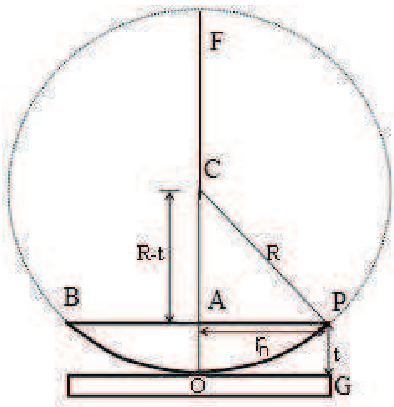


Figure 5.9

$$AP \times AB = AO \times AL$$

$$r^2 = t \times (2R - t) \quad \because AL=OL-OA$$

Where R is the radius of curvature of lens.

$$r^2 = 2Rt - t^2$$

Since R is very large and t is very small, we can write

$$r^2 = 2Rt \qquad \text{or} \qquad t = \frac{r^2}{2R}$$

Substituting this value of t in equation (5.16), we get,

$$2\mu \frac{r^2}{2R} = \left(\frac{2n-1}{2}\right)\lambda$$

or

$$r^2 = \left(\frac{2n-1}{2}\right)\frac{\lambda R}{\mu}$$

This expression contains n, *i.e.,* r is a function of n. Thus it is better to use r_n in place of r. If D_n is the diameter of nth bright ring then we have $r = r_n = D_n / 2$ and can write

$$\frac{{D_n}^2}{4} = \frac{\left(\frac{2n-1}{2}\right)\lambda R}{\mu}$$

or

$$ {D_n}^2 = \frac{2(2n-1)\lambda R}{\mu} \qquad \text{....... (5.18)}$$

Where n= 1, 2, 3……. Similarly for dark rings

$$2\mu t = n\lambda \qquad \text{or} \qquad 2\mu \frac{r^2}{2R} = n\lambda \qquad \text{or} \qquad r^2 = \frac{n\lambda R}{\mu}.$$

If D_n is diameter of nth dark ring then

$$\frac{{D_n}^2}{4} = \frac{n\lambda R}{\mu}$$

or

$$ {D_n}^2 = \frac{4n\lambda R}{\mu} \qquad \text{....... (5.19)}$$

Where n = 1, 2, 3……

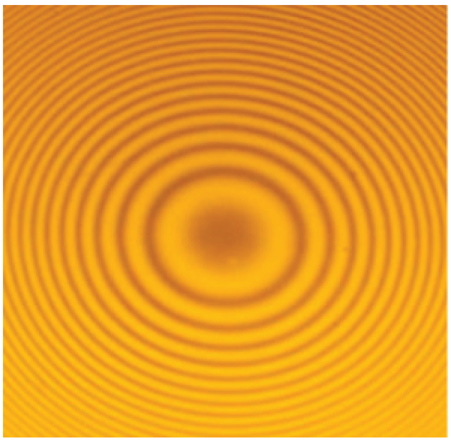


Figure 5.10

The alternate bright and dark rings are formed as shown in figure 5.10. The spacing between two consecutive rings can be given as

$r_{n+1}^2 - r_n^2 = \left(\sqrt{n+1} - \sqrt{n}\right) \lambda R$ (in case of air film μ =1)

Spacing between 1st and 2nd rings = $\left(\sqrt{2} - \sqrt{1}\right) \lambda R$ = 0.4142 λR

Spacing between 2nd and 3rd rings = $\left(\sqrt{3} - \sqrt{2}\right) \lambda R$ = 0.3178 λR

Spacing between 4th and 3rd rings = $\left(\sqrt{4} - \sqrt{3}\right) \lambda R$ = 0.21 λR

Thus it is clear that the spacing between successive rings decreases with increase in order.

5.8.4 Determination of Wave Length of a Monochromatic Light Source

In Newton's experiment if we use a light source of unknown wave length (say sodium lamp) then we can determine the wavelength of light source by measuring the diameters of Newton's ring.

If D_n is diameter of nth dark ring formed due to air film then

$$D_n^2 = 4n\lambda R$$

Where n is any integer number.

Similarly if $D_{(n+p)}$ is the diameter of $(n+p)^{th}$ ring

$$D_{n+p}^2 = \mu\,(n+p)\,\lambda R$$

Using this equation, we can write

$$D_{n+p}^2 - D_n^2 = 4\,(n+p)\,\lambda R - 4n\lambda R = 4\,P\,\lambda R$$

or

$$\lambda = \frac{D_{n+p}^2 - D_n^2}{4pR} \qquad \text{....... (5.20)}$$

Where p is any integer number and R is radius of curvature of plano-convex lens.

5.8.5. Determination of Refractive Index of a Liquid by Newton's Rings Experiment

In Newton's rings experiment the diameter of n^{th} dark ring in case air film is

$$D_n^2 = 4n\lambda R \qquad (\because \mu = 1)$$

The diameter of $(n+p)^{th}$ ring

$$D^2_{n+p} = 4(n+p)\lambda R$$

If a liquid of refractive index μ is filled between the plane glass plate and convex lens then

$$D^2_{n} = \frac{4n\lambda R}{\mu} \quad \text{and} \quad D^2_{n+p} = \frac{4(n+p)\lambda R}{\mu}$$

Thus we can write

$$\frac{[D_{n+p}^2 - D_n^2]air}{[D_{n+p}^2 - D_n^2]liquid} = \frac{4p\,\lambda R}{\frac{4p\,\lambda R}{\mu}} = \mu$$

or $$\mu = \frac{[D^2_{n+p} - D^2_n]air}{[D^2_{n+p} - D^2_n]liquid} \qquad .... (5.21)$$

Example 5.7: In Newton's rings experiment if the radius of curvature of plano-convex lens in 200 cm and wavelength of the light used is 5890 Å, calculate the diameter of 10^{th} bright ring.

Solution: The diameter of n^{th} bright ring is given as (μ=1 for air film) is given by

$$D_n^{\,2} = 2(2n-1)\,\lambda\, R$$

or $$D_{10}^{\;2} = 2 \times (20\text{-}1) \times 5890 \times 10^{-8} \times 200\ cm^2 = 6.69mm$$

The diameter of 10^{th} bright ring is 6.69 mm.

Example 5.8: In a Newton's ring experiment the diameter of 15^{th} dark ring and 5^{th} dark ring are 0.59 cm and 0.33cm respectively. If the radius of curvature of the convex lens is 100cm calculate the wave length of light used.

Solution: The wave length of unknown light source is Newton's rings experiment is given as

$$\lambda = \frac{[D^2_{n+p} - D^2_n]}{4pR}$$

Here $D_{n+p} = D_{15} = 0.59$ cm, $D_n = D_5 = 0.33$cm, $p = 10$, R = 100cm

$$\lambda = \frac{(0.59)^2 - (0.33)^2}{4 \times 10 \times 100} = 5980\ Å$$

Example 5.9: Newton's rings are formed by using a monochromatic light of 6000Å. When a liquid is introduced between the convex lens and plane glass plate the diameter of 6^{th} bright ring becomes 3.1mm. If the radius of curvature of lens is 1mt, calculate the refractive index of liquid.

Solution: Given that, n= 6, D_n = 3.1mm = 3.1×10^{-3} m, λ = 6000Å = 6×10^{-7} m, R = 1m

$$\mu = \frac{2(2n-1)\lambda R}{D_n^2} = \frac{2 \times 11 \times 6 \times 10^{-7} \times 1}{(3.1 \times 10^{-3})^2} = 1.37$$

Example 5.10: In Newton's ring experiment two light sources of wavelength 6000Å and 4500Å are used to form rings. It is observed that n^{th} dark ring due to 6000Å light coinside with $(n+1)^{th}$ dark ring due to 4500Å. If the radius of curvature of the plano convex lens is 100cm, calculate the diameter of n^{th} dark ring due to λ_1 and λ_2.

Solution: For n^{th} dark ring due to λ_1, $D^2_n = 4n\,\lambda_1 R$

Similarly for $(n+1)^{th}$ dark ring due to λ_2, $D^2_{n+1} = 4\,(n+1)\,\lambda_2 R$

Since n^{th} dark ring due to λ_1 co-inside with $(n+1)^{th}$ dark ring due to λ_2 therefore.

$4n\lambda_1 R = 4(n+1)\,\lambda_2 R$ or $n\lambda_1 = (n+1)\,\lambda_2$ or $n\lambda_1 - n\lambda_2 = \lambda_2$ or $n = \frac{\lambda_2}{\lambda_1 - \lambda_2}$

Here λ_1= 6000Å, λ_2 =4500Å

∴ $$n = \frac{4500}{6000 - 4500} = 3$$

Now the diameter of n=3rd dark ring due to λ_1

$$D_3^2 = 4n\lambda_1 R = 4\times3\times6000\times10^{-10}\times1 \text{ m}$$

Or $D_3 = 2.68$ mm.

Similarly diameter of n=3rd dark ring due to λ_2

$$D_3^2 = 4n\lambda_2 R = 4\times3\times4500\times10^{-10}\times1 \text{ m}$$

or $D_3 = 2.32$ mm.

Same relation can also be obtained for bright rings.

5.8.6 Newton's Rings in Case of Transmitted Light

The Newton's rings can also be formed in case of interference due to transmitted light as shown in figure 5.11. In this case the transmitted rays 1 and 2 interfere, and we can observe the rings in the field of view. In this case the net path difference between the rays is $\Delta = 2\mu t$. since we will not consider the path difference arises due to reflection from denser medium. Therefore this is net path difference.

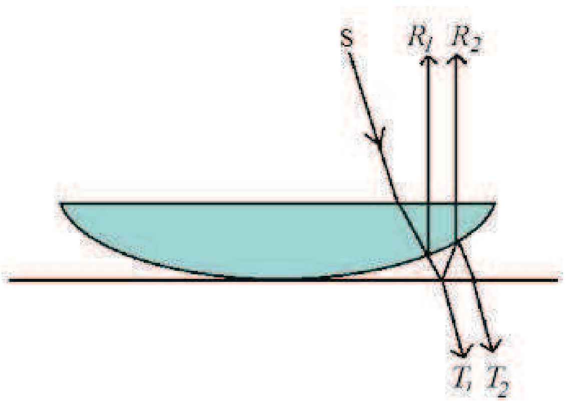


Figure 5.11

The condition for maxima (bright rings) is given by

$$2\mu t = n\lambda$$

And we know that in case of reflected light, $t = \frac{r^2}{2R}$

$$2\mu \frac{r^2}{2R} = n\lambda$$

Now if D_n is the diameter of nth bright ring then, $\frac{D_n}{2} = r$ and thus

$$Dn^2 = \frac{4n\lambda R}{\mu}$$

In case of air film, $Dn^2 = 4n\lambda R$

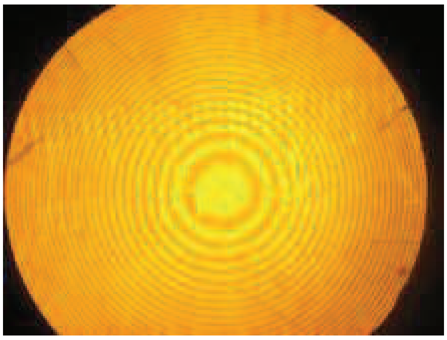


Figure 5.12

Similarly in case of minima (dark ring) the diameter nth dark ring is given by

$$Dn^2 = 2(2n-1)\, \lambda R$$

We can see that, this is an opposite case of reflected light. In case at point of contact the path difference is zero which is condition corresponding to bright fringe thus the centre point is bright. The rings system in this case is shown in figure 5.12.

5.9 SUMMARY

1. A thin film is any transparent material in a shape of thin sheet of order 1μm to10 μm. When a beam of light is incident on this sheet the interference may take place after reflection or transmission of light. In case of interference due to reflected light, the path difference

$$\Delta = 2\mu t \cos r - \frac{\lambda}{2}$$

The condition of bright fringes (maxima)

$$2\mu t \cos r = \left(\frac{2n+1}{2}\right)\lambda \quad \text{(where n= 0,1,2,3…..)}$$

Similarly condition of dark fringes (minima) is

$$2\mu t \cos r = n\lambda \qquad \text{(where n= 0,1,2,3….)}$$

2. In case of interference due to transmitted light, the path difference become $\Delta = 2\mu t \cos r$

The condition of bright fringe (maxima)

$$2\mu t \cos r = n\lambda$$

Similarly the condition of dark fringes (minima)

$$2\mu t \cos r = \left(\frac{2n+1}{2}\right)\lambda$$

3. In case of wedge shaped film the net path difference is given as

$$\Delta = 2\mu t \cos (r + \Theta) - \frac{\lambda}{2}$$

where Θ is angle of wedge and other symbols have their usual meaning. For bright fringes

$$2\mu t \cos (r + \Theta) = \left(\frac{2n-1}{2}\right)\lambda \qquad \text{(where n=1, 2, 3…..)}$$

For dark fringes

$$2\mu t \cos (r + \Theta) = n\lambda \qquad \text{(where n=1, 2, 3…..)}$$

If x_n is the distance of n^{th} fringe from the edge then

$$\tan \Theta = \frac{t}{x_n}$$

$$x_n = \frac{(2n-1)\lambda}{4\mu\, Cos\,(r+\Theta)\tan\Theta}$$

$$x_1 = \frac{\lambda}{4\mu\, Cos\,(r+\Theta)\tan\Theta}\quad , \quad x_2 = \frac{3\lambda}{4\mu\, Cos\,(r+\Theta)\tan\Theta}\,, \ldots\ldots\ldots\ldots$$

Fringe width $\omega = x_{n+1} - x_n$

For normal incident $r = 0^0$ and for small value of Θ ($\tan\Theta \approx \Theta$)

$$\omega = \frac{\lambda}{2\,\mu\Theta}$$

4. In case of interference due to thin film the extended source of light is more beneficial. In extended source of light, a large number of rays are available for production of interference pattern and larger area of the film can be seen by our eye in the field of view.

5. On the basis of variation in two parameters *t* and *r*, the fringes are two types ray fringes of equal thickness and fringes of equal inclination.

In case of fringes of equal thickness, the thickness of film is varying and light coming at same angle of incident then fringes are formed due to variation is thickness. The fringes are formed on the locus of points of equal thickness. Examples are thin films and Newton's rings.

On the other hand, in case of inclination, the thickness becomes constant. Now the fringes are formed at the locus of points of constant. Such fringes are called fringes of equal inclinations. Examples are fringes formed in Michelson interferometer.

6. When a plano-convex lens of large focal length is place on a plane glass plate, an air film is formed between the lens and glass plate. When a beam of light normally incident on this film the interference takes place between the reflected rays and we observe alternate dark and bright rings and called Newton's rings.

7. In Newton's rings the condition for bright rings is given by

$$2\mu t = \left(\frac{2n-1}{2}\right)\lambda \qquad \text{(where n=1, 2, 3….)}$$

Similarly condition for dark rings

$$2\mu t = n\lambda \qquad \text{(where n=1, 2, 3….)}$$

The diameter of n^{th} bright ring is given by

$$D^2_n = \left(\frac{2(2n-1)\lambda\, R}{\mu}\right)$$

Similarly if D_n is diameter of n^{th} dark ring then

$$D_n^2 = \frac{4n\,\lambda R}{\mu}$$

8. By using Newton's rings experiment, the wave length of a unknown light source can be determined as

$$\lambda = \frac{D^2_{n+p} - D^2_n}{4pR}$$

Where D_{n+p} is diameter of $(n+p)^{th}$ bright or dark ring and D_n is the diameter of nth bright or dark ring.

9. Newton's rings may also be observed in case of transmitted light. In this case if D_n is the diameter of n^{th} dark ring then it can be given as

$$D_n^2 = = \left(\frac{2(2n-1)\lambda\, R}{\mu}\right)$$

Similarly if D_n is diameter of n^{th} bright ring then

$$D_n^2 = \frac{4n\,\lambda R}{\mu}$$

5.10 GLOSSARY

Thin film: A thin sheet of thickness of the order of 1-10 μm.
Wedge shaped film: A film of unequal thickness which gradually changes.
Newton's rings: Circular bright and dark fringes formed in Newton's experiment.
Narrow source: Point source
Extended source: A broader source

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5.12. SUGGESTED READING

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2. Fundamentals of optics, Francis A. Jenkines and Harvey E. White, Tata MaGraw Hill, Publisher Limited N. Delhi, India.

5.13. TERMINAL QUESTION

Short Answer Type Questions

1. Explain why different colours are exhibited by a thin film when illuminated in white light.
2. With the help of diagram explain why an extended source of light is needed to observe the interference in thin film.
3. Discuss the phase change in reflection of light from a denser medium.
4. Explain the interference in a thin film of uniform thickness.
5. Calculate the path difference between the light ray reflected from the upper and lower surface of a thin film.
6. Find out the condition of maxima and minima in reflected light in case of thin film.
7. Why a thick film does not show colours when white light is incident on it.
8. What are Newton's rings?
9. Obtain the path difference between the reflected rays in Newton's rings experiment.
10. Find out the condition for bright and dark rings in Newton's ring experiment.
11. Explain why Newton's rings are circular?
12. Explain the difference in Newton's rings formed in case of reflected and refracted light.

Long Answer Type Question

1. Discuss the formation of bright and dark fringes formed by a thin film. Explain why different colours are exhibited by thin film in white light.
2. Explain the formation of interference fringes in wedge shaped film. Obtain the condition for bright and dark fringes, and fringe width.
3. What are Newton's rings? Draw a ray diagram for Newton's rings experiment. Find out the diameter of bright and dark rings.
4. What are Newton's rings? Derive the expression for diameter of bright and dark rings.
5. Give the theory of Newton's rings and describe how the wave length of a unknown light source can be determine with the help of these rings.
6. Describe the interference fringes observed when a thin wedge shaped film is observed by reflected light. Calculate the separation between two consecutive bright and dark fringes.
7. Show that in Newton's rings experiment, the diameter of dark rings are proportioned to root of natural numbers.
8. Explain the formation of Newton's ring. How the refractive index of a given liquid can be determined with the help of Newton's rings.
9. Describe the fringes of equal thickness and fringes of equal inclination.
10. What are Haidiger's and Newton's fringes?

Numerical Type Questions

1. A beam of monochromatic light of wavelength 5890Å is incident on a thin glass plate of refractive index 1.50 with the angle of refraction in the glass plate is 60^0. Calculate the smallest thickness of the plate which will make it appears dark by reflection.

2. Light of wave length 5000Å is incident on a soap film of refractive index 1.33 at an angle 60^0. When the reflected light is observed, a dark band is seen. If the thickness of the film is 1μm, calculate the order of the fringe dark band.

3. Calculate the thickness of a wedge shaped film at a point where the 4^{th} bright fringe is observed. The experiment is performed with a light source of wavelength 5890Å.

4. A wedge shaped film of angle 6×10^{-3} degree is illuminated normally with a monochromatic light source. If the reparation between two consecutive fringes is 3.00mm, find out the wave length of light source used.

5. In a Newton's rings experiment the diameter of 5^{th} and 12^{th} dark rings are 0.42 cm and 0.726cm. The radius of curvature of plano convex lens is 2.00m. Calculate the wavelength of light source.

6. In Newton's ring experiment a light source of wavelength 5890Å is used. If the radius of plano-convex lens is 2m and water is filled between the glass plate and plano convex lens, calculate the diameter of 5^{th} dark ring.

7. A wedge shaped film is formed with air between two glass plates, which touch each other at one point and separated by a wire of diameter 0.05 mm at a distance of 15cm. If a light of wave length 6000Å is used, calculate the fringe width.

8. In Newton's rings experiment the diameter of 4^{th} bright ring is 2.52cm. If a liquid of unknown refractive index is filled in place of air between lens and plane glass plate, the diameter becomes 2.21cm. Find out the refractive index of liquid.
9. Show that in Newton's rings experiment, the difference of square of diameters of two consecutive rings remains constant.
10. Newton's rings are formed with the help of a light source of wavelength 5890Å. If the diameter of 10^{th} dark ring is 0.5cm, calculate the radius of curvature of plano convex lens.
11. A thin equiconvex lens of focal length 4m and refractive index of 1.5 is place on a plane glass plate. A light of wave length 5890Å falls normally on it. What will the diameter of 10^{th} dark ring?

Objective Type Question

1. If the thickness of the parallel film increases, the path difference

(a) increases (b) decreases

(c) remains same (d) none of these

2. When a light wave is reflected from a surface of an optically denser medium, then the phase difference involved is

(a) $\pi/4$ (b) $\pi / 2$ (c) π (d) 2π

3. When a light wave is reflected from a surface of an optically denser medium, then the path difference involved is

(a) $\lambda/4$ (b) $\lambda / 2$ (c) λ (d) 2λ

4. In case of the thin film, the condition for constructive interference in reflected light, the path difference should be equal to

(a) $2\mu t \, Cos \, r - \frac{\lambda}{2}$ (b) $\frac{\lambda}{2}$ (c) $2\mu t \, Cos \, r + \frac{\lambda}{2}$ (d) λ

5. In Newton's rings experiment the diameter of nth bright ring is given by

(a) $D_n{}^2 = \frac{2(2n-1)\lambda R}{\mu}$ (a) $D_n{}^2 = \frac{(2n-1)\lambda R}{\mu}$

(c) $D_n{}^2 = \frac{4\lambda R}{\mu}$ (d) $D_n{}^2 = \frac{2\lambda R}{\mu}$

6. The lens used in Newton's rings experiment, which is placed on a plane glass plate to trap air film is

(a) concave (b) plano convex

(c) plano concave (d) none of these

7. In Newton's rings experiment, the diameter of bright rings is proportional to

(a) odd natural numbers (b) natural numbers

(c) even natural numbers (d) square root of odd natural numbers

Answer of Numerical Type Question

1. 0.39μm, **2.** 4^{th}, **3.** 1.02 μm, **4.** 6.28×10^{-5}cm, **5.** 4.87×10^{-5}cm **7.** 0.9 mm, **8.** 1.3, **10.** 1.06 m.

11. Hint: The focal length is given as

$$\frac{1}{f} = (\mu - 1)(\frac{1}{R_1} - \frac{1}{R_2})$$

Here, $\mu = 1.5\ , R_1 = R\ and\ R_2 = -R \Longrightarrow R = 4\ m$

$D^2_n = 4n\lambda R$

Therefore, $D_{10} = \sqrt{4 \times 10 \times 5890 \times 10^{-10} \times 4} = 9.70\ mm$

Answer of objective Type Question

1. (a), **2.** (c), **3.** (b), **4.** (a), **5.** (a), **6.** (b), **7.** (a)

UNIT 6: MULTIPLE BEAM INTERFEROMETRY

CONTENTS

6.1 INTRODUCTION

In first unit of interference, we understood the basic principle of interference, condition required for interference and experiment like Young double slit experiment and biprism experiment which show interference. In second unit of interference, we understood different types of thin films like wedge shaped or air films which cause interference under certain conditions. Further, we understood the fringes of equal thickness and fringes of equal inclinations.

Now in this unit of interference we are going to understand different types of interferometers, especially Michelson's interferometer. In interferometer we observe the fringes occur due to equal inclination which are called Haidinger fringes. In an interferometer, we study the different techniques of fringes formation and calculate the fringe width with great accuracy. The interferometers like Michelson interferometer have a lot of significant applications in the field of optics and other branches of physics.

6.2. OBJECTIVES

After reading this unit you will be able to understand

- Interferometry
- Haidinger fringes observed in interferometers
- Michelson interferometer
- Application and significance of Michelson's interferometer

6.3. INTERFEROMETRY

Interferometry is a branch of science in which optical waves or any other electromagnetic waves are superimposed on each other and interference phenomenon occurs. Interferometry plays important role to study in the field of optics, astronomy, fiber optics, spectroscopy, cosmology, remote sensing, particle physics plasma physics, velocity measurements and bio-molecular interactions. In present unit we only discuss the optical interferometry. Interferometers are devices use for different measurement of path difference, fringe widths, refractive index and many other parameters with the help of interference phenomenon.

6.4. FRINGES OF EQUAL INCLINATION (HAIDINGER FRINGES)

Before going ahead, we should understand the fringe formation in a interferometer. As we know the interference fringes are formed due to a path difference $\Delta = 2\mu t \, Cos \, r$ between the overlapping rays. Now for a particular wavelength, the path difference may occur due to variation of thickness t and angle of inclination r.

$$\delta\Delta = 2\mu\Delta t \cos r + 2\mu t\ \delta\ (Cos\ r) \qquad .... \ (6.1)$$

In case of a film with constant thickness then variation in path difference occurs as

$$\delta\Delta = 2\mu t\ \delta\ (Cos\ r) \qquad .... \ (6.2)$$

Thus the path difference occurs with the variation in the angle of inclination r. If we use an extended source of light, we have a large numbers of rays comes with equal angle of inclination r, which produces a particular path difference and fringes are observed corresponding to this path difference. Such fringes are called fringes of equal inclination. In case of Michelson interferometer, the thickness of film remains constant then the fringes are formed due to equal inclination and hence called fringes of equal inclination or Haidinger fringes.

6.5 MICHELSON INTERFEROMETER

Michelson interferometer is a device used for the formation and study of interference fringes by a monochromatic light. In this apparatus, a beam of light coming from an extended source of light is divided into two parts, one is reflected part and another is refracted part after passing through a partially polished glass plate. These two beams are brought together after reflected from plane mirrors, and finally interference fringes are produced in the field of view.

6.5.1 Construction

The apparatus is shown in Figure 6.1. The main part of the apparatus is a half silvered glass plate P, on which a beam of monochromatic light is incident. The plate P inclined at an angle 45^0 with incident light as shown in figur6.1, the incident light then divided into two parts, one is reflected part and another is transmitted part. The transmitted light is then passes through another glass plate Q which is of equal thickness as of P, and parallel to plate P, this pate Q is called compensating plate. The transmitted and reflected parts of light are normally incident on two mirrors M_2 and M_1 respectively. The mirror M_1 and M_2 are perpendicular to each other as shown in figure. The mirror M_1 is fixed in a carriage and can be moved to and fro with help of a screw and micro scale. Therefore mirror M_1 is movable and the mirror M_2 is fixed. A telescope is also fixed as shown in figure. The light reflected from mirror M_1 and M_2 are superimposed and interference fringes are formed in the field of view.

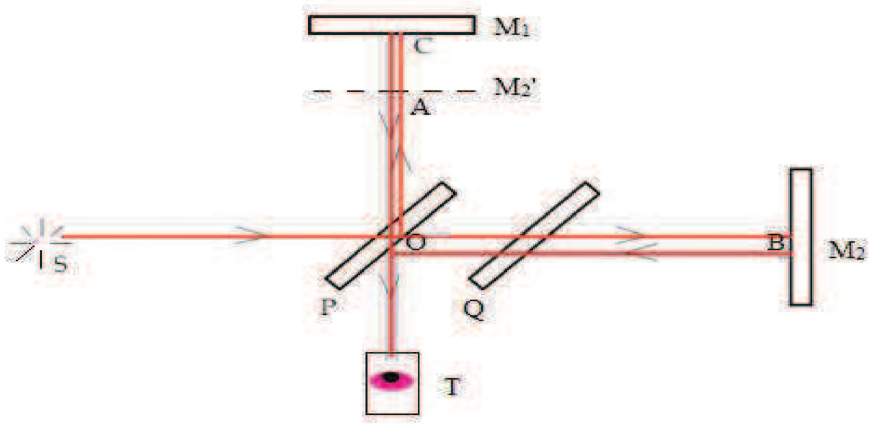

Figure 6.1

6.5.2. Working

S is a source of monochromatic light; the light coming from this source is rendered parallel by mean of a convex lens L, and after passing through Less L the light falls on plate P. Since plate P is partially polished, some part of light reflected back from P and going toward direction AC and incident on mirror M_1.

Similarly the light transmitted from plate P passing through compensating plate Q and then incident on mirror M_2. The compensating plate is used to compensate the optical path travelled by transmitted light. The beam of light reflected by P, crosses plate P two times, for transmitted light this optical path is compensated by using plate Q in which the transmitted light crosses Q two time. Thus by using compensating plate Q, the reflected and transmitted light travel equal optical path lengths.

Now the reflected light is incident on mirror M_1 and reflected back towards the telescope T. Similarly the transmitted light incident normally on mirror M_2 and reflected back towards plate P, and at P some part of this light again reflected toward the telescope. Now in the direction of telescope we have two coherent beams of light reflected from mirror M_1 and M_2, and interference takes place and we observed interference pattern/beam in the field of view.

6.5.3 Formation of Fringes

Since the fringes are form by the light reflected from mirror M_1 (movable) and M_2 (fixed) and we can consider a virtual image of M_2 called M_2' in the field of view as shown in figure 6.1. Further we can consider the interference fringes are now formed due to light reflected from the surface of air film formed between mirror M_1 and M_2'. Now it is clear that the shapes of fringes are depend upon the inclination of mirror M_1 and M_2. Since M_2 fixed therefore the shape are depends upon the inclination of M_1. Since OA = OB, therefore the path difference between two rays is simply the path traveled in air film before reaching to telescope. If t is the thickness of air film then path difference between light reflected from M_1 and M_2 is $2t$.

Condition for maxma $\Delta = 2t = n\lambda$

$$2t = n\lambda$$

If the movable mirror M_1 moved by a distance x and we observed fringes shift of N fringes then

$$2(t + x) = (n + N)\lambda$$

or $$2x = N\lambda$$

or $$\lambda = \frac{2x}{N} \quad \text{....... (6.3)}$$

It is clear that if M_1 and M_2 are exactly perpendicular to each other, then M_1 and M_2' are parallel to each other and air film between M_1 and M_2' is of equal thickness in this case we observed fringes of equal inclination or Haidinger's fringes of circular shape. If however, the two mirror M_1 and M_2 are not exactly perpendicular to each other then the shape of the air film formed between mirror M_1 and M_2' is of wedge shaped and the fringes are now of straight line parallel to the edge of wedge. This straight line fringes are because of the focus of constant thickness in a wedge shape film is a straight line.

Thus the shapes of fringes are depends on the inclination. The fringes are in general curved and convex toward the edge of wedge as shown in figure 6.2. These fringes are called localized fringes.

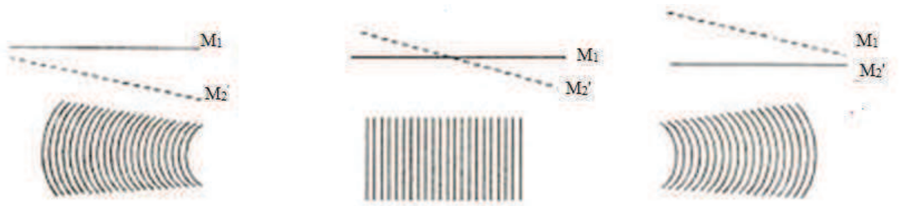

Figure 6.2

6.5.4 Determination of Difference of Wavelengths between Two Neighboring Wavelengths

Let us consider a source of light which emits two very close wavelengths. Sodium light is an example of such case. In sodium light, there are two wavelength D_1 *and* D_2 lines with wavelength λ_1 = 5890Å and λ_2 = 5896Å. By using Michelson interferometer we can determine the difference between these two wavelengths. In this case first we adjust the aperture for circular fringes. We know that each wavelength produce its own ring spectrum. Now the mirror M_1 is moved in such a way that when the position of very bright fringes are obtained. In this position the bright fringes due to λ_1 coincident with the bright fringes due to λ_2 and we observe distinct fringes of order n.

Now the mirror M_1 is further moved to a very small displacement, and the fringes are disappeared. This case occurs when the maxima due to λ_1 coincident on minima due to λ_2. This is the position of minimum intensity or uniform illumination with no clear fringes. In this case we observed indistinct fringes of order (n+1). If we moved a distance x between such two points of most bright and most indistinct fringes then

$$2x = n\lambda_1 = (n+1)\lambda_2$$

or $$n = \frac{\lambda_2}{\lambda_1 - \lambda_2}$$

or $$2x = \frac{\lambda_1\lambda_2}{\lambda_1 - \lambda_2}$$

or $$\lambda_1 - \lambda_2 = \frac{\lambda_1\lambda_2}{2x} \qquad \text{....... (6.4)}$$

If λ_1 and λ_2 are very close to each other then

$$\lambda_1 . \lambda_2 = \lambda^2$$

Where λ is the mean value of λ_1 and λ_2

$$\lambda = \frac{\lambda_1 + \lambda_2}{2}$$

Then $$\Delta\lambda = \lambda_1 - \lambda_2 = \frac{\lambda^2}{2x} \qquad \text{....... (6.5)}$$

6.5.5 Determination of Refractive Index of a Material

In Michelson interferometer, the two interfering beam of light travel in different directions, one is toward mirror M_1 and second one is toward mirror M_2. It is very easy to introduce a thin transparent sheet of a material of refractive index is and thickness t, in the path of one of the interfering beams of light. After introducing a sheet, the optical path of that beam increases by μt. Now the net increase in the path is $(\mu t - t)$. Since the beam crosses the sheet twice, the net path difference becomes $2(\mu t\text{-}t)$.

If n is the number of fringes be which the fringe system is displaced, then

$$2(\mu t - t) = n\,\lambda$$

or $$2(\mu - 1)t = n\,\lambda \qquad \text{....... (6.6)}$$

In experiment we first locate the central dark fringe by using while light. The cross wire of telescope is adjusted in such a way that the cross wire of telescope is adjusted on central dark fringes. Now the light is replaced by a monochromatic light of wavelength λ. Now a thin sheet is introduced into the path of one beam. The position of movable mirror M_1 is adjusted in such a way that the dark fringe is again coincide with the cross wire of telescope. We note the distance d through which the mirror is moved and count number of fringes displaced. By using the relation given below we can determine the thickness of sheet.

$$t = n\,\lambda / 2(\mu - 1) \qquad \text{....... (6.7)}$$

Similarly if we know the thickness, we can determine the refractive index of material.

$$2(\mu - 1)\,t = n\,\lambda$$

$$\mu = (n\,\lambda / 2t) + 1 \qquad \text{....... (6.8)}$$

6.5.6 Michelson Morley Experiment and Its Result

In classical mechanics it was assumed that the preferred medium for light propagation is ether which filled in all space uniformly. The ether is perfectly transparent medium of light and material bodies may pass in this medium without any resistance. Ether remains fixed in space and consider as absolute frame of reference. In the 19th century this ether drag hypothesis of light was widely discuss.

Michelson interferometer was originally designed to verify the existence of hypothetical medium ether. The experiment performed to verify this hypothesis is called Michelson Morley experiment. In this experiment, it was assumed that the Michelson interferometer is moving along the earth direction of motion. Due to motion of apparatus with transmitted light are not same. Mathematically the path difference between two ray (transmitted and reflected) is lv^2/c^2 where l is distance between plate P and mirror M_1 and v is velocity of ether corresponding to this path difference there should be a fringe shift of n = 0.37. Thus if the apparatus is at rest and starts motion, there should be a fringe shift of n = 0.37. But it is not possible to make earth at rest. In this experiment we consider if the whole apparatus was turned by 90^o, the fringe shift should be observed.

The experiment was performed by many scientists, many times at different location on earth but fringe shift was not observed. This is called negative result of Michelson Morley experiment. The result shows the non existence of hypothetical medium of ether. After this experiment, a foundation of modern though way lay down which led to Einstein theory of relativity.

Self Assessment Questions

1. What is an interferometer?
2. What is the role of compensating plate in Michelson interferometer?
3. How the air film is formed in Michelson's interferometer?
4. How the path difference is calculated in Michelson's interferometer?
5. Why fringes are circular in Michelson's interferometer?
6. What is the meaning of localized fringes?
7. What happens when white light is used in Michelson's interferometer?
8. Determine the thickness of a thin transparent film with the help of Michelson's interferometer.
9. Determine the refractive index of a material with the help of Michelson's interferometer.
10. If the mirrors M1 and M2 of Michelson's interferometer are exactly perpendicular to each other, how will be the shape of fringes?
11. How you will find the wavelength of a monochromatic light with Michelson's interferometer.
12. Give the application of Michelson's interferometer.

6.6 SOLVED EXAMPLES

6.1. In Michelson interferometer, when movable mirror M_1 is shifted by a distance 0.030mm, a fringe shift of 100 fringes is observed. Calculate the wavelength of light used.

Solution: In Michelson interferometer if the mirror is displaced by a distance x, the corresponding fringe shift N is

$$2x = N\lambda \qquad \text{or} \qquad \lambda = 2x/N = 2(0.030)/100 = 6000Å$$

6.2. The difference between two wavelengths of sodium light lines D_1 *and* D_2 is determined with the help of Michelson intereferometer. If the distance travelled by movable mirror for two successive position of most distinct and most indistinct position is 0.2945 mm calculate the difference between two wavelengths D_1 *and* D_2, the mean wavelength of two lines is 5893Å

Solution: If the displacement between two position of mirror for two successive position of most distinct and most indistinct position is x then

$$\lambda_1 - \lambda_2 = \frac{\lambda^2}{2x} = (5893 \times 5893)/(2 \times 0.2945 \times 10^7) = 6Å$$

6.3. Reflective index of a glass plate is to be determined by the help of Michelson interferometer. If is observed that when the glass plate is introduced, a fringe shift of 140 is observed. If the length of glass plate is 20cm and the wavelength of light is 5460Å, calculate the refractive index of material.

Solution: when a glass plate is introduce in one of the interfering ray of Michelson's interferometer then a fringe shift is observed as

$2(\mu - 1)\, t = n\lambda$ or $\mu = (n\,\lambda/2t) + 1 = [(140\times5460\times10^{-10}) \div (2\times20\times10^{-8})] + 1 = 1.0029$

6.4. In Michelson interferometer 790 fringes cross the field of view when the movable mirror is displaced through a distance 0.233mm. Calculate the wavelength of light used.

Solution: In Michelson interferometer if movable mirror is displaced through a distance x, the corresponding fringe shift n is given as

$$2x = n\lambda \qquad \text{or} \qquad \lambda = 2x/n = 2\times 0.233/790 \text{ mm} = 5896Å$$

6.7 SUMMARY

1. Interferometer is a device used for measurement of path difference, fringe width, refractive index, wavelength of a monochromatic light source and many other parameters with the help of interference phenomenon.
2. In Michelson's interferometer, an air film is formed with the help of two perpendicular mirrors. The light reflected from two mirrors M_1 and M_2 is equivalent to light reflected from the upper and lower surface of air film formed between mirror M_1 and M_2'.
3. The condition for bright fringes is given as $2x = N\lambda$
   Where, x = displacement of mirror M_1, N = number of fringe shifts on displacement of x, λ = wavelength of light used.
4. In Michelson interferometer if M_1 and M_2 mirror are exactly perpendicular to each other, the shape of fringes are circular which are called fringes of equal inclination or Haidinger fringes. If however, two mirrors are not perpendicular to each other, the shape of film formed between M_1 and M_2' is of wedge shape and the fringes are straight line or localised.
5. The difference between two neighboring wavelength of a source is given as
   $$\Delta\lambda = \lambda_1 - \lambda_2 = \frac{\lambda^2}{2x}$$
6. The refractive μ index of a medium can be determine by
   $2(\mu - 1)\, t = n\lambda$ or $\mu = (n\,\lambda/2t) + 1$
7. The thickness t can be determine by, $t = n\,\lambda/2\,(\mu - 1)$

6.8 GLOSSARY

Interferometer: A device used for measurement of path difference, fringe width, wavelength of light, refractive index etc. with the help of interference phenomenon.

Inclination: Degree of sloping, slope

Haidinger fringes: The fringes of equal of inclination.

Compensating plate: A plate used in Michelson interferometer for compensating the path difference in transmitted light raised due to glass plate.

6.9 REFERENCE

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2. C.L. Arora and P.S. Hemene, Physics for Degree Students, S. Chand and Company, New Delhi.

3. http://wikipedia.org

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6.10 SUGGESTED READING

1 Prank S.J. Pedrotte, Introduction to Optics, Pentice Hall India limited

2. Ajay Ghtak, Optics, McGraw Hill Company, NEW Delhi.

6.11 TERMINAL QUESTIONS

Short Answer Type Questions

1. Describe the construction of Michelson interferometer.
2. Describe the working of Michelson interferometer.
3. How Michelson's interferometer may be used to obtain circular and streight line fringes.
4. Explain why circular fringes shift in the field of view when we move the mirror M_1.
5. Outline the theory of Michelson's interferometer.
6. With the help of Michelson interferometer how the D_1 and D_2 lines of sodium light can be distinguished. Find out the difference between D_1 and D_2 lines of sodium light.
7. How the refractive index of a medium can be determined with the help of Michelson interferometer.
8. Explain the method of determine the thickness of sheet/fill with the help of Michelson interferometer.
9. Explain the role of compensation plate in Michelson's interferometer.
10. What are localised fringes in Michelson's interferometer?
11.

Long Answer Type Questions

1. 1 With the help of neat diagrams, describe the construction and working of Michelson's interferometer.
2. Explain the working of Michelson's interferometer. How the interferometer produces straight line and circular fringes.
3. Give the applications of Michelson's interferometer in detail.
4. Explain how circular fringes are produced in Michelson's interferometer. Show that the radii of circular fringes obtained by the Michelson's interferometer are proportional to the square root of natural number.

Numerical Type Questions

1. 1. Calculate the displacement between two successive positions of movable mirror giving the best fringes in case of sodium light. [Answer: 0.029cm]
2. 2. In Michelson's interferometer when movable mirror is displaced through a distance 0.589mm, a fringe shift of 200 is observed across the cross wire in the field of view. Calculate the wavelength of light used. [Answer: 5890A]
3. Determine the difference between the wavelengths of two D_1 and D_2 lines in sodium light. The wavelengths of D_1 and D_2 lines are 5896 Å and 5890 Å respectively. The scale reading of two successive distinct and indistinct points are 0.6939mm and 0.9884mm. [Answer: 6Å]
4. In Michelson's Interferometer when movable mirror is displaced through a distance 0.844mm a fringe shift of 300 is observed. Calculated the wave length of light used. [Answer: 562Å]
5. 3. Determine the difference between the wavelengths of two D_1 and D_2 lines in sodium light. The wave length of D_1 and D_2 are 5896 Å and 5890 Å respectively. The scale readings of two successive distinct and indistinct points are 0.6939mm and 0.9884mm. [Answer: 6 Å]
5. In Michelson's Interferometer when movable mirror is displaced through a distance 0.844mm a ping shift of 300 is observed. Calculated the wave length of light used.

[Answer: 562 Å]

Objective Type Questions

1. In Michelson interferometer, when mirror M_1 and M_2 are perpendicular to each other, then the shape of the fringes are
   (a) Straight line (b) Circular
   (c) elliptical (d) inclined
2. The use of compensating plate in the Michelson Interferometer is
   (a) To make path difference equal between light beams reflected from mirror M_1 and M_2
   (b) To make frequency equal between light beams reflected from mirror M_1 and M_2
   (c) To make path difference $\frac{\lambda}{2}$ between light beams reflected from mirror M_1 and M_2

(d) To make path difference λ between light beams reflected from mirror M_1 and M_2

[Answers 1(b), 2(a)]

UNIT-7: DIFFRACTION OF LIGHT WAVES AND FRESNEL DIFFRACTION

CONTANTS

7.1 INTRODUCTION

In the preceding units we have read, that the interference phenomenon arises when two or more coherent light beams, obtained either by division of wavefront or by division of amplitude, meet each other. In this unit we shall discuss the interference effect of secondary wavelets originating from the same wavefront or from single aperture. This is called diffraction. The wave nature of light was further confirmed by the phenomenon of diffraction.

Diffraction refers to various phenomena which occur when a wave encounters an obstacle or a slit (or aperture). Since at the atomic level, physical objects have wave-like properties, they can also exhibit diffraction effects. The diffraction of light was first observed and characterized by an italian mathematician Francesco Maria Grimaldi. The word diffraction originated from Latin word 'diffractus' which means 'to break into pieces'. Thus he referred this phenomenon as breaking up of light into different directions. Isaac Newton attributed them to inflexion of light rays. James Gregory used a bird feather and observed the diffraction patterns. This was effectively the first diffraction grating to be discovered. Augustin-Jean Fresnel did more studies and calculations of diffraction and thereby gave great support to the wave theory of light that had been advanced by Christiaan Huygens.

The effects of diffraction are often seen in everyday life. For example, the closely spaced tracks on a CD or DVD act as a diffraction grating for incident light and form a rainbow like pattern when seen at it. The hologram on a credit card is another example. Almost the same colourful pattern is formed due to the diffraction of light. A bright ring around a bright light source like the sun or the moon is because of the diffraction in the atmosphere by small particles.

7.2 OBJECTIVES

Upon completion of this unit you will be able to

- State the diffraction of light and the necessary conditions for producing this effect
- Differentiate the phenomena interference and diffraction
- Describe the Fresnel and Fraunhofer classes of diffraction
- Define the construction of half period zones and compute their radii and area
- Find the resultant amplitude at a point on the screen due to a number of zones
- Prove that the light propagate along a rectilinear path
- Describe a zone plate, its construction, its action and theory.
- List the similarities and dissimilarities between a zone plate and a lens
- To understand 'what kind of diffraction effect is produced by a sharp straight edge at various points on the screen'
- Find the expressions for the positions of maxima and minima, and the intensity distribution due to diffraction effect produced by a sharp edge

7.3 DIFFRACTION OF LIGHT

As per the rules of geometric optics, the light should caste a well defined and distinct shadow of an object placed in its path. If the direction of incidence of light is perpendicular to the length of obstacle then due to its rectilinear propagation, the size of the image should be equal to the size of the object (fig. 7.1). No light should reach into the regions of shadow. The same thing happens with aperture. Light enters from the open region of aperture and reaches to the screen (fig.7.2). When the direction of incidence is not normal to length of obstacle (or aperture), the size of image (or shadow) will be different from that of obstacle or aperture (fig.7.3 and 7.4).

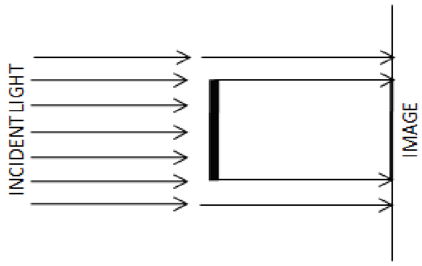

Figure 7.1

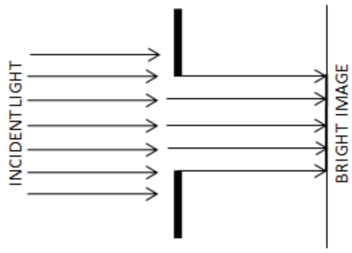

Figure 7.2

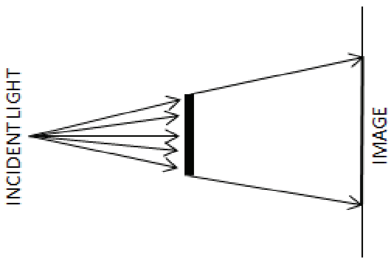

Figure 7.3

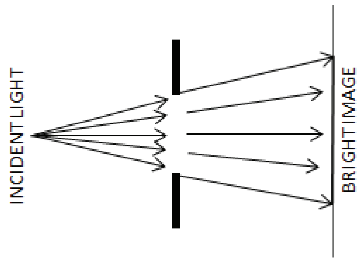

Figure 7.4

A very close and careful observation of light distribution reveals that there are dark and bright fringes near the edges. As the size of the aperture is decreased the fringes become more and more distinct. When the size of aperture becomes comparable to the wavelength of incident light the fringes become broad and practically cover the entire shadow region, so instead of a sharp shadow we obtain bright and dark fringes on the screen.

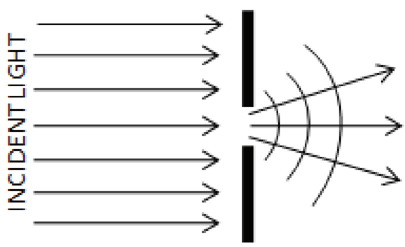

Figure 7.5

In simple language we can say that 'when the size of the opaque obstacle (or aperture) is small enough and is comparable to the wavelength of incident light, the light bends round the corners'. If the opening is much larger than the light's wavelength, the bending will be almost unnoticeable. The phenomenon of bending of light round the corner or edge and spreading into the geometrical shadow region of the obstacle (or aperture), placed in its path, is known as diffraction. The bending of light for a small slit is shown in figure 7.5. The formation of alternate bright and dark fringes, by the redistribution of light intensity, is called the diffraction pattern. The amount of bending depends on the relative size of the wavelength of light to the size of the opening.

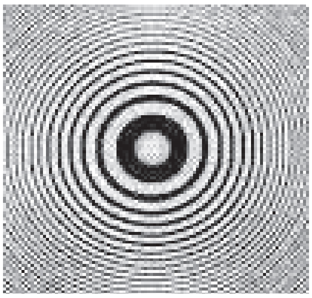

Figure 7.6

Dominique Arago placed a small circular disc in between a point light source and screen and obtained almost a regular pattern of alternate dark and bright rings. There was a bright circular spot at the centre of this pattern. The formation of this kind of diffraction pattern could not be explained on the basis of rectilinear propagation of light. Thus wave theory of light was used to explain the bending of light into the regions of geometrical shadow. One such pattern is depicted in figure 7.6.

Self Assessment Question (SAQ) 1: What do you understand by the term diffraction? What is the condition of obtaining observable diffraction pattern?

7.4 DIFFERENCE BETWEEN INTERFERENCE AND DIFFRACTION

(i) The interference occurs between two separate wavefronts originating from two coherent sources while in the phenomenon of diffraction the interference occurs between the secondary wavelets originating from different points of the exposed part of same wavefront.

(ii) In the interference pattern all the maxima are of the same intensity but in diffraction pattern the intensity of central maximum is maximum and goes on decreasing as we move away.

(iii) The interference fringes are usually equally spaced while the diffraction fringes are never equally spaced.

(iv) In interference the minima are perfectly dark but it is not so in diffraction pattern.

7.5 FRESNEL AND FRAUNHOFER CLASSES OF DIFFRACTION

The diffraction phenomenon is usually divided into two classes; the Fresnel diffraction and Fraunhofer diffraction. Following are the main differences between these two types of diffractions.

(i) In Fresnel diffraction either the source of light or the screen or both are in general at finite distance from the diffracting element (obstacle or aperture) whereas in Fraunhofer diffraction both the source of light and the screen are at infinite distance from diffracting element.
(ii) In Fresnel diffraction no lenses are used for rendering the rays parallel or convergent therefore the incident wavefront is divergent either spherical or cylindrical. In Fraunhofer class of diffraction generally two convergent lenses are used; one to make the incoming light parallel and other to focus the parallel diffracted rays on the screen. The incident wavefront is, therefore, plane.
(iii) In Fresnel diffraction the phase of secondary wavelets is not the same at all points in the plane of aperture while converse is true for Fraunhofer diffraction.
(iv) Depending on the number of Fresnel's zones formed, the centre of the diffraction pattern may be either dark or bright in Fresnel diffraction but in Fraunhofer diffraction it is always bright for all paths parallel to the axis of lens.
(v) In Fresnel class of diffraction the lateral distances are important while in Fraunhofer diffraction the angular inclination plays important role in the formation of diffraction pattern.
(vi) In Fresnel diffraction the diffraction pattern formed is a projection of diffracting element modified by the diffracting effects and the geometry of the source and in Fraunhofer diffraction the diffraction pattern is the image of the source modified by the diffraction at diffracting element.

SAQ 2: How will you differentiate the interference and diffraction phenomenon?
SAQ 3: Write any four differences between Fresnel and Fraunhofer class of diffraction.

7.6 FRESNEL'S HALF PERIOD ZONES

According to Huygens principle each point on a wavefront acts as a source of secondary disturbance. When a wavefront is made to incident on a slit, most of it is obstructed by the slit. The small portion of the wavefront passed through the slit is, thus, equivalent to a string of coherent point sources. The intensity at any point on the screen may be obtained by suitably summing the intensities of wavelets originating from those point sources at the slit and superposing at that point of screen. Thus diffraction pattern is formed at screen due to the interference of secondary wavelets.

Since the coherent sources are located at different distances from any point on the screen, the waves reach that point with differing phases. Their superposition produces interference pattern with maxima and minima formation. Therefore, the diffraction of light is due to the superposition of waves from coherent sources of the same wavefront after the wavefront is obstructed by obstacle or aperture.

7.6.1. Construction of Zones

For the qualitative understanding of the diffraction pattern, Fresnel introduced the idea of half period zones. The wave-front originated from the source and striking the obstacle or aperture is divided into a number of the circular and the concentric zones. Zone is the small area on the plane wave-front with reference to the point of the observation such that all the waves from the area reach the point without any path difference. The paths of light rays from the successive zones differ by *λ/2*. Since path difference of *λ/2* corresponds to half time period, these zones are known as half period zones.

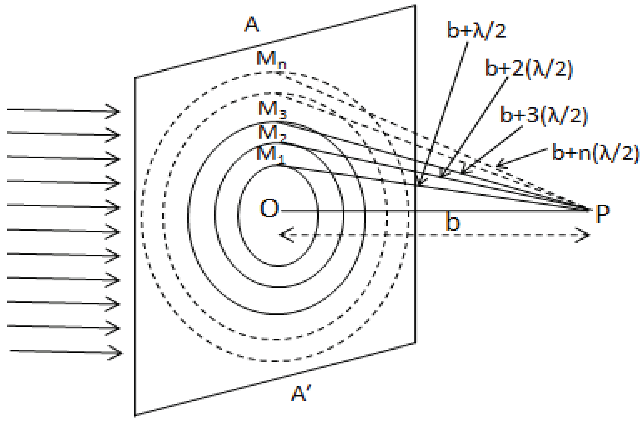

Figure 7.7

In order to understand the construction of half period zones taking a plane wavefront AA' and droping a perpendicular PO on the wavefront from an external point P. If the distance PO is *b* then taking *P* as a centre draw spheres of radii *b+λ/2*, *b+2(λ/2), b+3(λ/2) etc.* The spheres will cut the wavefront AA' in circles of radii OM_1, OM_2, OM_3 etc as shown in figure 7.7. The annular regions between two consecutive circles are called *half period zones,* e.g., the annular region between $(n\text{-}1)^{th}$ circle and n^{th} circle is called the n^{th} half period zone.

7.6.2. Radii and Area of Zones

From simple geometry the radius of n^{th} such circle, OM_n, can be written as

$$OM_n = r_n = \left[\left(b + n\frac{\lambda}{2}\right)^2 - (b^2)\right]^{1/2}$$

$$= \sqrt{n\lambda b}\left[1 + \frac{n\lambda}{4b}\right]^{1/2} = \sqrt{n\lambda b} \qquad \text{....... (7.1)}$$

Here we have assumed *b>> λ,* which is true in most of the experiments using visible light. We have also assumed here that *n* is not a very large number. From expression given by equation (7.1), it is clear that the radii of half period zones are proportional to the square roots of natural numbers. Therefore, the radii of first, second, third etc. half period zones are $\sqrt{\lambda b}$, $\sqrt{2\lambda b}$, $\sqrt{3\lambda b}$ etc

With the help of equation (7.1), the area of n^{th} half period zone is given by

$$A_n = \pi r_n^2 - \pi r_{n-1}^2 = \pi[n\lambda b - (n-1)\lambda b] = \pi\lambda b \qquad \text{.......} \ (7.2)$$

Thus for $b >> \lambda$ and n not very large, the areas of half period zones are independent of n and are approximately equal for fixed value of λ and b. The area of the zone may be varied by varying the wavelength of light used and the distance of the point from the wavefront.

Example 7.1. A screen is placed at a distance of 100 cm from a circular hole illuminated by a parallel beam of light of wavelength 6400 Å. Compute the radius of fourth half period zone.

Solution: If b is the distance of the point of consideration from the pole on the wavefront then the radii of the spheres whose sections cut by by the wavefront from the half period zones are $b + \frac{\lambda}{2}$, $b + \frac{2\lambda}{2}$, $b + \frac{3\lambda}{2}$ etc. Hence the radius of fourth half period zone is given by

$$r_4 = \sqrt{\left(b + \frac{4\lambda}{2}\right)^2 - b^2} = \sqrt{(4\lambda^2 + 4b\lambda)} \cong \sqrt{4b\lambda}. \text{ Because } 4b\lambda \gg \lambda^2$$

It is given that, $b = 100\ cm$ and $\lambda = 6400$ Å $= 6400 \times 10^{-8}\ cm$

$\therefore \qquad r_4 = \sqrt{4 \times 100 \times 6400 \times 10^{-8}} = 0.16\ cm$

Example 7.2: A plane wavefront of light of wavelength 1000 Å is allowed to pass through an aperture and a diffraction pattern is obtained on the screen placed at a distance of 1m from aperture find the radius and area of 1000th half period zone.

Solution: Given that $\lambda = 1000 \times 10^{-10}$ m $= 10^{-7}$ m, b = 1m and n =1000

We know that the radius of nth zone is given by, $r_n = \sqrt{nb\lambda}$

$\therefore \qquad r_{1000} = \sqrt{1000 \times 1 \times 10^{-7}} = 10^{-2}\ m = 1.0\ cm$

The area of zone = $\pi b\lambda = 3.14 \times 1 \times 10^{-7} = 3.14 \times 10^{-7}\ m^2$

Example 7.3: A light of wavelength 5×10^{-7} m is made to incident on a hole. Calculate the number of half period zones lying within the hole with respect to a point at a distance of 1.0 m from the hole if the radius of hole is (i) 10^{-3} m and (ii) 10^{-2} m.

Solution: It is given that $\lambda = 5 \times 10^{-7}$ m, b = 1 m. If A_n is the area of hole of radius r_n containing n-half period zones each of area $\pi b\lambda$ then, we have, $A_n = \pi r_n^2 = n.\ \pi b\lambda$

(i) For $r_n = 10^{-3}$ m,

Substituting in the above equation, we get,

$$\pi \times (10^{-3})^2 = n \times \pi \times 1 \times 5 \times 10^{-7}$$

$$\therefore \qquad n = \frac{10^{-6}}{5 \times 10^{-7}} = 2$$

(ii) For $r_n = 10^{-2}$ m

$$\pi \times (10^{-2})^2 = n \times \pi \times 1 \times 5 \times 10^{-7}$$

$$\therefore \qquad n = \frac{10^{-4}}{5 \times 10^{-7}} = 200$$

SAQ 4: The radius and area of n^{th} zone are 1.0 cm and $3.14\text{x}10^{-7}$ m^2. Find the value of n.

SAQ 5: Light of 5000 Å is passed through a hole and two half period zones are formed with respect to a point at a distance of 1.0 m from the hole. Calculate the diameter of the hole.

7.6.3. Resultant Amplitude at Point P

According to Fresnel the resultant amplitude at any point due to whole of the wavefront will be the combined effect of all the zones, while the amplitude produced by a particular zone is proportional to the area of the zone and inversely proportional to the distance of the zone from the point of consideration, *P*. This amplitude also varies with obliquity factor $\frac{1}{2}(1 + cos\theta)$. Where θ is the angle between the normal *PO* to the wavefront and the line *QP*. Thus if u_n represents the amplitude produced by the secondary wavelets emanating from the n^{th} zone then we can write

$$u_n = (Constant) \times \frac{A_n}{Q_n P} \times \frac{(1+cos\theta_n)}{2} \qquad \text{....... (7.3)}$$

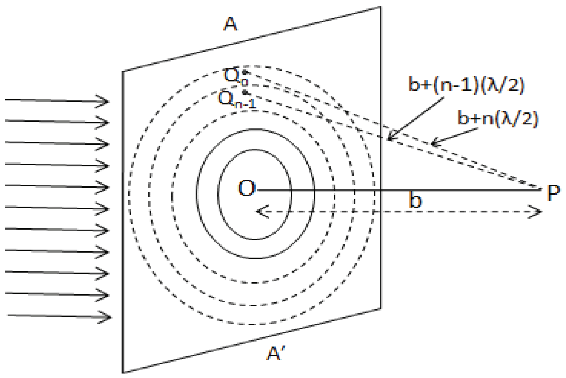

Figure 7.8

Where θ_n is the value of θ for n^{th} zone. If we take infinitesimal areas around point Q_n in the n^{th} half period zone and around a corresponding similar point Q_{n-1} in $(n-1)^{th}$ half period zone as shown in the figure 7.8 such that

$$Q_n P - Q_{n-1} P = \lambda/2 \qquad \text{....... (7.4)}$$

This path difference of $\lambda/2$ corresponds to a phase difference of π. Although the areas of the zones are almost the same but the distance of the zone from point *P* and the value of θ increases as we move from lower to higher *n*. The amplitudes u_1, u_2, u_3 etc. of 1^{st}, 2^{nd}, 3^{rd} etc. zones at point *P* will be, therefore, in gradually decreasing order as shown in figure 7.9. The opposite directions of alternate amplitudes correspond to the phase change of π between consecutive zones.

Thus the resultant amplitude at *P* can be written as

$$u_p = u_1 - u_2 + u_3 - u_4 + \ldots\ldots\ldots + (-1)^{n+1}u_n \qquad \ldots\ldots. (7.5)$$

The positive and negative signs on the right hand side between alternate terms of this equation may be ascribed to the fact that the disturbances produced by two consecutive zones at P will be out of phase by π radians.

As the disturbances at P due to various zones are of gradually decreasing magnitudes, the amplitude due to any zone may be taken approximately equal to the average of the amplitudes due to the preceding zone and the succeeding zone. That is, we can take

$$u_2 = \frac{u_1+u_3}{2},\ u_4 = \frac{u_3+u_5}{2} \text{ etc.} \qquad \ldots\ldots. (7.6)$$

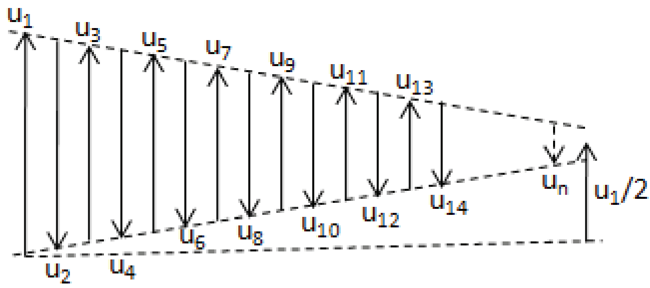

Figure 7.9

In equation (7.5), the last term on right hand side will be positive if n is odd and negative if it is even. We can rewrite equation (7.5) as

$$u_p = \frac{u_1}{2} + \left(\frac{u_1}{2} - u_2 + \frac{u_3}{2}\right) + \left(\frac{u_3}{2} - u_4 + \frac{u_5}{2}\right) + \ \ldots \qquad \ldots\ldots. (7.7)$$

Thus if n is odd we have, $u_p = \frac{u_1}{2} + \left(\frac{u_1}{2} - u_2 + \frac{u_3}{2}\right) + \ \ldots\ \ldots + \left(\frac{u_{n-2}}{2} - u_{n-1} + \frac{u_n}{2}\right) + \frac{u_n}{2}$

Using equation (7.6), we get, $u_p = \frac{u_1}{2} + \frac{u_n}{2}$ .... (7.8)

And if n is even then,

$$u_p = \frac{u_1}{2} + \left(\frac{u_1}{2} - u_2 + \frac{u_3}{2}\right) + \ \ldots\ \ldots + \left(\frac{u_{n-3}}{2} - u_{n-2} + \frac{u_{n-1}}{2}\right) + \frac{u_{n-1}}{2} - u_n$$

Using equation (7.6), we have, $u_p = \frac{u_1}{2} + \frac{u_{n-1}}{2} - u_n$ .... (7.9)

If the number of half period zones formed is large enough then due to gradually decreasing amplitudes of zones, the values of u_n and u_{n-1} may be neglected as compared to u_1, and therefore we can write

$$u_p \cong \frac{u_1}{2} \qquad \ldots\ldots. (7.10)$$

And the intensity at point P, therefore, may be given by

$$I_p \cong u_p^2 = \frac{u_1^2}{4} \qquad \ldots\ldots. (7.11)$$

Thus the resultant amplitude produced by whole of the wavefront is equal to one half of that produced by the first zone and the intensity due to the entire wavefront is the one fourth of that by the first zone.

Example 7.4: A plane wavefront of light of wavelength $5x10^{-5}$ cm falls on a circular hole and is received at a point 200 cm away from that hole. Calculate the radius of the hole so that the amplitude of light on the screen is two times the amplitude in the absence of hole.

Solution: It is given that λ = $5x10^{-5}$cm = $5x10^{-7}$ m and b = 200 cm = 2.0 m

We know that the amplitude due to the whole wavefront is only half to that due to first half period zone, therefore

Radius of hole = Radius of first half period zone = $\sqrt{b\lambda} = \sqrt{(2.0 \times 5 \times 10^{-7})} = 10^{-3}m = 1.0\ mm$

SAQ 6: The radius of an opening is $4.47x10^{-2}$ cm. The light of wavelength λ is passed through that opening and collected at a distance of 40 cm from opening. Calculate the wavelength of light so that the intensity of light on the screen is four times the intensity in the absence of the opening.

7.7. RECTILINEAR PROPAGATION OF LIGHT

With the help of the theory discussed so far we can explain the rectilinear propagation of light. Suppose a plane wavefront of monochromatic light is made to incident on a screen with square aperture *ABCD* and whole of the wavefront except *ABCD* portion is blocked by the screen as shown in the figure 7.10. Let *P* be a point at which the intensity of the light is required and its pole *O* with respect to the aperture *ABCD* is well inside from the edges. Taking *O* as centre if we draw the half period zones in the incident wavefront then the number of the wavefronts will be quite large before they intersect the edges *AB*, *BC*, *CD*, and *DA*. Thus practically all the effective zones are exposed and the resultant amplitude at *P* due to aperture *ABCD* is given by equation (7.10). This amplitude is equal to the one half that due to the first zone and since the areas of these zones are extremely small, we can consider the light to be travelling along a straight line along *OP*. This condition is the same as if the screen with square aperture *ABCD* was removed.

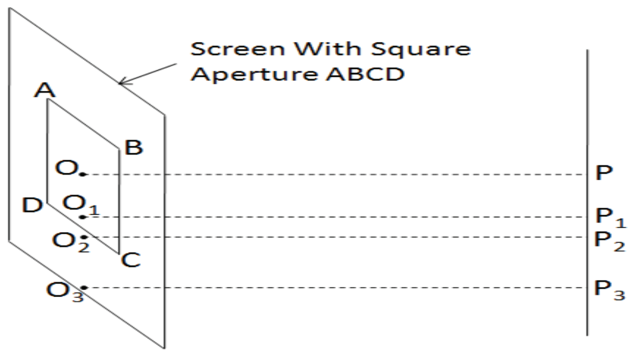

Figure 7.10

The poles O_1 and O_2 of the points like P_1 and P_2 on the screen lie very close to edges of the aperture *ABCD*. If we draw the half period zones around these poles then some of the zones are obstructed and some are exposed. Thus there will be neither uniform illumination nor complete darkness at points P_1 and P_2. For the points near the edges the light, therefore, enters into the geometrical shadow region. The point P_3 is well inside the geometrical shadow region and its pole is O_3. Since the amplitude at a point due to a zone decreases on increasing its order, almost all the effective zones around O_3 are cut off. The amplitude reaching at P_3 is nearly zero and there is a complete darkness. This is possible only when light travels along a straight line.

From the above mentioned facts this may be concluded that there is almost uniform illumination at the points whose poles lie well inside the edges of the aperture and complete darkness at the points whose poles lie well outside the edges. This strongly supports the rectilinear propagation of light. There is a slight deviation from the rectilinear path for the points whose poles lie very close to the edges. However due to very small value of the wavelength of light this region is very small as compared to whole of the aperture. Thus as a whole the propagation of the light may be considered along a rectilinear path.

7.8. ZONE PLATE

A zone plate is a device used to focus light; however zone plates use diffraction instead of refraction or reflection as in case of lenses and curved mirrors. It is a specially designed diffraction screen consisting of a large number of half period zones. In the honor of Augustin-Jean Fresnel they are sometimes called Fresnel zone plates. It is constructed in such a way that every alternate zone blocks the light incident on it. In other words we can say that it consists of alternate opaque and transparent set of radially symmetric rings (zones).

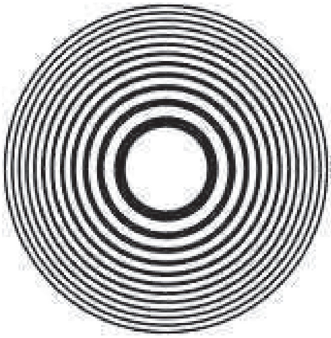

Figure 7.11

Figure 7.12

The zones can be spaced so that the diffracted light constructively interferes at the desired focus. The light may be cut off either by even numbered zones or by odd numbered zones. When the light is obstructed by even numbered zones the plate is known as positive zone plate and when obstructed by odd numbered zones it is called negative zone plate. These two kinds of zone plates are shown in figures 7.11 and 7.12.

7.8.1. Construction and Theory of Zone Plate

From equation (7.1) of section 7.6.2, it is evident that the radii of half period zones are proportional to square roots of natural numbers. Thus to construct a zone plate, we draw the concentric circles of the radii proportional to square roots of natural numbers on a white paper. The alternate regions between the circles are painted black. If the odd numbered zones are painted black then drawings appears like figure 7.12 and if even numbered zones are covered with black ink then the drawing looks like figure 7.11. Suppose the drawing resembles with figure 7.11. If we take a reduced photograph of it then the developed negative resembles with figure 7.12. This negative is then used as a zone plate.

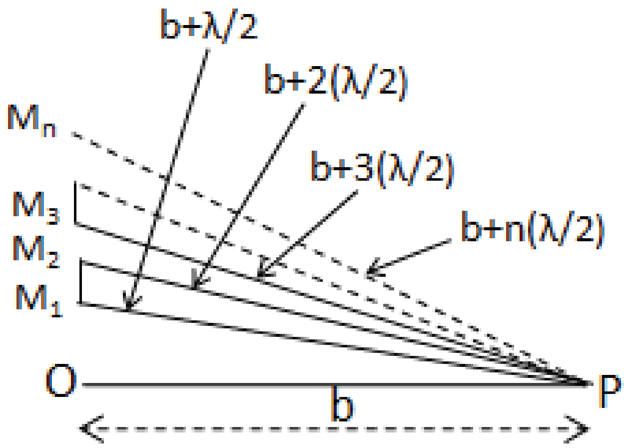

Figure 7.13

If a beam of light is made to incident on such a zone plate normally and a screen is placed on the other side of this plate to get an image then the maximum brightness is obtained at a particular point of the screen. Suppose this point is P at a distance of b units from the zone plate as shown in figure 7.13. Only upper half portion of the zone plate is shown in this figure. If λ is the wavelength of light used then radius of the first zone ($OM_1=r_1$), second zone ($OM_2=r_2$) etc are given by $r_1 = \sqrt{b\lambda}$ and $r_2 = \sqrt{2b\lambda}$ etc.

The general expression for radius may be written as

$$r_n = \sqrt{nb\lambda} \text{ or } b = \frac{r_n^2}{n\lambda} \qquad \text{....... (7.12)}$$

Since the wavelength of light has a small value, the sizes of the zones are usually very small as compared to the distance of the light source from the zone plate. Hence OM_1, OM_2, OM_3 etc are extremely small as compared to distance *a* (source *S* to zone plate *AB* separation). But to make the points M_1, M_2, M_3 etc distinct and to show the complete figure the distances are not taken in this ratio in figure 7.14. Because of this reason the incident wavefront may be taken as a plane wavefront.

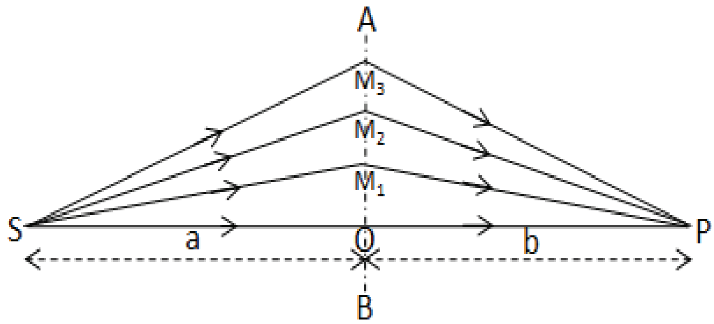

Figure 7.14

Now suppose even numbered zones are opaque to incident light then from equation (7.5), the resultant amplitude reaching at *P* may be written as (*n* is odd)

$$u_p = u_1 + u_3 + u_5 + \ldots\ldots\ldots + u_n \qquad \text{....... (7.13)}$$

In this case if all the zones are transparent to light then from equation (7.5), the resultant amplitude at *P* is given by

$$u_p = u_1 - u_2 + u_3 - u_4 + \ldots\ldots\ldots + u_n \qquad \text{....... (7.14)}$$

For large value of *n*, from equation (7.10), we have,

$$u_p = \frac{u_1}{2} \qquad \text{....... (7.15)}$$

If we compare the values of the resultant amplitudes from equations (7.13) and (7.15), we find that, when the even numbered zones are opaque the intensity at point *P* is much greater than that when all the zones are transparent to incident light. Again from the above discussion we can state that a zone plate behaves like a converging lens. The focal length of the zone plate may be given by

$$f_n = b = \frac{r_n^2}{n\lambda} \qquad \text{....... (7.16)}$$

Therefore, the focal length of a zone plate varies with the wavelength of incident light that is why it is called a multi foci zone plate. For this reason if white light is made to incident on a zone plate different colours come to focus on screen at different points and it shows chromatic aberration.

7.8.2. Action of a Zone Plate

Refer to figure 7.14; AB is the section of zone plate perpendicular to the plane of paper, *S* is the point light source at a distance *a* from zone plate and point *P* is on the screen placed at a distance *b* from the zone plate. As compared to the radii of zones, the distance of source from the zone plate is extremely large and therefore we can take approximation as $SO \approx SM_1 \approx SM_2 \ldots\ldots = a$. The position of the screen is chosen such that the light rays reaching at *P* from successive zones have a path difference of $\lambda/2$. We can write

$$SO + OP = a + b \qquad \ldots\ldots\ (7.17)$$

$$SM_1 + M_1P \approx SO + (OP + \lambda/2) = a + b + \lambda/2 \qquad \ldots\ldots\ (7.18)$$

Similarly, $$SM_2 + M_2P = a + b + 2\,\lambda/2 \qquad \ldots\ldots\ (7.19)$$

Now from right angle triangle ΔSOM_1, we have,

$$(SM_1)^2 = (SO)^2 + (M_1O)^2 \quad \text{or} \quad SM_1 = (a^2 + r_1^2)^{1/2} = a(1+\frac{r_1^2}{a^2})^{1/2}$$

Since $a >> r_1$, expanding above and neglecting higher order terms, we get,

$$SM_1 = a(1+\frac{r_1^2}{2a^2}) = (a+\frac{r_1^2}{2a}) \qquad \ldots\ldots\ (7.20)$$

Proceeding in a similar way we can obtain,

$$M_1P = (b+\frac{r_1^2}{2b}) \qquad \ldots\ldots\ (7.21)$$

Substituting values of SM_1 and M_1P from equations (7.20) and (7.21) in the left hand side of equation (7.18), we get,

$$(a+\frac{r_1^2}{2a}) + (b+\frac{r_1^2}{2b}) = a + b + \lambda/2$$

or $$r_1^2\left(\frac{1}{a}+\frac{1}{b}\right) = \lambda$$

From equation (7.19), we have, $$r_2^2\left(\frac{1}{a}+\frac{1}{b}\right) = 2\lambda$$

Proceeding similarly for higher order zones, we obtain

$$r_n^2\left(\frac{1}{a}+\frac{1}{b}\right) = n\lambda \qquad \ldots\ldots\ (7.22)$$

Now comparing the zone plate with converging device like convex lens and using similar sign convention for the distances of the object and image from the lens, the equation (7.22) may be modified as

$$\left(\frac{1}{b}-\frac{1}{a}\right)=\frac{n\lambda}{r_n^2} \qquad \text{....... (7.23)}$$

This equation is similar to the lens equation $\left(\frac{1}{v}-\frac{1}{u}\right)=\frac{1}{f}$. Thus a zone plate behaves like a converging lens of focal length, $f_n=\frac{r_n^2}{n\lambda}$. Thus the focal length of zone plate depends on the number of zones and the wavelength of light used.

7.8.3. Multiple Foci of Zone Plate

A zone plate has a multiple foci. In order to prove this, taking an object at infinity, i.e. at $a=\infty$ in equation (7.23), we get, $r_n^2=bn\lambda$ and therefore, the area of n^{th} zone is given by

$$A_n=\pi r_n^2-\pi r_{n-1}^2=\pi[n\lambda b-(n-1)\lambda b]=\pi\lambda b \qquad \text{....... (7.24)}$$

Since the object is at infinity, the light rays will be parallel to principal axis and the image will be formed at the principal focus at a distance $b=\frac{r_n^2}{n\lambda}$ from the zone plate.

If we take a point P_3 at a distance $b/3$ from the zone plate somewhere in between O and P then the area of each half period zone with respect to P_3 will now becomes $\pi\lambda(b/3)$, that is, one third to the previous case. Thus each zone, in this case, can be assumed to contain three half period elements corresponding to P_3. If the amplitude due to these elements are represented by m_1, m_2, m_3 etc. then the first zone (amplitude u_1) will consist of the first three elements (amplitudes m_1, m_2 and m_3), second zone (amplitude u_2) will consist of the next three elements (amplitudes m_4, m_5 and m_6) etc. Again similar to half period zones there will be a phase difference of π between the successive elements. Thus while adding the amplitudes; the m_1 will be taken positive, m_2 as negative etc. Substituting the values of u_1, u_2, u_3 etc. with m_1, m_2, m_3 etc., equation (7.13) changes to

$$u_{p_3}=(m_1-m_2+m_3)+(m_7-m_8+m_9)+(m_{13}-m_{14}+m_{15})+\ \dots\dots\dots$$

$$=\left(m_1-\frac{m_1+m_3}{2}+m_3\right)+\left(m_7-\frac{m_7+m_9}{2}+m_9\right)+\left(m_{13}-\frac{m_{13}+m_{15}}{2}+m_{15}\right)+\dots\dots$$

$$=\frac{1}{2}(m_1+m_3+m_7+m_9+m_{13}+m_{15}+\ \dots\dots) \qquad \text{....... (7.25)}$$

Here it should be noted that each of the amplitudes m_1, m_2, m_3 etc is one third of u_1, u_2, u_3 etc.

If we compare the equations (7.13) and (7.25), we find that the intensity reaching at P_3 is sufficiently large but is less than that reaching at P. Thus the image of S is also formed at P_3 and therefore, it may be taken as the second focal point. The second focal length is given by

$$f_3=\frac{r_n^2}{3n\lambda} \qquad \text{....... (7.26)}$$

Similarly the images of S can be formed on points P_5, P_7, P_9 etc. but with decreasing intensity. The distance of these points from the zone plate are $\frac{r_n^2}{5n\lambda}, \frac{r_n^2}{7n\lambda}, \frac{r_n^2}{9n\lambda}$ etc. Thus a zone plate has multiple foci given by $f_1 = \frac{r_n^2}{n\lambda}$, $f_3 = \frac{r_n^2}{3n\lambda} = \frac{f_1}{3}$, $f_5 = \frac{r_n^2}{5n\lambda} = \frac{f_1}{5}$ etc.

7.8.4. Comparison of Zone Plate and Lens

Some of the features of zone plate are similar to a lens and in some it has dissimilarity. The following are the resemblance and differences between the two.

(i) Similar to a lens, a zone plate forms an image of an object placed on its axis. The same sign convention is used while representing the distance of the object and image in both the cases.

(ii) The focal length formula in terms of distance of object and image for zone plate is $\left(\frac{1}{b} - \frac{1}{a}\right) = \frac{1}{f}$ and for the convex lens is $\left(\frac{1}{v} - \frac{1}{u}\right) = \frac{1}{f}$, which are identical.

(iii) The image due to a convex lens is more intense as compared to that due to a zone plate.

(iv) The convex lens has a focal length given by $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ which depends on wavelength (refractive index varies with wavelength) and the focal length of zone plate $f_n = \frac{r_n^2}{n\lambda}$ also varies with wavelength. Hence both exhibit chromatic aberration.The focal length of a zone plate is inversely proportional to the wavelength hence red rays come to focus at a smaller distance from the zone plate than violet rays. The reverse is true for convex lens. Thus $f_v > f_r$ in zone plate while $f_r > f_v$ in lens. The order of colours in chromatic aberration is therefore opposite in the two cases.

(v) A convex lens has one focal length for a fixed wavelength while a zone plate has a number of foci at which the images of diminishing intensities are formed.

Example 7.5: Calculate the focal length of the zone plate and the radius of the first zone when a point source of light of wavelength $6x10^{-7}$ m is placed at a distance of 100 cm from a zone plate. Its image is formed at a distance of 200 cm on the other side.

Solution: For a zone plate we have. $\left(\frac{1}{a} + \frac{1}{b}\right) = \frac{n\lambda}{r_n^2} = \frac{1}{f}$. Given that, a = 1 m, b = 2 m and $\lambda =$ $6x10^{-7}$ m. Thus $\frac{1}{f} = \frac{1}{1} + \frac{1}{2} = \frac{3}{2}$ or $f = \frac{2}{3}\ m$.

For first zone, $f = \frac{{r_1}^2}{1\times\lambda}$, thus $r_1^2 = f \times \lambda = \frac{2}{3} \times 6x10^{-7}$ or $r_1 = 6.32x10^{-4}$ m.

Example 7.6: A plane wavefront of light of wavelength $5x10^{-5}$ cm fall on a zone plate. The radius of the first half period zone is 0.5 mm. Where should a screen be placed so that the light is focused at the brightest spot?

Solution: We know that the brightest spot is formed at the first focus of the plate, i.e. at f_1. Given that $r_1 = 0.5\ mm = 5x10^{-2}\ cm$ and $\lambda = 5 \times 10^{-5}\ cm$

$$f_n = \frac{r_n^2}{n\lambda}\text{, Therefore, } f_1 = \frac{{r_1}^2}{\lambda} = \frac{\left(5\times10^{-2}\right)^2}{5\times10^{-5}} = 50\ cm$$

Example 7.7: Calculate the radius of 10th zone in a zone plate of focal length 0.2 m for light of wavelength $5x10^{-7}$ m.

Solution: From $f_n = \frac{r_n^2}{n\lambda}$, we have, $0.2 = \frac{{r_{10}}^2}{10\times5\times10^{-7}}$ or $r_{10} = 0.01\ m = 1.0\ cm$

Example 7.8: Calculate the radii of first three clear elements of a zone plate which is designed to bring a parallel light of wavelength 6000 Å to its first focus at a distance of two meters.

Solution: It is Given that, $f = b = 2.0\ m$, $\lambda = 6000\text{Å} = 6\times10^{-7}\ m$.

If odd number half period zones are clear (transparent) then taking n=1, 3, 5 in the expression $r_n = \sqrt{nb\lambda}$, we get $r_1 = \sqrt{f\lambda} = \sqrt{6\times10^{-7}\times2} = 10.95\times10^{-4}$ m.

$r_3 = \sqrt{3f\lambda} = \sqrt{3\times6\times10^{-7}\times2} = 1.9\times10^{-3}$ m.

$r_5 = \sqrt{5f\lambda} = \sqrt{5\times6\times10^{-7}\times2} = 2.45\times10^{-3}$ m.

SAQ 7: What is the radius of first zone in a zone plate of primary focal length 20 cm for a light of wavelength 5000 Å.

SAQ 8: If the focal length of zone plate is 1 m for light of wavelength $6.0x10^{-7}$ m. What will be its focal length for the wavelength $5x10^{-7}$m.

7.9. DIFFRACTION AT A STRAIGHT EDGE

To show the diffraction effect of a straight edge, the light from a monochromatic light source *S* is passed through a narrow slit *AB* and a sharp edge of an opaque obstacle like blade is placed in its path as shown in figure 7.15. The slit, opaque obstacle and screen $P'P''$ are parallel to each other and perpendicular to the plane of the paper. The sharp edge is placed in such a way that the line joining the slit to edge *O* when reproduced meet the screen at *P* and *OP* is normal to screen.

In the absence of diffraction of light due to sharp straight edge there should be a uniform illumination above point *P* and complete darkness below it. As we move towards *P'*, unequally spaced bright and dark bands are obtained near *P*. On further moving towards *P'*, i.e. with increasing value of *x* the intensity reaches a steady value I_o resulting a uniform illumination. Because of the diffraction effect, the light enters to a certain distance below *P* (towards *P''*) in the geometrical shadow region.

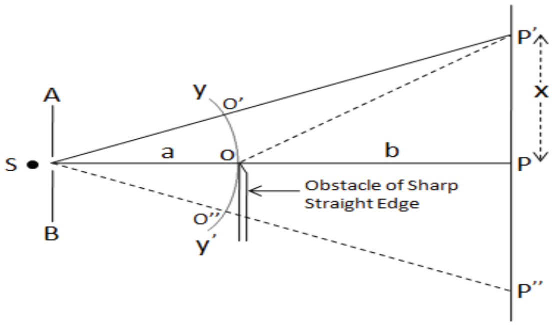

Figure 7.15

In this region the intensity of light decreases to zero very rapidly without forming maxima and minima in a small but finite distance as shown in intensity distribution curve of figure 7.16. If the average intensity is I_o then at point *P* on the screen (corresponding to the edge) it reduces to $I_o/4$. This all is due to the diffraction of light produced by sharp straight edge.

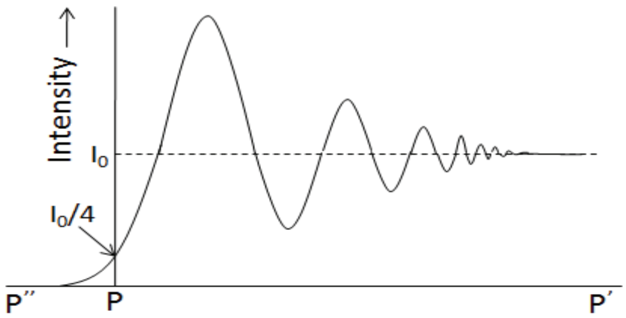

Figure 7.16

7.9.1. Theoretical Analysis

Refer to figure 7.15. Suppose we want to find the resultant at any point, say *P'*, on the screen. The pole of the wavefront *YY'* with respect to point *P'* will be *O'*. With *P'* as centre if we draw the circles of radii $O'P'+\lambda/2$, $O'P'+2\lambda/2$, $O'P'+3\lambda/2$ etc, the wavefront is divided into half period strips. Thus for point *P*', the wavefront is divided in two similar parts; one above point *O'* another below it. The light from entire upper half portion of the wavefront reaches to *P'*. The resultant due to this will be equivalent to one half to that due to first half period strip, i.e. $m_1/2$. Now the number of half period strips within the lower half portion of the wavefront, i.e. *O'O* will depend on the position of the point *P'* on the screen. Suppose the lower half portion contains only one half period strip then the amplitude due to it at *P'* will be only m_1 and therefore, the total amplitude at *P'* by whole of the exposed wavefront is given by $\frac{m_1}{2}+m_1$. This is the position of first maximum.

If *O'O* contains two, three, four etc half period strips then the resultant amplitude at *p'* is given by $\frac{m_1}{2}+m_1-m_2, \frac{m_1}{2}+m_1-m_2+m_3, \frac{m_1}{2}+m_1-m_2+m_3-m_4$ etc. and the position of *P’* gives the position of first minimum, position of second maximum and the position of second minimum respectively. Thus at point *P’*, a maximum or a minimum is formed according as *O’O* contains odd or even number of half period strips.

As we move away from *P* towards *P’* alternate maxima and minima are obtained. From the previous discussion we see that the amplitude or intensity of these maxima and minima are comparable, hence the bands have a poor contrast. If the point of consideration is at a sufficiently large distance from *P* then entire upper half and a large number of half period strips of the lower half are exposed. The diffraction bands merge together to produce uniform illumination. The resultant amplitude at the point of consideration, in this case, is therefore, $\frac{m_1}{2}+\frac{m_1}{2} = m_1$ and the intensity is $m_1{}^2$.

7.9.2. Positions of Maximum and Minimum Intensities

In figure 7.15, the path difference between the rays $O^{'}P^{'}$ and $OP^{'}$ is given by

$$\Delta = OP' - O'P' = (OP^2+PP'^2)^{1/2} - (SP' - SO') = (OP^2+PP'^2)^{1/2} - \left[\{SP^2+PP'^2\}^{1/2} - SO'\right]$$

$$= (b^2+x^2)^{1/2} - \left[\{(a+b)^2+x^2\}^{1/2} - a\right]$$

∵ Y*Y’* is the spherical wavefront of the point light source *S* with *S* as a centre, thus *SO’* = *SO* = a, is the radius of the sphere.

In actual experimental set up we have, *x*<<*b*. Thus taking *b* out (common) from the first term and *(a+b)* out from the second term on the right hand side of the above equation, expanding the series and neglecting higher order terms, we obtain

$$\Delta = b\left\{1+\frac{x^2}{2b^2}\right\} - (a+b)\left\{1+\frac{x^2}{2(a+b)^2}\right\} + a = \frac{x^2}{2}\cdot\frac{a}{b(a+b)} \quad .... \ (7.27)$$

Now if *O’O* contains an odd number of half period strips then a maximum will be formed at point *P’* and the path difference Δ, in this case, will be an odd number of half-wavelengths, and vice-versa. Thus for maxima we have,

$$\Delta = (2n-1)\frac{\lambda}{2} \quad .... \ (7.28)$$

For minima we have, $\Delta = 2n.\frac{\lambda}{2}$ .... (7.29)

On comparing equations (7.27) and (7.28), we get the position of n^{th} maximum as

$$x_n = \sqrt{\frac{(2n-1)(a+b)b\lambda}{a}} = K\sqrt{2n-1} \quad .... \ (7.30)$$

Where, $K = \sqrt{\frac{(a+b)b\lambda}{a}}$, is a constant.

Similarly the comparison of equations (7.27) and (7.29) gives the position of n^{th} minimum as

$$x_n = \sqrt{\frac{2n(a+b)b\lambda}{a}} = K\sqrt{2n} \qquad \text{....... (7.31)}$$

From equation (7.30), we have, $x_1 = K,\ x_2 = K\sqrt{3}, x_3 = K\sqrt{5}$ etc. Thus the separations between successive maxima are $x_2 - x_1 = 0.732K, x_3 - x_2 = 0.504K, x_4 - x_3 = 0.409$ etc. We see that with increasing order of maxima the separation between consecutive maxima decreases and the fringes come closer. The same is true for minima.

7.9.3. Intensities at Various Positions

The intensity variation curve is shown in figure 7.16. Now we will find out the value of intensity at some specific points.

(i) Intensity at the Edge of Geometrical Shadow

In figure 7.16 the edge of geometrical shadow is represented by *P*. The pole of this edge at wavefront is point *O*, which is nothing but the edge of sharp obstacle. Thus with respect to the edge of geometrical shadow region (point *P*), the incident wavefront can be divided in two parts; one above point *O* (*OY)* and other below point *O* (*OY'*). The light from the entire upper half portion of the wavefront reaches to point *P* while the light from the lower half portion of the wavefront is completely cut off by sharp edge obstacle. The resultant amplitude at *P*, in this case, is $m_P = m_1 - m_2 + m_3 + m_4 - \ldots\ldots\ldots$, *which is* $m_1/2$. Thus the resultant intensity at *P* is $m_1^2/4 = I_o/4$. Where I_o is the value of intensity at *P* in the absence of obstacle.

(ii) Intensity at a Point Inside the Geometrical Shadow

If the point of consideration is inside the geometrical shadow region then the pole of the point will be below point *O,* i.e. in the wavefront region *OY'*. Suppose we take a point *P''* then its pole will be *O''*. In this case the complete lower half portion and most of the upper half portion of the wavefront is obstructed by the obstacle. Only a small part of the upper half portion of the wavefront (*OY*) is exposed. As we move down gradually from point *P* inside geometrical shadow, the first, the first two, the first three etc. half period strips of the upper half of the wavefront are obstructed and the amplitudes are thus $m_2/2$, $m_3/2$, $m_4/2$ etc. respectively. The intensities, therefore, will be $(m_2/2)^2$, $(m_3/2)^2$, $(m_4/2)^2$ etc. respectively.

Since the amplitudes m_1, m_2, m_3 etc. are in decreasing order of magnitude, the intensity of light decreases rapidly as we move inside the geometrical shadow. This is because of the fact that most of the effective half period strips of the upper half portion of wavefront are cut off.

Example 7.9: A narrow slit illuminated by light of wavelength 4900 Å is placed at a distance of 3m from a straight edge. If the distance between the straight edge and screen is 6 m, calculate the distance between the first and fourth band.

Solution: For minima we have, $x_n = \sqrt{\frac{2n(a+b)b\lambda}{a}} = K\sqrt{2n}$, where $K = \sqrt{\frac{(a+b)b\lambda}{a}}$

It is given that b = 6 m, a = 3 m, λ = 4.9x10^{-7} m.

Therefore, $K = \sqrt{\frac{(3+6)\times 6\times 4.9\text{x}10^{-7}}{3}} = 2.97x10^{-3}$

For first minimum, $x_1 = K\sqrt{2} = 2.97\times 10^{-3}\times\sqrt{2} = 4.20\times 10^{-3}m$

For fourth minimum, $x_4 = K\sqrt{8} = 2.97\times 10^{-3}\times\sqrt{8} = 8.40\times 10^{-3}$

Separation between the two, $x_4 - x_1 = (8.40 - 4.20)\ x\ 10^{-3} = 4.20\times 10^{-3}m$

SAQ 9: In an experiment with straight edge diffraction, the slit to edge distance is 1.0 meter and the edge to screen distance is 2.0 m. If $\lambda = 6000$ Å, calculate the position of the first three maxima and their separation.

7.10. SUMMARY

In this unit you have studied that Huygens's principal is the basic principle to explain the diffraction phenomenon. Diffraction is mainly due to interference of the secondary wavelets. Diffraction pattern is formed whenever a wave encounters an object or aperture, the size of which is comparable to wavelength of light. To make the concept more clear the difference between interference and diffraction, construction and theory of half period zones and zone plate are explained. It is stated that for $b>>\lambda$ the radii of half period zones are proportional to square root of natural numbers and the zones have the same areas. The expressions for radius and area are given by $\sqrt{n\lambda b}$ and $\pi\lambda b$. If the incident wavefront contains a large number of half period zones and all zones are exposed then the resultant amplitude at a point on the screen will be equal to half of that due to first zone, i.e. $u_1/2$. With the help of zone theory it is proved that the light propagates along a rectilinear path. The zone plate may be used as a focusing device and the focal length of it is given by the expression $\frac{1}{f_n} = \left(\frac{1}{b} - \frac{1}{a}\right) = \frac{n\lambda}{r_n^2}$. It is a multiple foci device having focal lengths $\frac{r_n^2}{n\lambda}, \frac{r_n^2}{3n\lambda}, \frac{r_n^2}{5n\lambda}$ etc. In some of the features, the zone plate, resembles with a lens and has some dissimilarity.

The formation of diffraction pattern is explained by taking the obstacle in the form of a sharp and straight edge. If almost all the wavefront is exposed, the amplitude produced at a point on the screen is m_1 and the intensity is m_1^2. The maxima and minima formed are not equally spaced. Their position of maxima is given by $x_n = \sqrt{\frac{2n(a+b)b\lambda}{a}} = K\sqrt{2n}$ and that of minima is given by $x_n = \sqrt{\frac{(2n-1)(a+b)b\lambda}{a}} = K\sqrt{2n-1}$. If I_o is the value of intensity at a point on the screen in the absence of obstacle then $I_o/4$ will be the intensity at the edge of geometrical shadow. If we move inside the geometrical shadow region the intensity decreases and diminishes to zero rapidly.

7.11 GLOSSARY

Annular – ring-shaped, forming a ring.

Aperture – an opening, a gap or a space through which light passes in an optical or photographic instrument.

Ascribe – attribute or impute, regard as belonging.

Attribute – ascribe to or regard as the effect of (a stated cause).

Convention – general agreement, esp. on social behaviour etc. by implicit consent of the majority, a custom or customary practice esp. an artificial or formal one.

Converse – opposite, contrary, reverse.

Depict – to describe.

Distinct – not identical, separate, individual, different in kind or quality, unlike.

Emanate – issue, originate (from a source), proceed.

Evident – plain or obvious (visually or intellectually), manifest.

Illumination – an act to light up or to make bright.

Inflexion – the act or condition of inflecting or being inflected, an instance of this.

Lateral – of, at, towards, or from the side or sides, in direct line.

Monochromatic – light or other radiation of single wavelength, containing only one colour.

Obstruct – block up, make hard or impossible to pass along or through.

Opaque – not transmitting light, impenetrable to light.

Rectilinear – bounded or characterized by straight lines, in or forming a straight line.

Render – cause to be or become, make.

Respectively – in the order mentioned, for each separately or in turn.

Reveal – display or show, allow to appear, disclose, divulge, betray.

Vary – undergo change (become or be different).

7.12 TERMINAL QUESTIONS

1. Calculate the radii and areas of the first two half period zones for a plane wavefront. The point of observation is at a distance of 1.0 m from theb wavefront and wavelength of light is 4900 Å.

2. The diameter of the first ring of a zone plate is 1.1 mm. If plane waves (6000 Å) fall on the plate, where should the screen be placed so that light is focused to a brightest spot?

3. A light of wavelength 5000 Å is allowed to fall on a zone plate for which the radius of the first zone is $3\text{x}10^{-2}$ cm. Find the first three focal lengths for this zone plate.

4. Light of wavelength 5896 Å is made to incident on a zone plate placed at a distance of 150 cm from it. The image of the point source is obtained at a distance of 3 m on the other side. What will be the power of equivalent lens which may replace the zone plate withought disturbing the set up? Also calculate the radius of the first zone of the plate.

5. For axial point source for a zone plate, a series of images is obtained. If the sharpest image is obtained at 30 cm and the next sharpest at 6 cm on the other side of the source, calculate the distance of the source from the zone plate.

6. For a light of wavelength 4000Å, the brightest image is formed by a zone plate at a distance of 20 cm for an object placed at a distance of 20 cm from it. Calculate the number of Fresnel's zones in a radius of 1 cm of that plate.

7. A point source of $\lambda = 5.5 \times 10^{-7} m$ is placed 2 meters away along the axis of a circular aperture of radius 2 mm. On the other side a screen is moved along the axis from infinity to closer distances. Calculate the first three positions where minima are observed.

8. A parallel beam of wavelength $6\text{x}10^{-7}$ m falls normally on a narrow circular aperture of radius 0.9 mm. At what distance along the axis will the first maximum intensity be observed?

9. A straight edge is placed at a distance of 50 cm from a slit illuminated by monochromatic light of wavelength 5000 Å. If the distance of the screen from the edge is 1.50 m, calculate the positions of first, second, third and tenth bright fringe from the edge of the geometrical shadow. Also find the separation between first-second and second-third bright fringes.

7.13 OBJECTIVE TYPE QUESTIONS

Q1. The bending of light rays round the corners of an obstacle is called

(a) interference (b) polarization

(c) dispersion (d) diffraction

Q2. For obtaining the diffraction pattern the size of the obstacle should be

(a) 10 mm (b) 10^{-1} mm

(c) 10^{-4} mm (d) 0.1 cm

Q3. The phenomenon of diffraction was discovered by

(a) Francesco Maria Grimaldi (b) Isaac Newton

(c) Fraunhofer (d) Huygen

Q4. The tip of a needle does not give a sharp image on the screen because of the following

(a) reflection (b) diffraction

(c) polarization (d) refraction

Q5. Fresnel half period zones differ from each other by a phase difference of

(a) 2π (b) π

(c) $\pi/2$ (d) $\pi/4$

Q6. For a light of wavelength $5x10^{-7}$m, a zone plate of focal length 0.5 m is to be constructed. The radius of first zone will be

(a) 0.25 cm (b) $2.5x10^{-2}$ cm

(c) 0.5 cm (d) $5x10^{-2}$ cm

Q7. The constant area of half period zone is given by

(a) $\pi b\lambda$ (b) $\pi b/\lambda$

(c) $\lambda/\pi b$ (d) $2\lambda/\pi b$

Q8. The first (principal) focal length of a zone plate has least value for the following colour

(a) red colour (b) green colour

(c) violet colour (d) yellow colour

Q9. The focal length of a zone plate is given by the expression

(a) $\frac{r_n}{n\lambda}$ (b) $\frac{r_n^2}{n}\lambda$

(c) $\frac{r_n^2}{n\lambda}$ (d) $\frac{r_n^2}{\lambda}n$

Q10. A zone plate behaves like

(a) concave lens (b) convex lens

(c) plane mirror (d) glass plate

7.14 ANSWERS/HINTS

7.14.1 Self Assessment Questions

1. Refer article 7.3, **2.** Refer article 7.4, **3.** Refer article 7.5,

4. It is given that the radius of n[th] zone is given by $r_n = \sqrt{nb\lambda} = 1.0\ cm = 10^{-2}m$ and the area of zone, $A_n = \pi b\lambda = 3.14 \times 10^{-7}\ m^2$

Thus, $\frac{r_n^2}{A_n} = \frac{nb\lambda}{\pi b\lambda} = \frac{n}{\pi}$ or $n = \pi\,\frac{r_n^2}{A_n} = 3.14 \times \frac{(10^{-2})^2}{3.14\times10^{-7}} = 1000$

5: It is given that $\lambda = 5 \times 10^{-7}$ m, b = 1 m and n = 2. If A_n is the area of hole of radius r_n containing n-half period zones each of area $\pi b\lambda$ then, we have, $A_n = \pi r_n^2 = n.\ \pi b\lambda$

Substituting the given values in the above equation, we get,

$\pi r_n^2 = 2 \times \pi \times 1 \times 5 \times 10^{-7}$ or $r_n = 10^{-3}$ m, thus diameter, $d_n = 2 \times 10^{-3}$ m.

6: The intensity due to whole wavefront is only one fourth to that due to first half period zone, therefore, Radius of opening = Radius of first half period zone = $\sqrt{b\lambda} = 4.47 \text{x} 10^{-2}$ cm, b = 40 cm (given).

Thus, $\lambda = \frac{(4.47\text{x}10^{-2})^2}{b} = \frac{(4.47\text{x}10^{-2})^2}{40} = 5 \times 10^{-5}\ cm$

7: Hint: $f_n = \frac{r_n^2}{n\lambda}$, $\therefore\ r_1 = \sqrt{f\lambda} = 3.16 \times 10^{-4}\ m.$

8: Hint: If *f* and *f'* are the focal lengths for the wavelengths λ and λ' then we have

$f = \frac{r_n^2}{n\lambda}$ and $f' = \frac{r_n^2}{n\lambda'}$. Dividing we get, $f' = f\frac{\lambda}{\lambda'} = 1 \times \frac{6\times10^{-7}}{5\times10^{-7}} = 1.2\ m$

9: For maxima we have, $x_n = \sqrt{\frac{(2n-1)(a+b)b\lambda}{a}} = K\sqrt{2n-1}$, where $K = \sqrt{\frac{(a+b)b\lambda}{a}}$

It is given that b = 2 m, a = 1 m, $\lambda = 6000$ Å $= 6 \times 10^{-7}$ m.

Therefore, $K = \sqrt{\frac{(1+2)\times 2\times 6\text{x}10^{-7}}{1}}$=*1.897x10^-3^m*

For first maximum, $x_1 = K\sqrt{1} = 1.897\times10^{-3}\times\sqrt{1} = 1.897\times10^{-3}m$

For second maximum, $x_2 = K\sqrt{3} = 1.897\times10^{-3}\times\sqrt{3} = 3.286\times10^{-3}m$

For third maximum, $x_3 = K\sqrt{5} = 1.897\times10^{-3}\times\sqrt{5} = 4.243\times10^{-3}m$

Separation between the two, $x_4 - x_1 = (8.40 - 4.20)\ x\ 10^{-3} = 4.20\times10^{-3}m$

7.14.2 Terminal Questions

1. Radii are $7\text{x}10^{-4}$m and $9.9\text{x}10^{-4}$m respectively, and area of each is $1.54\text{x}10^{-6}\text{m}^2$, **2.** 50 cm, **3.** 18 cm, 6 cm, 3.6 cm, **4.** 1.0 dioptre, 0.0768 cm, (**Hint:** Power, $P = \frac{1}{f}$ dioptre where $\frac{1}{f} = \frac{1}{a} + \frac{1}{b}$ and $r_n = \sqrt{fn\lambda}$, **5.** a=30 cm (**Hint:** $\left(\frac{1}{a}+\frac{1}{b}\right) = \frac{n\lambda}{r_n^2}$, Thus $\left(\frac{1}{a}+\frac{1}{30}\right) = \frac{n\lambda}{r_n^2}$ and $\left(\frac{1}{a}+\frac{1}{6}\right) = \frac{3n\lambda}{r_n^2}$), **6.** 2500 (**Hint:** $n = \frac{r_n^2}{f\lambda}$ where f can be calculated by $\left(\frac{1}{a}+\frac{1}{b}\right) = \frac{1}{f}$), **7.** 19.98 m, 3.076 m, 1.664 m (**Hint:** $n = \frac{r^2}{\lambda}\left(\frac{1}{a}+\frac{1}{b}\right)$, $\therefore n = 3.636 + \frac{7.273}{b}$, For first three positions of minima, *n=4, 6, 8)*, **8.** 1.35 m (**Hint:** For parallel beam, $a=\infty$, and for first maximum *n=1*), **9.** $x_1 = 0.173$ cm, $x_2 = 0.300$ cm, $x_3 = 0.66$ cm, $x_{10} = 0.533$ cm, $x_2\text{-}x_1 = \beta_{12} = 0.127$ cm, $\beta_{23} = 0.066$ cm,

7.14.3 Objective Type Questions

1. (b), **2.** (c), **3.** (a), **4.** (b), **5.** (b), **6.** (d), **7.** (a), **8.** (a), **9.** (c), **10.** (b)

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UNIT 8: FRAUNHOFER DIFFRACTION

CONTANTS

8.1 INTRODUCTION

If an opaque obstacle is placed between a source of light and a screen then light bends around the corner of the obstacle into the geometrical shadow. This bending of light is called diffraction. The phenomenon of diffraction depends on the size of the obstacle and the wavelength of the light beam.

Diffraction is one particular type of wave interference, caused by the partial obstruction or lateral restriction of a wave. Not all interferences are diffraction; for example, sound waves emitted by two stereo speakers will interfere with each other if they are of the same frequency and have a definite phase relationship, but this is not diffraction. Diffraction will not occur if the wave is not coherent, and diffraction effects become weaker (and ultimately undetectable) as the size of obstruction is made larger and larger compared to the wavelength. In well-defined cases, a diffraction pattern may be observed. It is necessary to mention here that diffraction is not the same as refraction, although both are phenomena in which a wave does not propagate in a single direction.

8.2 OBJECTIVES

After studying this unit, you will be able to

- have the basic idea of diffraction and its various classes.
- know the diffraction output at various structure like single, double and multiple slit.
- introduce the plane diffraction grating.
- determine the missing orders for diffraction spectra.

8.3 CLASSES OF DIFFRACTION

Based on the distance between source, aperture and screen, and also on the shape of wavefront, diffraction pattern is classified into two classes

1. **Fresnel Diffraction-**If the source of light and the screen are at finite distances from the diffracting aperture, then the wavefront falling on the aperture will not be plane (spherical or cylindrical). The diffraction obtained under this type of arrangement is called Fresnel Diffraction. This type of diffraction is also called near-field diffraction. No lenses are used to make the rays parallel or convergent.

   Fresnel Diffraction is obtained when light suffers diffraction at a straight edge, a thin wire, a narrow slit etc. Both the size and shape of the pattern depends on the distance between the diffracting aperture and the screen.

2. **Fraunhofer Diffraction-**If both the source of light and the screen are effectively far enough from the aperture so that the wavefronts reaching the aperture and the screen can be considered plane. Then the source and the screen are said to be at infinite distances from the aperture. This kind of diffraction is called Fraunhofer Diffraction. This is also called far-field diffraction.

Fraunhofer Diffraction is encountered in the case of gratings that contain number of slits. When the screen is moved, the size of the diffraction pattern changes uniformly while the shape of the pattern does not change.

8.4 FRAUNHOFER DIFFRACTION DUE TO A SINGLE SLIT

Let AB is a slit of width b, the diffracted beam through the slit is tilted at an angle θ with respect to straight direction.

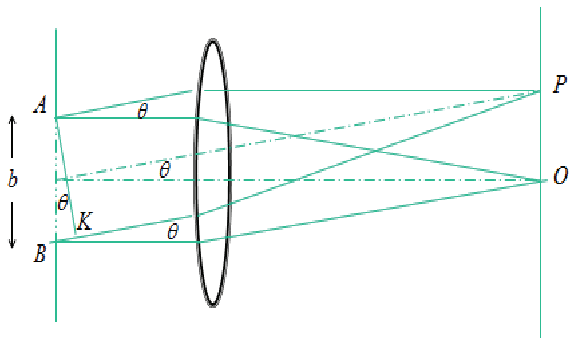

Figure 8.1

Path difference between two rays diffracted from two extreme points of slit

$$= BK = AB\, sin\theta = b\, sin\theta$$

Phase difference $\qquad = \dfrac{2\pi}{\lambda}$ x path difference $= \dfrac{2\pi}{\lambda}(b \sin\theta)$

Let the width AB of the slit be divide into n equal parts. The amplitude of vibration at P due to the waves from each part will be same, say *a*. The phase difference between the waves from any two consecutive parts is

$$\frac{1}{n}\left(\frac{2\pi}{\lambda} b \sin\theta\right) = 2\beta \text{, say}$$

Then the resultant amplitude at P is given by

$$R = \frac{a \sin(nd/2)}{\sin(d/2)} = \frac{a \sin\left(\dfrac{\pi b \sin\theta}{\lambda}\right)}{\sin\left(\dfrac{\pi b \sin\theta}{n\lambda}\right)}$$

Let us put $\qquad \left(\dfrac{\pi}{\lambda} b \sin\theta\right) = \alpha$

Then $\qquad R = \dfrac{a \sin\alpha}{\sin(\alpha/n)} = \dfrac{a \sin\alpha}{\alpha/n} = \dfrac{na \sin\alpha}{\alpha}$ .... (8.1)

When $n \to \infty$, $a \to 0$, but the product *na* remains finite.

Let $\qquad na = A$

The resultant intensity at P, being proportional to the square of the amplitude, is

$$I = R^2 = A^2\left(\frac{\sin\alpha}{\alpha}\right)^2 \qquad \text{.......} \quad (8.2)$$

Condition for Maxima

$$R = \frac{A\sin\alpha}{\alpha} = \frac{A}{\alpha}\left[\alpha - \frac{\alpha^3}{3!} + \frac{\alpha^5}{5!} - \frac{\alpha^7}{7!} + \text{......}\right]$$

$$R = \frac{A\sin\alpha}{\alpha} = A\left[1 - \frac{\alpha^2}{3!} + \frac{\alpha^4}{5!} - \frac{\alpha^6}{7!} + \text{......}\right] \qquad \text{.......} \quad (8.3)$$

For $\alpha = 0$, $R = A$

This is the intensity of central maximum

$$\alpha = \left(\frac{\pi}{\lambda} b\sin\theta\right) = 0 \text{ or } \sin\theta = 0$$

Condition for Minima

$$\frac{\sin\alpha}{\alpha} = 0 \text{ or } \sin\alpha = 0\text{, but } \alpha \neq 0$$

$\alpha = \pm m\pi$, Where m has an integral value 1, 2, 3 except zero

So $$\left(\frac{\pi}{\lambda} b\sin\theta\right) = \pm m\pi \Rightarrow b\sin\theta = \pm m\lambda \qquad \text{.......} \quad (8.4)$$

This equation gives the position of first, second, third etc. minima for m = 1, 2, 3 etc

Secondary Maxima

$$\frac{dI}{d\alpha} = 0$$

or $$\frac{d}{d\alpha}\left[A^2\left(\frac{\sin\alpha}{\alpha}\right)^2\right] = 0$$

or $$A^2\left(\frac{2\sin\alpha}{\alpha}\right)\frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2} = 0$$

$$\frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2} = 0$$

$$\alpha\cos\alpha - \sin\alpha = 0$$

$$\alpha = \tan\alpha = y\ (say)$$

$$y = \alpha \text{ and } y = \tan\alpha$$

The maxima will occur when

$$\alpha = \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$$

or $$\alpha = (2n+1)\frac{\pi}{2} \quad n = 1,2,3..... \qquad .... (8.5)$$

These are points of secondary maxima

$$I = I_0\left(\frac{\sin\alpha}{\alpha}\right)^2 \qquad .... (8.6)$$

Put $$\alpha = \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2} \; etc.$$

$$I_1 = \frac{4}{9\pi^2} I_0, \; I_2 = \frac{4}{25\pi^2} I_0, \; I_3 = \frac{4}{49\pi^2} I_0 \; etc$$

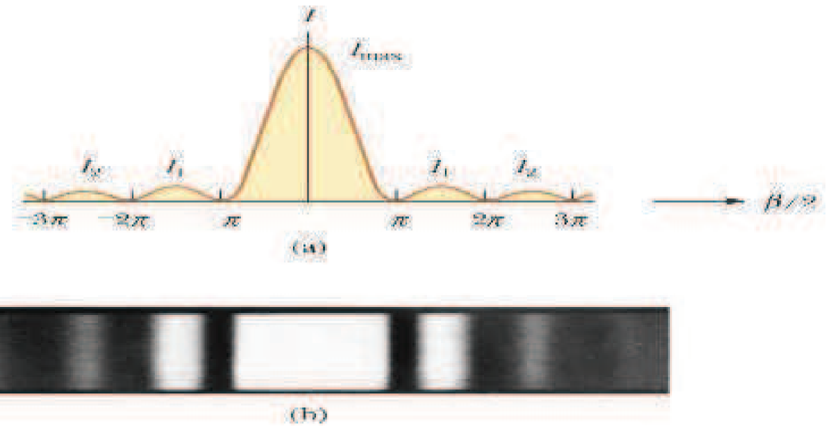

Figure 8.2

8.5 FRAUNHOFER DIFFRACTION DUE TO DOUBLE SLIT

Let a parallel beam of monochromatic light of wavelength λ be incident normally upon two parallel slits AB and CD, each of width *b* and their separation as *d*. The distance between the corresponding points of two slits will be *(b+d)*.

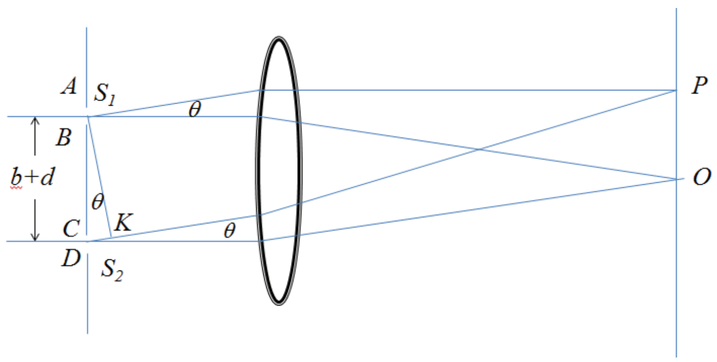

Figure 8.3

Suppose each slit diffracts the beam in a direction making an angle θ with the direction of incident beam. From the theory of diffraction at single slit, the resultant amplitude will be

$$\frac{A \; Sin\,\alpha}{\alpha}$$

Where $$\alpha = \frac{\pi b Sin\ \theta}{\lambda}$$

Now consider the two slits equivalent to two coherent sources, placed at the middle points S_1 and $S_2$of the slits and each sending a wavelet of amplitude $\frac{A\ Sin\ \alpha}{\alpha}$.

Therefore, the resultant amplitude at point P on the screen will be the result of the interference between two waves of same amplitude $\frac{A\ Sin\ \alpha}{\alpha}$ and having a phase difference δ.

∴ Path difference between the wavelets coming from S_1 and S_2in direction θ is given by

$$S_2K = (b+d)\ sin\theta$$

Phase difference = $\frac{2\pi}{\lambda}$ x path difference = $\frac{2\pi}{\lambda}(b+d)\sin\theta) = 2\beta$

Resultant amplitude R at point P can be obtained by vector addition method as

$$I = R^2 = 4A^2 \frac{Sin^2\alpha\ Cos^2\beta}{\alpha^2} \qquad \text{....... (8.7)}$$

Here $\frac{Sin^2\alpha}{\alpha^2}$ gives the diffraction pattern due to each individual slit and $Cos^2\beta$ gives the interference pattern due to double slit. $\frac{Sin^2\alpha}{\alpha^2}$ gives a central maximum in the direction β= 0, having alternate minima and secondary maxima of decreasing intensity on either side.

The minima are obtained in the directions given by

$$Sin\alpha = 0 \qquad \text{or} \qquad \alpha = \pm m\pi$$

$$\because \qquad \alpha = \frac{\pi b\ Sin\ \theta}{\lambda}$$

$$\therefore \qquad b\,Sin\theta = \pm m\pi \qquad \text{....... (8.8)}$$

Where m = 1, 2, 3…. (except zero).

The term $Cos^2\beta$ in the intensity pattern gives a set of equidistant dark and bright fringes.

$$Cos^2\beta = 1$$

$$\therefore \qquad \beta = \pm\ n\pi$$

$$\frac{\pi}{\lambda}(b+d)\sin\theta = \pm\ n\pi$$

$$(b+d)\sin\theta = \pm\ n\lambda \qquad \text{....... (8.9)}$$

Where n = 0, 1, 2, 3…., correspond to zero-, first-, second- etc. order Maxima.

8.5.1 Missing Orders

In the output intensity pattern of a double slit, for certain values of d, few interference maxima become absent.

As, the directions of interference maxima are given by

$$(b + d) \sin \theta = n\lambda \qquad \text{....... (8.10)}$$

The directions of diffraction minima are given by

$$b \sin \theta = m\lambda \qquad \text{....... (8.11)}$$

If the values of *b* and *d* are such that both the equations are satisfied for the same value of a, then a certain interference maximum will overlap the diffraction minimum and hence the spectrum order will be missing (absent).

Dividing equation (8.10) by equation (8.11), we get,

$$\frac{b+d}{b} = \frac{n}{m} \qquad \text{....... (8.12)}$$

If *b=d*

$$\frac{n}{m} = 2 \quad or \ n = 2m. \ If \ m = 1,2,3.......etc., \ then \ n = 2,4,6.......etc$$

This means that the 2, 4, 6 etc. orders of interference maxima will be missing in the diffraction pattern. Thus the central diffraction maxima will have three interference maxima (the zero order and two first-orders).

If d=2b

$$\frac{b+2b}{b} = \frac{n}{m} \quad or \ n = 3m. \ If \ m = 1,2,3.......etc., \quad n = 3,6,9.......etc$$

This means that 3rd, 6th, 9th etc, orders of interference maxima will be missing in the diffraction pattern. On both sides of the central maximum, the number of interference maximum is 2 and hence there will be five interference maxima in the central diffraction maximum.

8.6 FRAUNHOFER DIFFRACTION AT CIRCULAR APERTURE

The problem of diffraction at a circular aperture was first solved by Airy in 1835. The amplitude distribution for diffraction due to a circular aperture forms an intensity pattern with a bright central band surrounded by concentric circular bands of rapidly decreasing intensity (Airy pattern). The 1st maximum is roughly 1.75% of the central intensity. 84% of the light arrives within the central peak called the airy disk

Let us consider a circular aperture of diameter *d* is shown as AB in figure below. A plane wave front WW' is incident normally on this aperture. Every point on the plane wave front in the aperture acts as a source of secondary wavelets. The secondary wavelets spread out in all directions as diffracted rays in the aperture. These diffracted secondary wavelets are converged on the screen SS′ by keeping a convex lens (L) between the aperture and the screen. The screen is at the focal plane of the convex lens. Those diffracted rays traveling normal to the plane of aperture [i.e., along CP_0] are get converged at P_0.

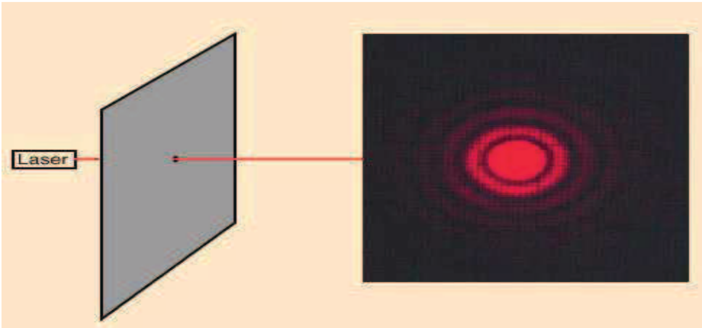

Figure 8.4

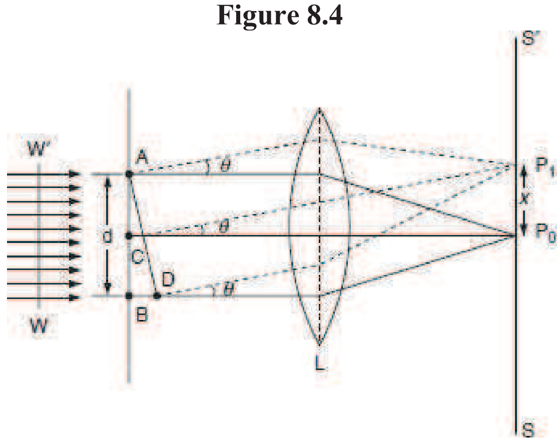

Figure 8.5

All these waves travel some distance to reach P_0 and there is no path difference between these rays. Hence a bright spot is formed at P_0 known as Airy's disc. P_0 corresponds to the central maximum.

Next consider the secondary waves traveling at an angle θ with respect to the direction of CP_0. All these secondary waves travel in the form of a cone and hence, they form a diffracted ring on the screen. The radius of that ring is x and its center is at P_0. Now consider a point P_1 on the ring, the intensity of light at P_1 depends on the path difference between the waves at A and B to reach P_1. The path difference is BD = AB sin θ = d sin θ. The diffraction due to a circular aperture is similar to the diffraction due to a single slit. Hence, the intensity at P_1 depends on the path difference d sin θ. If the path difference is an integral multiple of λ then intensity at P_1 is minimum. On the other hand, if the path difference is in odd multiples of images, then the intensity is maximum.

i.e., $d \sin\theta = n\lambda$, for minima .... (8.13)

and $d \sin\theta = (2n-1)\dfrac{\lambda}{2}$, for maxima .... (8.14)

Where n = 1, 2, 3… etc. n = 0 corresponds to central maximum.

The Airy disc is surrounded by alternate bright and dark concentric rings, called the Airy's rings. The intensity of the dark ring is zero and the intensity of the bright ring decreases as we go radially from P_0 on the screen. If the collecting lens (L) is very near to the circular aperture or the screen is at a large distance from the lens, then

$$Sin\,\theta \approx \theta \approx \frac{x}{f} \qquad \text{.......} \ (8.15)$$

Where, f is the focal length of the lens.

Also from the condition for first secondary minimum [using equation (8.13)]

$$Sin\,\theta \approx \theta \approx \frac{\lambda}{d} \qquad \text{.......} \ (8.16)$$

Equations (8.15) and (8.16) are equal

$$\frac{x}{f} = \frac{\lambda}{d} \ or \ x = \frac{f\lambda}{d} \qquad \text{.......} \ (8.17)$$

But according to Airy, the exact value of x is

$$x = \frac{1.22\,f\lambda}{d} \qquad \text{.......} \ (8.18)$$

Using equation (8.18) the radius of Airy's disc can be obtained. Also from this equation we know that the radius of Airy's disc is inversely proportional to the diameter of the aperture. Hence by decreasing the diameter of aperture, the size of Airy's disc increases.

8.7 DIFFRACTION DUE TO A PLANE DIFFRACTION GRATING OF N PARALLEL SLITS

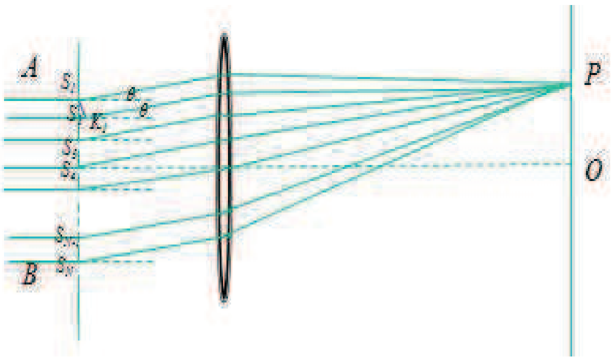

Figure 8.6

Here, S_1, S_2, S_3….. $S_N$are N narrow slits, in between points A and B. Let b= width of slit, d= width of opaque part between two slits.

The amplitude from each slit in the direction θ is

$$R_0 = \frac{A\sin\alpha}{\alpha}$$

Where $\alpha = \frac{\pi b}{\lambda}\sin\theta$ (As derived, in case of Single slit Fraunhofer diffraction)

The path difference between the wavelets from S_1 and S_2 in the direction θ is

$$S_2K_1 = (b+d)\sin\theta$$

Hence the phase difference between them

$$\frac{2\pi}{\lambda}(b+d)\sin\theta = 2\beta \text{, say}$$

If N be the total number of slits in the grating, the resultant amplitude in the direction of θ will be

$$R = R_0\frac{\sin N\beta}{\sin\beta} = \left(\frac{A\sin\alpha}{\alpha}\right)\frac{\sin N\beta}{\sin\beta} \qquad \text{.......} \quad (8.19)$$

Thus, the resultant intensity at point P is

$$I = R^2 = A^2\left(\frac{\sin\alpha}{\alpha}\right)^2\left(\frac{\sin N\beta}{\sin\beta}\right)^2 \qquad \text{.......} \quad (8.20)$$

The factor $A^2\left(\frac{\sin\alpha}{\alpha}\right)^2$ gives the intensity distribution due to single slit, while $\left(\frac{\sin N\beta}{\sin\beta}\right)^2$ gives the distribution of intensity in the diffraction pattern due to the interference in the waves due to N slits.

Principal Maxima

$$I = R^2 = A^2\left(\frac{\sin\alpha}{\alpha}\right)^2\left(\frac{\sin N\beta}{\sin\beta}\right)^2$$

The intensity will be maximum when

$$\sin\beta = 0 \Rightarrow \beta = \pm n\pi$$

$Where,\ n = 0,1,2,3.....$

This result in

$$\frac{\sin N\beta}{\sin\beta} = \frac{0}{0} \text{ (Indeterminate)}$$

Applying L' Hospital rule

$$\lim_{\beta\to\pm n\pi}\frac{\sin N\beta}{\sin\beta} = \lim_{\beta\to\pm n\pi}\frac{\frac{d}{d\beta}(\sin N\beta)}{\frac{d}{d\beta}(\sin\beta)}$$

$$\lim_{\beta\to\pm n\pi}\frac{N\cos N\beta}{\cos\beta} \Rightarrow \pm N$$

This result in

$$I = A^2\left(\frac{\sin\alpha}{\alpha}\right)^2 N^2$$

The condition for principal maxima is

$$sin\,\beta = 0 \qquad \text{or } \beta = \pm n\pi$$

$$\frac{\pi}{\lambda}(b+d)Sin\,\theta = \pm n\pi$$

$$(b+d)\,Sin\theta = \pm n\lambda \qquad \text{....... (8.21)}$$

For n = 0, we get $\theta = 0$ and this gives the direction of zero order principal maxima. The value of n = 1, 2, 3 etc. gives the direction of first, second, third etc. order principal maxima.

Minima

$$I = R^2 = A^2\left(\frac{\sin\alpha}{\alpha}\right)^2\left(\frac{\sin N\beta}{\sin\beta}\right)^2$$

The intensity will be minimum when

$$\sin N\beta = 0 \text{ but } \sin\beta \neq 0$$

Therefore, $$N\beta = \pm m\pi \qquad \text{....... (8.22)}$$

8.7.1 Missing Orders

As the resultant intensity due to N-parallel slits (plane diffraction grating) is given by

$$I = R^2 = A^2\left(\frac{\sin\alpha}{\alpha}\right)^2\left(\frac{\sin N\beta}{\sin\beta}\right)^2$$

Where, $$\alpha = \frac{\pi b}{\lambda}\sin\theta$$

And $$\beta = \frac{\pi}{\lambda}(b+d)\sin\theta$$

Now the direction of principal maxima in grating spectrum is given as

$$(b+d)\,Sin\theta = n\lambda \qquad \text{....... (8.23)}$$

Further the direction of minima of a single slit pattern is

$$b\,Sin\,\theta = m\lambda \qquad \text{....... (8.24)}$$

Where *m= 1, 2, 3……*

If both the conditions are simultaneously satisfied, a particular maximum of order n will be absent in the grating spectrum, these are known as absent spectra (or missing order spectrum).

Dividing equation (8.23) by equation (8.24), we get

$$\frac{b+d}{b}=\frac{n}{m} \qquad .... \ (8.25)$$

If b = d, then 2^{nd}, 4^{th}, 6^{th} etc. orders maxima will be missing in the grating diffraction pattern.

If d = 2b, then 3^{rd}, 6^{th}, 9^{th} etc. orders maxima will be missing in the grating diffraction pattern.

8.7.2 Maximum Number of Order Available in a Grating

The grating equation is $(b+d)\,Sin\theta = n\lambda$

$$or \qquad n=\frac{(b+d)\,Sin\,\theta}{\lambda} \qquad .... \ (8.26)$$

Maximum possible value of θ is 90^0.

Therefore, Maximum possible order will be

$$n_{\max}=\frac{(b+d)\,Sin\ 90}{\lambda}=\frac{(b+d)}{\lambda} \qquad .... \ (8.27)$$

8.8 SOLVED EXAMPLES

Example 8.1: A single slit is illuminated by two wavelengths λ_1 and λ_2. One observes that due to Fraunhofer diffraction the first minimum for λ_1 coincides with the second diffraction minimum for λ_2. What is the relation between λ_1 and λ_2.

Solution: In a single slit diffraction pattern, the direction of minimum intensities are given as

$a\ \sin\theta = \pm m\lambda_n$, where m= 1, 2, 3 …..

Hence for m = 1, we have, $a\sin\theta = \pm\lambda_1$

and for m = 2, we have, $a\sin\theta = \pm 2\lambda_2$

Equating above two equations, we get, $\lambda_1 = \lambda_2$

Example 8.2: In a double slit Fraunhofer diffraction pattern, the screen is placed 170 cm away from the slits. The width of the slit is 0.08 mm and slits are 0.4 mm apart. Calculate of the wavelength of light, if the fringe width is 0.25 cm. Also find the missing order.

Solution: In a double slit Fraunhofer diffraction pattern, the fringe width is given by-

$$w=\frac{D\lambda}{2d}$$

Here D=170 cm=1.7 m, W= 0.25 cm= 2.5 x 10^{-3}m, a=0.08 mm = 8 x 10^{-5} m and b = 0.4 mm= 4 x 10^{-4} m, 2d = b = 4 x 10^{-4} m

$$\therefore \qquad \lambda=\frac{2dW}{D} = 0.5882 \text{ x } 10^{-6} = 5882 \text{ Å}$$

The condition for missing order is-

$$\frac{a+b}{a} = \frac{n}{m} \quad or \quad n = \left(\frac{a+b}{a}\right)m \quad = \left(\frac{8\times10^{-5} + 4\times10^{-4}}{8\times10^{-5}}\right)m = 6m$$

n=*6m*

Hence the missing orders are 6, 12, 18, 24, 30…..

8.9 SUMMARY

The basics of the diffraction phenomena along with various classes of diffraction have been discussed. The Fraunhofer diffraction for single slit, double slit, circular aperture and N slits (grating) have been discussed in the details. The calculation for the intensity of the principal maxima, secondary maxima and minima has been derived. Their relative comparison in terms of their intensities has also been made. Determination of missing orders in case of double slit and N slits (grating) diffraction pattern has also been made.

8.10 GLOSSARY

Fraunhofer Diffraction- Far field diffraction

Grating- Fine and equidistant slits in large number

Missing Order- Absent maxima

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2. Optics and Atomic Physics by D. P. Khandelwal, Himalaya Publishing House, New Delhi, 2015

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2. Introduction to Optics- Frank S. J. Pedrotti, Prentice Hall, 1993

8.13 TERMINAL QUESTIONS

Objective Type

1. Grating element is equal to

A. nλ/sinθ B. nλ C. sinθ D. cosθ

2. In Fraunhofer's diffraction, incident light waves have________ type of wavefront.

A. Circular B. Spherical C. Cylindrical D. Plane

3. In single-slit experiment, if the red color is replaced by blue then_____________.

A. The diffraction pattern becomes narrower and crowded together

B. The diffraction bands become wider

C. The diffraction pattern does not change

D. The diffraction pattern disappears.

4. On increasing the width of a single slit, the width of the central maximum

A. increases B. remains constant C. decreases D. becomes zero

5. Maximum number of orders possible with a grating is

A. Independent of grating element

B. Inversely proportional to grating element

C. Directly proportional to grating element

D. Directly proportional to wavelength.

6. When white light is incident on a diffraction grating, the light diffracted more will be

A. Blue B. Yellow C. Violet D. Red

7. Diffraction phenomena are usually divided into __________ classes.

A. One B. Two C. Three D. Four.

8. Light of Wavelength 5000 Å is incident on a single slit of width 0.1 mm. The screen is at a distance of 2 m from the slit. The width of the central bright fringe on the screen will be

A. 18 mm B. 36 mm C. 20 mm D. 6 mm

9. Light of Wavelength 6000 Å is incident normally on a single slit of width 24 x 10^{-5} cm. The angular position of the second minimum from the central minimum from the central maximum will be -

A. 30^0 B. 60^0 C. 90^0 D. 45^0

10. In a diffraction grating, the condition for principal maxima is

A. $b \sin\theta = n\lambda$ B. $(b + d) \sin\theta = n\lambda$

C. $d \sin\theta = n\lambda$ D. $\sin\theta = n\lambda$.

Long Answer Type

1. Define diffraction phenomena. What do you mean by the Fresnel class and Fraunhofer class of diffraction?
2. Describe Fraunhofer diffraction due to single slit for central maxima and prove that the relative intensities of the successive maximum are nearly 1:1/22:1/61…

3. What are missing orders in double slit Faunhofer diffraction? Further in a grating, if the widths of transparencies and opacities are equal.
4. Give an account of the diffraction effects produced by a slit. Explain what happens when the slit width is gradually increased and also when the screen is gradually moved away from the slit.
5. Discuss Fraunhofer diffraction at a circular slit; describe the formation of Airy's disc.
6. Give the theory of a plain transmission grating. What particular spectra would be absent if the widths of transparencies and opacities of the grating are equal.

Numerical Questions

1. A circular aperture of 1.2 mm diameter is illuminated by a plane wave of monochromatic light. The diffracted light is received on a distant screen which is gradually moved towards the aperture. The center of the circular path of the light first becomes dark when the screen is 30 cm from the aperture. Calculate he wavelength of light.
2. Light of wavelength 5500 Å falls normally on a slit of width 22 × 10-5 cm. Calculate the angular position of the first two minima on either side of the central maximum.
3. Plane wave of wavelength 6×10^{-5} cm fall normally on a slit of width 0.2 mm. Calculate (i) the total angular width of the central maximum (ii) the linear width of the central maximum on a screen placed 2 m away.
4. Calculate the angle at which the first dark band and the next bright band are formed in the Fraunhofer diffraction pattern of a slit 0.3 mm wide (λ = 5890 Å).
5. In a single slit diffraction pattern the distance between the first minimum on the right and first minimum on the left is 5.2 mm. The screen on which the pattern is displayed is 80 cm from the slit and the wavelength is 5460 Å. Calculate the slit width.
6. Calculate the wavelength of light whose first diffraction maximum in the diffraction pattern due to a single slit falls at $\theta = 30^0$ and coincides with the first minimum for the red light of wavelength 6500 Å.
7. Light of wavelength 600 nm is incident normally on a diffraction grating. Two adjacent maxima occur at angles given by $\sin\theta= 0.2$ and $\sin\theta = 0.3$. The fourth-order maxima are missing. (a) What is the separation between adjacent slits? (b) What is the smallest slit width this grating can have? For that slit width, what are the (c) largest, (d) second largest, and (e) third largest values of the order number m of the maxima produced by the grating?
8. A diffraction grating is made up of slits of width 300 nm with separation 900 nm. The grating is illuminated by monochromatic plane waves of wavelength λ= 600 nm at normal incidence. How many maxima are there in the full diffraction pattern?

8.14 ANSWERS

Objective Type

1 (A), 2 (D), 3 (A), 4 (C), 5 (C), 6 (C), 7 (C), 8(C), 9(A), 10(B)

Numerical Questions

1. 6000 Å
2. 14^0 29' & 30^o
3. 6×10^{-3} radians & 1.2 cm
4. 0.112^0& 0.168^0
5. 1.68×10^{-4} cm
6. 4333.3 Å

a. 6μm (b) 1.5 μm (c) 9 (d) 7 (e) 6

7. 3

UNIT 9: RESOLUTION AND RESOLVING POWER

CONTANTS

9.1 INTRODUCTION

When the two objects are very near to each other or they are at very large distance from our eye, the eye may not be able to see them as separate. If we want to see them separate, optical instruments such as telescope, microscope etc. (for close objects) and prism and grating etc. (for spectral lines) are employed. Even if we assume that the instruments employed are completely free from all optical defects, the image of a point object or line is not simply a point or line but it is a diffraction pattern with a bright central maximum and other secondary maxima, having minima in between of rapidly decreasing intensity. Thus an optical instrument is said to be able to resolve two point objects if the corresponding diffraction patterns are distinguishable from each other.

The ability of an optical instrument to resolve (i.e. view separately) the images of two close point source is known as resolving power.

Limit of Resolution: The minimum separation between two objects that can be resolved by an optical instrument is called the limit of resolution (or just resolution).

9.2 OBJECTIVE

After studying this unit, you will be able to –

- have the basic idea of resolution.
- know the Rayleigh criterion of resolution.
- calculate the resolving power of various instruments/ accessories like grating, prism, telescope and microscope.

9.3 RAYLEIGH CRITERION OF RESOLUTION

According to Rayleigh, two close point objects are said to be just resolved if the principal maxima of one coincides with the first minima of the other and vice-versa.

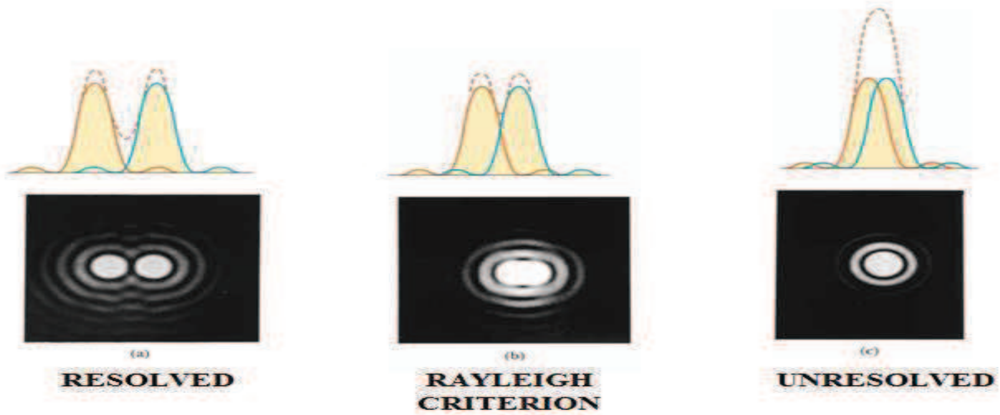

Figure 9.1

9.4 RESOLVING POWER OF TRANSMISSION GRATING

Let λ andλ+dλ are two closely spaced spectral lines (wavelengths).The resolving power of the grating is defined as the ratio of wavelength (λ) to the difference dλ of the wavelengths to be resolved

$$\mathrm{R}.P. = \frac{\lambda}{d\lambda}$$

The direction of nth principal maxima for wavelength λ is given by

$$(b+d)\sin\theta = n\lambda$$

The direction of nth principal maxima for wavelength λ+dλ is given by

$$(b+d)\sin(\theta+d\theta) = n\,(\lambda+d\lambda) \qquad \text{.......}\ (9.1)$$

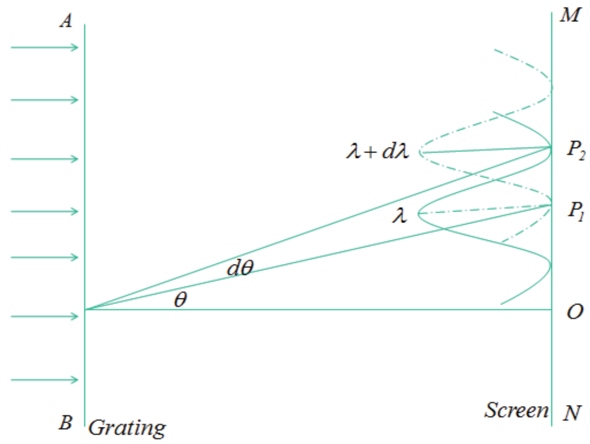

Figure 9.2

The minima in the direction θ is given by (due to wavelength λ+dλ)

$$N(b+d)\sin(\theta+d\theta) = m(\lambda+d\lambda)$$

Here *m* can have all integral values except *0, N, 2N, 3N……*

(Because for these values of m the condition of maxima is satisfied)

The first minimum adjacent to nth principal maxima in the direction (θ+dθ) can be obtained by putting m as (nN+1) (due to wavelength λ)

$$N(b+d)\sin(\theta+d\theta) = (nN+1)(\lambda)$$

$$(b+d)\sin(\theta+d\theta) = \frac{(nN+1)\lambda}{N} \qquad \text{.......}\ (9.2)$$

Comparing equation (1) and (2)

$$n(\lambda+d\lambda) = \frac{(nN+1)\lambda}{N}$$

or $$n\lambda + nd\lambda = n\lambda + \frac{\lambda}{N}$$

$$nd\lambda = \frac{\lambda}{N} \Rightarrow \frac{\lambda}{d\lambda} = nN \qquad \text{....... (9.3)}$$

Since resolving power is directly proportional to N, it means that larger will be the number of lines per cm of a grating greater will be the resolving power.

9.5 RESOLVING POWER OF PRISM

An example of the use of Rayleigh criterion for the resolving power of a rectangular aperture is found in the prism spectroscope, if we assume that the face of the prism limits the refracted beam to a rectangular section. The resolving power of a prism is defined as its capacity to form separate spectral lines of two wavelengths which are very near to each other. It is measured by λ/dλ, where λ is the wavelength of either of them or mean wavelength and dλ is the difference in their wavelengths.

Expression for Resolving Power

Let ABC be the section of prism as shown in Fig. 3. A parallel beam of light consisting of wavelengths λ and λ + dλ is incident on the prism placed in the minimum position for these two wavelengths (this is possible because the wavelength difference dλ is very small). BD represents the incident plane wavefront. As the two wavelengths are refracted by different amounts in passing through the prism, therefore CE and CF represent the corresponding emergent wavefronts. Let dθbe thedifference in deviation for the light rays of wavelengths λ and (λ + dλ) respectively. The telescope objective of thespectrometer focuses the wavefront CE at I_1 and CF at I_2. Thus I_1 corresponds to the principal maximum for wavelength λ and λ + dλ.

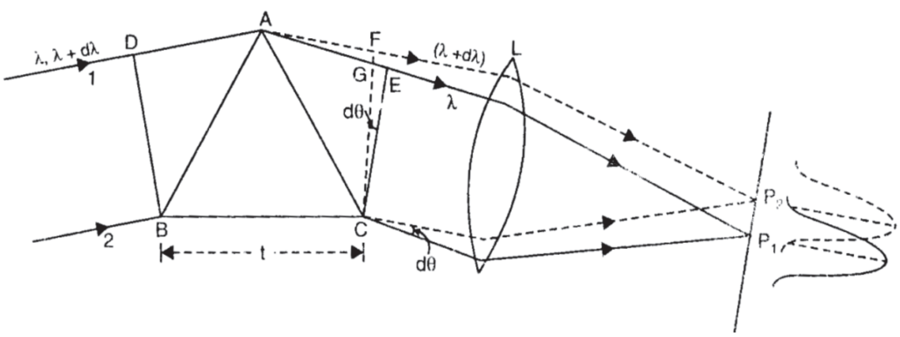

Figure 9.3

The face AC ofthe prism acts like a rectangular aperture. Hence the Rayleigh criterion can be applied here. According to Rayleigh's criterion, the two wavelengths can be resolved by the prism if the principal maximum of one falls on the first minimum of the other in the same direction.

Let μ and (μ – dμ) be the values of refractive indices of the prism, for wavelengths λ and λ+dλ respectively.

Equating the path covered by ray 1 and 2

$$DA + AE = \mu(BC) = \mu\, t \qquad (for\ \lambda) \qquad .... \ (9.4)$$

$$DA + AF = (\mu - d\mu)\, BC = (\mu - d\mu)t \qquad (for\ \lambda + d\,\lambda) \qquad .... \ (9.5)$$

Equations (9.4) and (9.5) are obtained by applying the Fermat's principle which states that for any wavelength all the actual optical paths between the incident and the emergent wavefronts must be equal. Subtracting equation (9.5) from equation (9.4), we get,

$$AE - AF = d\mu.(AC) = d\mu.t$$

From the geometry of the figure

$$AE - AF = AE - AG = d\mu.t \qquad \text{(Since } AF = AG, \text{ approximately)}$$

or $\qquad GE = d\mu.t$

If GE = λ, then according to the theory of Fraunhofer diffraction, Rayleigh criterion of resolution is satisfied and spectral lines of wavelengths λ and λ + dλ, will be just resolved.

Thus $\qquad \lambda = t.d\mu$

Dividing both sides by dλ, we obtain the expression for the resolving power of prism will be

$$\frac{\lambda}{d\lambda} = t\left(\frac{\mathrm{d}\mu}{\mathrm{d}\lambda}\right) \qquad .... \ (9.6)$$

From equation (3), it is evident that the resolving power of a prism varies directly as

(i) t, the width of the base of the prism, and
(ii) dμ/dλ, rate of change of refractive index with wavelength.

9.6 RESOLVING POWER OF TELESCOPE

A telescope is used to see the distinct objects. The details which it gives depend on the angle subtended at its objective by two point objects and not on the linear separation between them. The resolving power of a telescope is defined as the reciprocal of the smallest angle subtended at the objective by the two distinct object points which can be just seen as separate ones through the telescope.

Expression for the resolving power: Let d isthe diameter of the objective of the telescope (Fig. 4). Consider the incident rays of light from two neighboring points(say two stars lying very close to each other, not shown in the figure). Suppose dθ is the angle subtended by the two distant objects at the objective of the telescope. The ring supporting the telescope objective and the lens itself serveas a circular aperture and produce Fraunhofer diffraction patterns in the focal plane of the objective.

Let P_1 and P_2 be the positions of the central maxima of the two images. The pattern will be very close to each other with a large amount of overlapping. If the overlapping is too much, the telescope may not be able to distinguish them as separate. According to Rayleigh's criterion, the patterns will be just resolved if the central maxima of one just falls on the first minima of the other.

Now the secondary waves travelling in direction AP_2 and BP_2 meet at P_2 and have a path difference equal to $(BP_2-AP_2) = BC = d.d\theta$

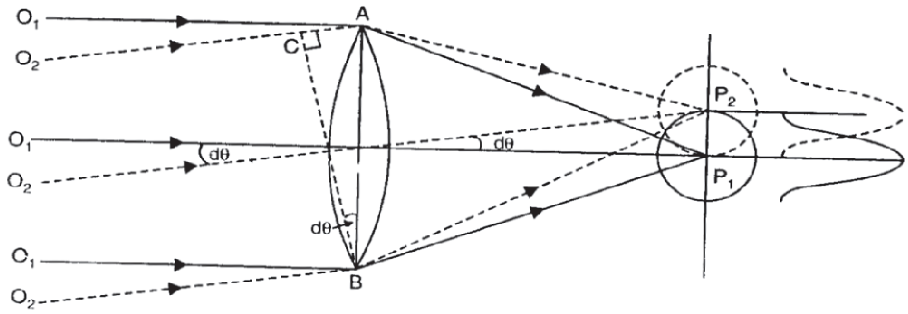

Figure 9.4

$$BC = AB \sin d\theta = AB.d\theta \text{ (for small angles)}$$

If this path difference $d.d\theta=\lambda$, the position of P_2 corresponds to the first minimum of first image and we have,

$$d.d\theta = \lambda \text{ or } d\theta = \frac{\lambda}{d} \qquad .... (9.7)$$

The above idea may be understood in the following way:

If we consider that the whole wavefront AB isdivided into two halves AO and OB, then the path difference between the secondary waves from the corresponding points in the two halves is $\lambda/2$. All the secondary waves from the two halves interfere destructively with one another and hence P_2 corresponds to the first minimum of the first image.

The condition (9.7) holds good for rectangular aperture. According to Airy this condition in case of a circular aperture can be expressed as

$$d\theta = \frac{1.22\lambda}{d} \qquad .... (9.8)$$

Here $d\theta$ represents the minimum resolvable angle between the two distant point objects or this gives the limit of resolution of the telescope. The reciprocal of $d\theta$ measures the resolving power of the telescope. Hence

$$\frac{1}{d\theta} = \frac{d}{1.22\lambda} \qquad .... (9.9)$$

Thus a telescope with large diameter of objectivehas a higher resolving power.

9.7 RESOLVING POWER OF MICROSCOPE

The function of a microscope is to magnify an object and give its finer details which cannot be observed by naked eye. The ability of a microscope to form distinctly separate images of two neighboring small objects is known as its resolving power. It is measured by the smallest linear separation between two point objects whose images are just resolved by the objective of the microscope. The smaller is the linear separation which can be resolved, the higher is said to be the resolving power.

Expression for Resolving Power: In Figure 9.5, AB is the aperture of the objective of the microscope; O_1 and O_2 are the self-luminous point objects very close to each other and separated at a distance d. The periphery of the objective acts as a circular aperture and as a result the images of O_1 and O_2 are Fraunhofer diffraction patterns. The patterns consisting of a central bright disc surrounded by a series of alternate dark and bright rings.P_1 represents the central maximum of the diffraction pattern of the point object O_1. Similarly P_2 represents the central maximum of the diffraction pattern of the other point object O_2.

According to the Rayleigh's criterion the two objects may be resolved if the central maximum of one pattern falls on the first minimum of the other. In this case the two objects may be resolved if P_1 islocated at the first minima of the diffraction pattern centered at P_2.

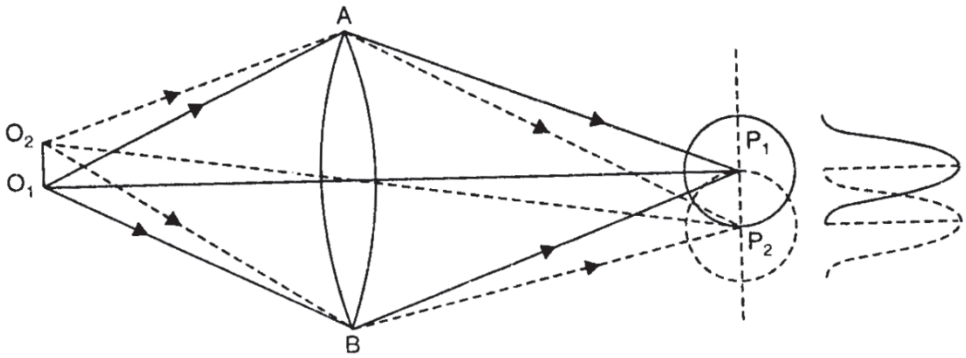

Figure 9.5

According to the Rayleigh's criterion the two objects may be resolved if the central maximum of one pattern falls on the first minimum of the other. In this case the two objects may be resolved if P_1 islocated at the first minima of the diffraction pattern centered at P_2. Thus we have to find out condition under which the first minima of the diffraction pattern due to O_2 lies at the central maxima of diffraction pattern due to O_1. This will happen when the path difference between the extreme rays O_2BP_1 and O_2AP_2 is equal to λ. To consider this path difference, the magnified view of O_1O_2 and the rays starting from them are shown in Fig. 6. The path difference is given by

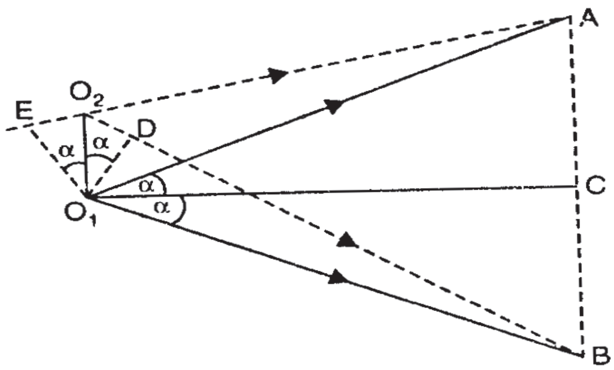

Figure 9.6

$$(O_2B+BP_1)-(O_2A+AP_1)=O_2B-O_2A \qquad (\text{Since } BP_1=AP_1)$$

In Figure 9.6, O_1C is perpendicular to CA and O_1D is perpendicular to O_2B.

$$O_2B-O_2A=(O_2D+DB)-(EA-EO_2)=O_2D+EO_2 (\text{As } DB=O_1B=O_1A=EA)$$

Therefore, path difference = $O_2D + EO_2 = 2d \sin \alpha$

If the path difference $2d \sin \alpha = 1.22\lambda$, then P_1 corresponds to the first minimum of the image P_2 and the two images appear just resolved.

$$2d \sin \alpha = 1.22\lambda$$

or

$$d = 1.22\lambda / 2\sin \alpha \qquad \text{.......} \quad (9.10)$$

The result is derived on the assumptions that the objects viewed with microscope are self-luminous and emitting light of wavelength λ.

In case of objects illuminated by some external source of light of wavelength λ, Abbe showed that the factor 1.22 may be omitted and we can write

$$d = \lambda / 2\sin \alpha \qquad \text{.......} \quad (9.11)$$

The high resolution power microscopes are generally oil immersion types in which the space between the object and objective is filled with an oil of refractive index μ. In this case as the path difference will then be multiplied with the factor μ

$$d = \lambda / 2\mu\sin \alpha \qquad \text{.......} \quad (9.12)$$

Here, the factor $\mu\sin \alpha$ is known as the numerical aperture of the microscope.

$$\therefore \text{Resolving power of the microscope} = \frac{1}{d} = \frac{2\mu \sin \alpha}{1.22\lambda} \qquad \text{.......} \quad (9.13)$$

Thus, using small wavelengths (UV) and using quartz lenses, the resolving power of the microscope can be increased. Such microscopes are known as the ultra-microscope.

9.8 SOLVED EXAMPLES

Example 9.1: Calculate the minimum number of lines in a grating which will just resolve the sodium lines in the first order spectrum. The wavelengths are 5890 and 5896 Å.

Solution: We know that resolving power $\lambda/d\lambda = nN$

Here n=1, $\lambda_1 = 5890$ Å $= 5890\times10^{-8}$ cm

$\therefore$ $d\lambda = 5896 \times 10^{-8} - 5890 \times 10^{-8} = 6 \times 10^{-8}$ cm.

Now, $N=(1/n) \times (\lambda/d\lambda) = (5890 \times 10^{-8})/(1 \times 6 \times 10^{-8})$ =982 approximately

Example 9.2: A grating has 15 cm of the surface ruled with 16000 lines per cm. What is the resolving power of the grating in the first order?

Solution: The resolving power of a grating is given by-

$\lambda/d\lambda = nN$, here n = 1, N = 15 x 6000 = 90000

$\lambda/d\lambda$ = 1 x 90000 =90000

Example 9.3: A prism spectrometer uses a prism of base 5 cm and material whose dispersion $\frac{d\mu}{d\lambda}$ is 200 in the range λ = 5000Å. What is the smallest difference of the wavelength in this range which this spectrometer may resolve?

Solution: The expression for the resolving power of prism is

$$\frac{\lambda}{d\lambda} = t\left(\frac{\mathrm{d}\mu}{\mathrm{d}\lambda}\right),$$

Here, t= 5 cm, λ = 5000Å = 5 x 0^{-5} cm, $\frac{d\mu}{d\lambda} = 200$

Putting the values, we get, $d\lambda = 5 \times 10^{-8}$ cm =5 Å

Example 9.4: Two pin holes 1.5 mm apart are placed in front of a source of light of wavelength 5.5 x 10^{-5} cm and seen through a telescope with objective diameter of 0.4 cm. Find the minimum distance from the telescope at which the pin holes can be resolved.

Solution: We know that $d\theta = \frac{1.22\lambda}{d}$ and also $d\theta = \frac{x}{a}$

$$\therefore \quad \frac{1.22\lambda}{d} = \frac{x}{a} \quad \text{or} \quad a = \frac{xd}{1.22\lambda} = \frac{0.15\times 0.4}{1.22\times 5.5\times 10^{-5}} = 894.2 \text{ cm}$$

Example 9.5: The smallest object detail that can be resolved with a certail microscope with light of wavelength 6000 Å is 3.5 x 10^{-5} cm. Find (i) The numerical aperture of the objective when used dry, and (ii) The numerical aperture obtained if an immersion oil of refractive index 1.5 is used.

Solution: The resolving power of microscope is –

$$d = \frac{\lambda}{2Sin\alpha} = \frac{\lambda}{2\,NA},$$ where NA is numerical aperture

(i) NA = $NA = \frac{\lambda}{2d} = \frac{6000\times 10^{-8}}{2\times 3.5\times 10^{-5}} = 0.86\ approx.$

(ii) Oil immersion numerical aperture= μ x dry aperture = 1.5 x 0.86 =1.44

9.9 SUMMARY

The resolving power of an optical instrument is defined as its ability to just resolve the images of two close point sources or small objects. The Rayleigh Criterion gives a quantitative account of the phenomena of resolution. The definitions and physical meanings for the resolving powers of diffraction grating, prism, telescope and microscope were discussed. Their mathematical expressions have also been derived in the present chapter.

9.10 GLOSSARY

Resolving Power: Ability of an optical instrument to see the close objects separately

Limit of resolution: Minimum resolvable distance

Principal maxima: Central maxima

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9.12 SUGGESTED READINGS

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9.13 TERMINAL QUESTIONS

Objective Type

1.The maximum resolving power of a microscope can be obtained with

(A) Violet light (B) Yellow Light (C) Red Light (D) Green Light

2.What will be limit of resolution of a microscope if its numerical aperture is 0.5 and the wavelength of light used 5000 Å

(A) 6100 mm (B) 6100 cm (C) 6100 m (D) 6100 Å

3.Two stars distant eight light years are just resolved by a telescope. The diameter of the telescope lens is 26 cm. If the wavelength of the light used is 5000 Å , the minimum distance between the stars will be

(A) 1.95×10^{12} M (B) 1.95×10^{11} M (C) 1.95×10^{10} M (D) 1.95×10^{9} M

4.The resolving power of a telescope can be increased by having a

(A) Large focal length of eyepiece (B) Small focal length of eyepiece (C)Large aperture of objective lens (D) Small aperture of objective lens

5.The Resolving power of a grating having N slits in nth order will be

(A) (n+N) (B) (n-N) (C) nN (D) n/N

6.The resolving power of a prism is

(A) Directly proportional to the rate of change of refractive index with wavelength
(B) Inversely proportional to rate of change of refractive index with wavelength
(C) Inversely proportional to the thickness of the prism
(D) Independent of thickness of prism

Long Answer Type

1. Discuss Rayleigh criterion for resolution. What is limit of resolution? Determine an expression for the resolving power of a grating.
2. Explain clearly, what is meant by the resolving power of an optical instrument and deduce an expression for the resolving power of a prism.
3. Explain what do you understand by the limit of resolution of a telescope and obtain an expression for it. What is the effect of the size of the image of a star as aperture of the objective increases?
4. On the basis of diffraction theory, explain the need of large apertures for telescopes used for astronomical purposes.
5. Define the resolving power of a microscope. Deduce an expression for it and discuss it.

Numerical Questions

1. Find the separation of the two points that can be resolved by a 500 cm telescope. The distance of the moon is 3.8×10^5 KM. The eye is highly sensitive to light of wavelength of 5500 Å.
2. Show that for a transmission grating with 1 inch ruled space, the resolving power cannot exceed 5×10^4 at normal dence for $\lambda = 5080$ Å.
3. A microscope objective gathers light over a cone of semi-angle 30^0 and uses visible light of 5500 Å. Estimate its resolving limit.
4. Calculate the minimum thick ness of the base of a prism which will just resolve the D1 and D2 lines of sodium. Given μ for wavelength 6563 Å = 1.6545 and for wavelength 5270 Å = 1.6635.
5. Calculate the resolving power of a prism which has a dispersion $\frac{d\mu}{d\lambda} = 600$ per cm and a base of 3 cm. Will this be adequate to resolve two spectral lines (i) 5890 Å (ii) 5230 Å
6. A diffraction grating with a width of 2.0 cm contains 1000 lines/cm across that width. For an incident wavelength of 600 nm, what is the smallest wavelength difference this grating can resolve in the second order?
7. How many rulings must a 4.00-cm-wide diffraction grating have to resolve the wavelengths 415.496 and 415.487 nm in the second order? (b) At what angle are the second-order maxima found?

9.14 ANSWERS

Objective Type

1.(A), **2**. (D), **3**. (B), **4**. (C), **5**. (C), **6**. (A)

Numerical Questions

1.61 m, **3.** 6.1 x 10^{-5} cm, **4.** 1.41,**5.(i)** Prism will resolve this line **(ii)** Prism will not resolve this line, **6.**$\Delta\lambda$= 0.15 nm, **7.** (a) 23100, (b) 28.7°

UNIT 10: POLARIZATION

CONTANTS

10.1 INTRODUCTION

Waves are basically of two types; transverse waves and longitudinal waves. A wave in which particles of the medium oscillate to and fro, in the form of compression and rarefaction, along the direction of wave propagation is called a longitudinal wave, e.g., waves produced on string and sound waves. On the other hand waves produced in ripples on water waves and waves on a rope, in which every particle of the medium oscillates up and down, in the form of trough and crest, at right angles to the direction of wave propagation is called a transverse wave. Interference, diffraction and polarization are three major phenomenon exhibited by waves, out of these three interference and diffraction are shown by any type of wave whether longitudinal or transverse but polarization is shown by only transverse wave. As light shows all three phenomenon it is clear that light is a transverse wave.

10.2 OBJECTIVE

After the study of this unit the student will be able

- To understand the concept of polarization
- To explain different types of polarization
- To differentiate between plane, circular and elliptically polarized light
- To describe different methods of plane polarized light
- To explain Brewster's law
- To explain Malus law

10.3 POLARIZATION

The phenomenon of interference and diffraction has proved the wave nature of light but it doesn't tell us regarding the character (whether longitudinal or transverse?) and nature of vibration of light (whether linear, circular, elliptical or torsional). It is the phenomenon of polarization that shows that light wave is definitely transverse in nature. Light wave is a transverse electromagnetic wave made up of mutually perpendicular, fluctuating electric and magnetic fields vibrating perpendicular to each other as well as direction of propagation too.

In general, natural light is unpolarized in nature, i.e., it consists of a very large number of wavelength with electric vector vibrates in all possible planes with equal probability when ordinary light is allowed to pass through tourmaline crystal, the vibration of electric field are confined only to one direction in a plane perpendicular to the direction of propagation of light. This light which has acquired the property of one sidedness and whose electric field vibration lacks in symmetry called polarized light. This polarization of light means departure from complete symmetry about the direction of propagation. Let us explain the concept of polarization using two slit analogy and their optical equivalent.

10.3.1 Two Slit Analogy of Polarized Light

Let S_1 and S_2 be the two slit adjacent to each other (figure 10.1). A string AB passing through the slit and attached to a fixed point at B. Now if we move the end A of the string up and down perpendicular to AB, the string vibrates and a transverse wave propagates along CD. If slit S_2 is placed parallel to S_1, vibration passing through S_1 and S_2 reach the end B without any change in amplitude (figure 10.1(a)). Now let us rotate slit S_2 as S_2 becomes perpendicular to S_1, the vibration pass through the slit S_1 undisturbed as before but are not able to pass through S_2 and therefore the string does not vibrate between S_2 and B (figure 10.1(b)).

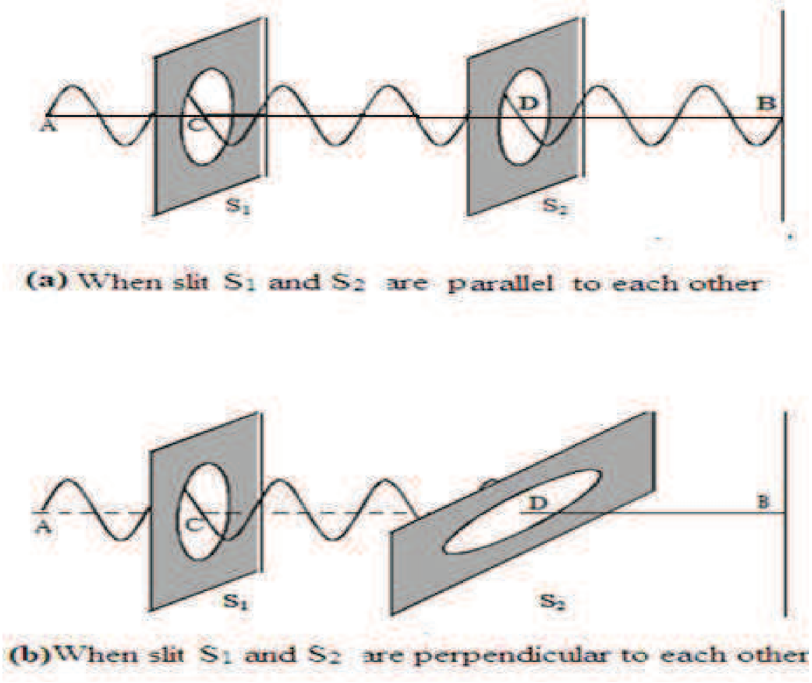


Figure 10.1

In the intermediate positions of the slit S_2, the vibration are partially transmitted and partly stopped, reaching the end B with diminished amplitude. The variation of amplitude as the rotating slit S_2 is only because of transverse vibration in the string. On the other hand the end A is moved to and fro parallel to the length of the string instead of up and down setting longitudinal vibration, we see that the rotation of any of the slide about AB as axis does not affect the passage of vibrations and hence vibration reach at B with undiminished amplitude. Therefore we can say that variation in amplitude of vibration passing through S_2 on rotation of S_2 signifies the transverse vibration of the string.

Now if we replace the slit with tourmaline crystal and string with a source of light, exactly similar phenomenon is observed (Figure 10.2). When light from source S falls in a tourmaline crystal A cut parallel to its crystal axis the emergent light is slightly coloured. Now if we place a similar cut crystal B in the path of beam partially to the axis of crystal A we observed that emerging light is still coloured and the intensity is maximum. If now keeping crystal ‘A’ fixed we rotate the crystal B about the axis the intensity decreases and becomes zero when B is perpendicular to A. By further rotation light reappears and becomes maximum again when ‘A’ and ‘B’ again becomes parallel.

This variation in amplitude proves that light is transverse in nature. Whereas, if it is longitudinal, there shouldn’t be any variation by rotating the crystals discussed in two slit

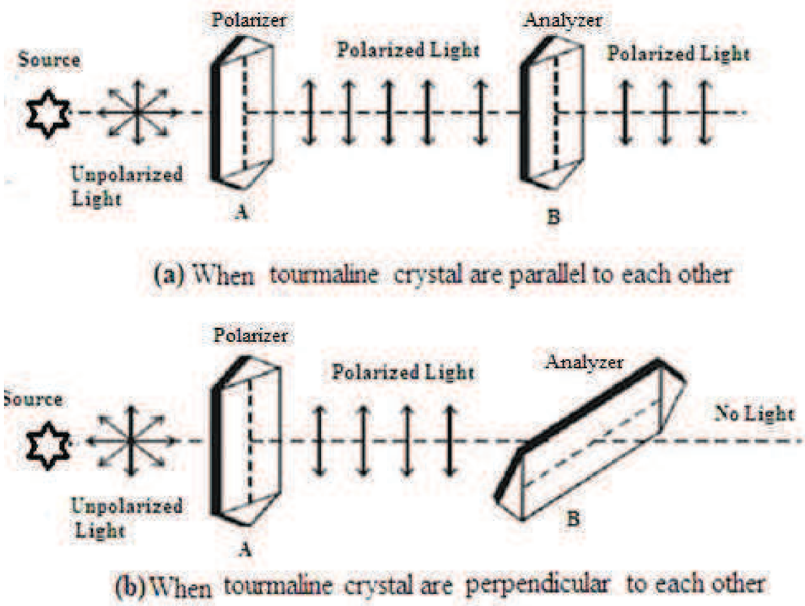


Figure 10.2

analogy. It also shows that light vibration after passing through rotating slit are not symmetrical about the direction of propagation.

10.3.2 Comparison of Unpolarized Light and Polarized Light

S.N.	Unpolarized Light	Polarized Light
1	Unpolarized light consists of waves with planes of vibration equally distributed in all directions about the direction of propagation.	Polarized wave consists of waves with light vector vibrating in a single plane perpendicular to the direction of propagation.
2	Unpolarized light is symmetrical about the ray direction.	Polarized light is asymmetrical about the ray direction.
3	Unpolarized light is produced by conventional light sources	Polarized light is generally obtained from unpolarized light with the help of polarizer.

10.4 TYPES OF POLARIZATION

The polarization of a light wave describes the shape and locus of the tip of the E vector at a given point in space as a function of time. Depending upon the locus of the tip of the E vector light may be exhibit three different states of polarization. They are

1. Plane polarized light

2. Circularly polarized light

3. Elliptically polarized light

Apart from these the light may also be partially polarized.

10.5 CONCEPT OF PLANE POLARIZED LIGHT, CIRCULARLY POLARIZED LIGHT AND ELLIPTICALLY POLARIZED LIGHT

As we know that light is an electromagnetic wave consist of mutually perpendicular electric and magnetic field vector both are vibrating perpendicular to the direction of propagation of light wave. Also electric vector is dominating and is responsible for optical effects of wave hence the electric vector is also called light vector.

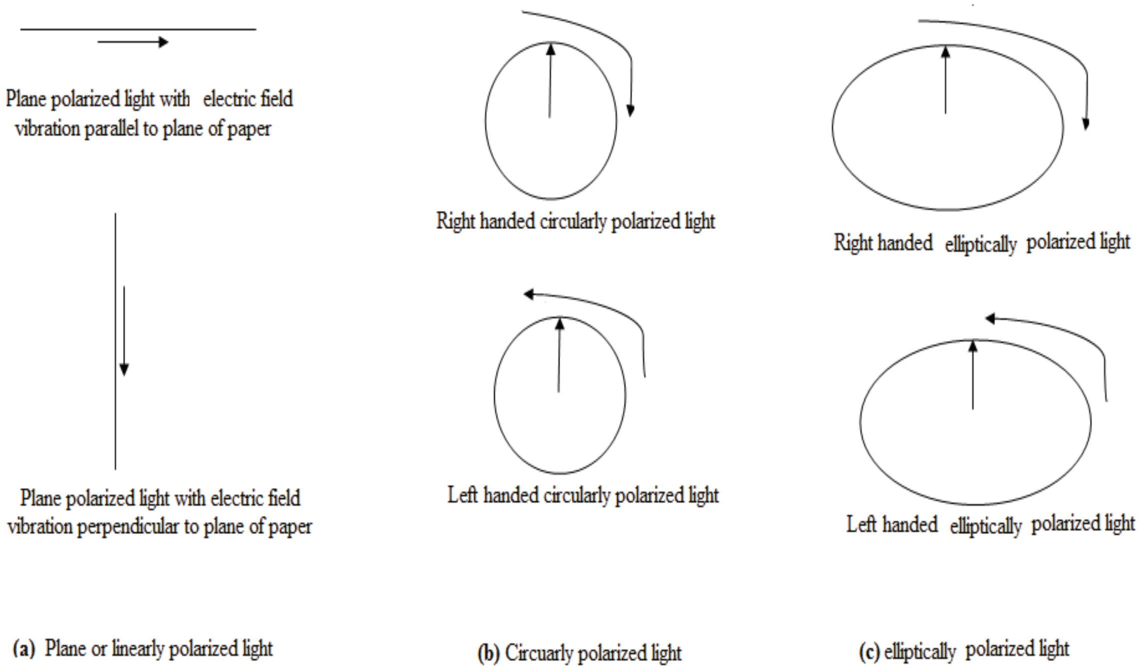


Figure 10.3

As mentioned earlier, unpolarized light have vibrations along all possible straight lines perpendicular to the direction propagation the light. Light which has acquired property of one sidedness is called polarized light. Therefore plane polarized light is not symmetrical about the direction of propagation but the vibrations of light vector (electric vector) are confined to a single direction i.e. along a line of course perpendicular to direction of propagation also known as linearly polarized light. We can also say that in plane or linearly polarized light the magnitude of light vector changes but its orientation remains unchanged. Usually light is a mixture of plane polarized light and unpolarized light, known as partially plane polarized.

On the other hand a light wave is circularly polarized if the magnitude of light vector remains constant but its orientation rotates at a constant rate about the direction of propagation so that the tip of the light vector traces a circle. It completes one evolution within one wavelength. Circularly polarized wave may be considered as the result of superposition of two mutually perpendicular plane polarized waves having equal amplitude but a phase difference of 90^0. If rotation of tip of light vector E is seen clockwise it is called right circularly polarized light if it rotates anticlockwise the wave is said to be left circularly polarized light.

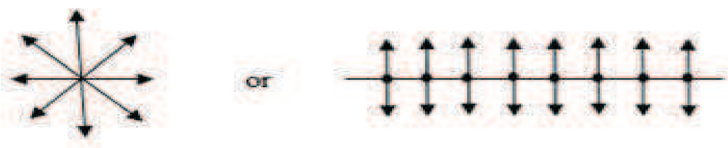
Similarly a light wave is called elliptically polarized if the magnitude of light vector as well as its orientation changes about the direction of propagation so that the tip of the light vector traces an ellipse. Elliptically polarized wave may be considered as the result of superposition of two mutually perpendicular plane polarized waves having different amplitude and not in same phase.

Like circularly polarized light if rotation of tip of light vector E is seen clockwise it is called right elliptically polarized light if it rotates anticlockwise the wave is said to be left elliptically polarized light.

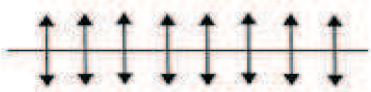
10.6 PICTORIAL REPRESENTATION OF PLANE POLARIZED LIGHT

Ordinary or unpolarized light obtained from any source consists of vibration of electric field vector in al possible plane perpendicular to the beam direction i.e. the electric field vibrations are symmetrical. Unpolarized light can be considered as consisting of two sets of vibrations-one set vibrating in one plane and other perpendicular to it. It may also be represented respectively by arrows and dots. Hence unpolarized lights pictorially represented end view would be as shown in Figure 10.4 (a).

In a plane polarized light the vibrations of electric vector are along a single straight line thus having departure of complete symmetry. When electric field vector or light vector of plane polarized light has vibration in the plane of the paper they are represented by arrows as shown in Figure 10.4 (b).When the vibrations of light vector are in a direction perpendicular to the plane of the paper they are represented by dots as shown in Figure 10.4 (c).



(a) Unpolarized light with symmetrical vibration of electric field vector



(b) Plane polarized light with electric field vibration parallel to plane of paper

(c) Plane polarized light with electric field vibration perpendicular to plane of paper

Figure 10.4

10.7 PLANE OF VIBRATION AND PLANE OF POLARIZATION OF PLANE POLARIZED LIGHT

As discussed earlier plane polarized light may be defined as the light in which the electric vector or light vector vibrates along a fixed straight line in a plane perpendicular to the direction of propagation. However to define the properties of plane polarized light completely we have to define two planes, one containing the vibrations and other perpendicular to it, as the properties of plane polarized light differ with respect to these two planes. The plane containing the direction of vibration and direction of propagation or the plane in which the vibration takes place is called the plane of vibrations and a plane perpendicular to plane of vibration is called plane of polarization. We can also define plane of polarization as the plane passing through the direction of propagation and containing no vibrations.

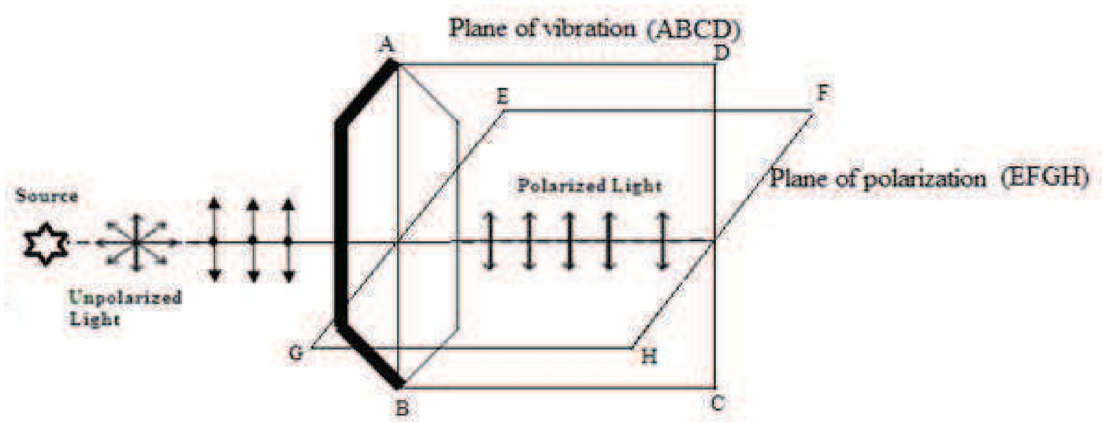


Figure 10. 5

As shown in Figure 10.5 the plane ABCD is the plane of vibration and the plane EFGH is the plane of polarization.

10.8 METHODS OF PRODUCTION OF PLANE POLARIZED LIGHT

Plane polarized light may be produced from unpolarized light using one of the five optical phenomena listed as below:

1. Polarization by reflection (example - Biot's polariscope)
2. Polarization by refraction (example - piles of plates method)
3. Polarization by double refraction (example-Nicol prism)
4. Polarization by selective absorption or dichroism (example-Polaroids)
5. Polarization by scattering (example-light from a blue sky)

10.9 PLANE POLARIZED LIGHT BY REFLECTION

Producing plane polarized light by reflection is the simplest way. In 1808 E.L. Malus noticed that when natural or unpolarized light is incident on a transparent medium like glass or water the reflected light is partially plane polarized. The degree of polarization depends upon the incident angle on the surface and upon the material of the surface. At a certain angle of the incidence depending upon the nature of the reflecting surface the reflected light is completely plane polarized. This angle of incidence is called angle of polarization or polarizing angle.

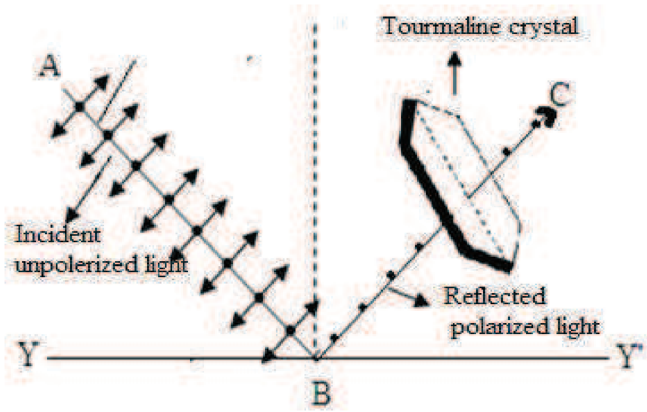


Figure 10.6

Here it must be noted that light reflected from the metallic surface contained a variety of vibration directions; i.e. reflected light from metallic surface is unpolarized. But if light is reflected from dielectric surface such as glass, water etc. is linearly polarized. If the extent of linear polarization is large enough as glare from field of snow on bright sunny day, its glare from the surface may be almost blinding to human eye.

When light wave is incident on a boundary between two dielectric materials, part of it is reflected and part of it is transmitted. Let a beam of unpolarized light incident along AB on a glass surface YY’ and reflected as BC. To show that reflected light is plane polarized a tourmaline crystal is placed in the path of reflected ray BC and rotated about BC as axis. It is observed that the transmitted light shows variation in intensity. It proves partial plane polarized nature of reflected ray. At polarizing angle of incidence, reflected ray from crystal is almost completely extinguished shows that maximum percentage of plane polarized light. Further rotation about BC as axis, the intensity of reflected beam through crystal is twice maximum and minimum in one complete rotation depending upon whether axis of crystal is perpendicular or parallel to plane of incidence respectively. It indicates that the light vibration in reflected beam is perpendicular to plane of incidence.

However this particular method of polarizing light is not very advantageous as only a small portion of incident beam is reflected therefore the intensity of the reflected beam is very small.

10.9.1 Biot’s Polariscope

It is a simple instrument for producing and detecting plane polarized light by reflection. In place of tourmaline crystal Biot’s polariscope consists of two glass plates P_1 and P_2. To avoid internal reflection and absorb refracted light both the plates are painted back on their back surfaces. A monochromatic light ray AB falls on plate P_1 at polarizing angle and reflected vertically upwards along BC and incident on other plate P_2, which is held parallel to P_1, also at polarizing angle and is reflected along CD. When the plate P_2 is gradually rotated about BC as axis the angle of incidence on plate P_2 is still same. Intensity of reflected ray CD decreases and becomes zero for a rotation of 90^0 of plate P_2. On further rotation of P_2 intensity of CD goes maximum at 180^0 and then decreases till 270^0 where it again becomes zero and becomes again maximum at 360^0.

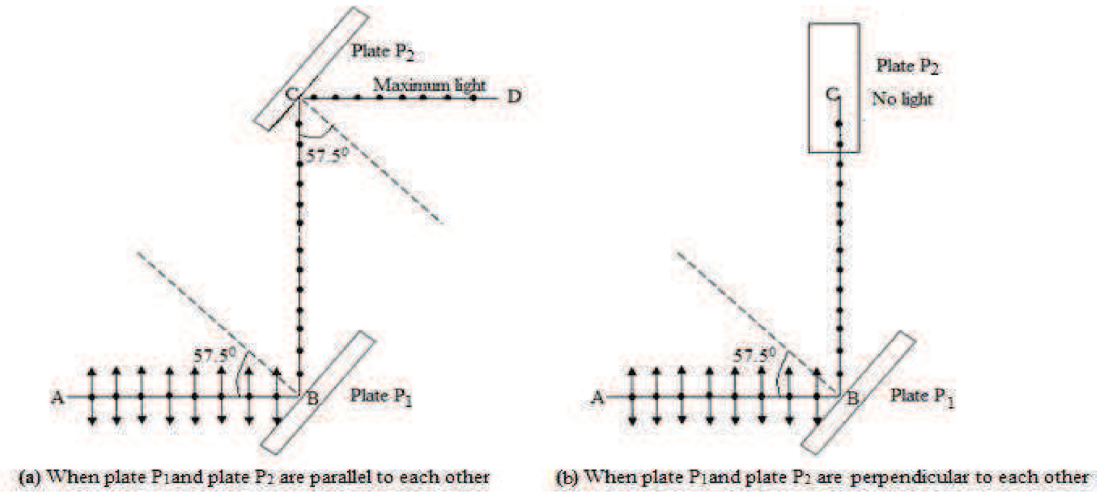


Figure 10.7

As the beam AB is incident on P_1 at polarizing angle, the reflected beam BC is completely plane polarized with its vibration perpendicular to the plane of incidence. When P_2 is parallel to P_1 (when P_2 is rotated through 180^0 or 360^0) the vibrations of BC are perpendicular to the plane of incidence therefore ray BC is completely reflected as Ray CD and intensity is maximum but when P_2 is perpendicular to P_1 (when P_2 is rotated through 90^0 or 180^0) vibrations of BC are parallel to the plane of incidence with respect to plate P_2 therefore no light is reflected.

As the plate P_1 causes the beam BC to be polarized and P_2 for analyzing the polarized light, plate P_1 is known as polarizer and P_2 as analyzer.

10.10 BREWSTER'S LAW

In 1811 A series of experiments on polarization of light by reflection from different reflecting surface was performed by Sir David Brewster. He observed that degree of polarization of ordinary light varies with angle of incidence and at a particular angle of incidence on the surface of a given transparent medium the reflected light is completely plane polarized. This particular angle of incidence is called polarizing angle or Brewster angle. However refracted light is still partially plane polarized. Further in case of reflected ray of light the plane of vibration is perpendicular to plane of incidence while it is parallel to plane of vibration in case of partially polarized transmitted light.

Brewster found that polarizing angle depends upon the refractive index of the medium. As a result of experiments he proved that the tangent of angle of polarization is numerically equal to the refractive index of the reflecting medium. If i_p is angle of polarization and μ is the refractive index of the medium then according to Brewster's law

$$\mu = \tan i_p \qquad \text{.......} \quad (10.1)$$

It must be noted here as refractive index depends upon the wavelength of incident light, angle of polarization varies with the wavelength hence complete polarization is possible with

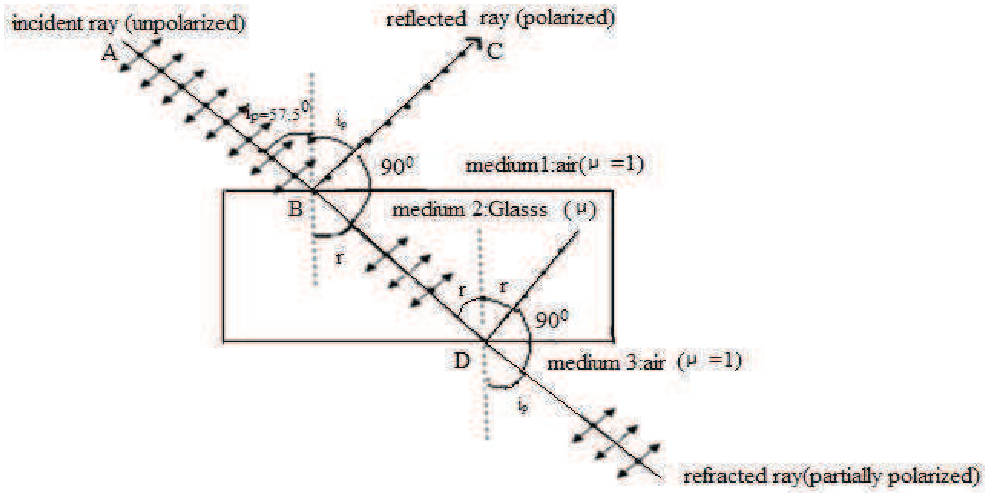


Figure 10.8

monochromatic light and not with white light. A direct consequence of Brewster's law is that when light is incident at the angle of polarization i_p, the reflected ray is at right angle to the refracted ray. This can be easily proved as follows

Let unpolarized beam AB is incident on a smooth glass surface at the polarizing angle i_p. It is reflected along BC and refracted ray along BD as shown in figure 10.8.

Then from Brewster's law $\quad \mu = \tan i_p = \frac{\sin i_p}{\cos i_p}$

Also from Snell's law $\quad \mu = \frac{\sin i_p}{\sin r}$

∴ $\quad \frac{\sin i_p}{\cos i_p} = \frac{\sin i_p}{\sin r}$

or $\quad \frac{\sin i_p}{\sin\left(\frac{\pi}{2} - i_p\right)} = \frac{\sin i_p}{\sin r}$

or $\quad \frac{\pi}{2} - i_p = r$

or $\quad i_p + r = \frac{\pi}{2}$

As $\quad \angle NBN' = \pi,$

$\quad i_p + \angle CBD + r = \pi$

or $\quad \angle CBD = \pi - (i_p + r) = \pi - \frac{\pi}{2} = \frac{\pi}{2}$

That is, at polarizing angle of incidence, the reflected ray is perpendicular to refracted ray.

Another important consequence of the law is that if unpolarized light is incident on the upper surface of a smooth parallel sided glass slab at the angle of polarization, the refracted ray also meets the lower surface of the slab at angle of polarization. As shown in the figure 10.8, the angle of incidence and refraction at the lower surface are r and i_p respectively. Hence at lower surface boundary between glass and air the refractive index of second medium (air) with respect to the first (glass)

$$\frac{1}{\mu} = \frac{\sin r}{\sin i_p}$$

Also $$\tan r = \frac{\sin r}{\cos r} = \frac{\sin r}{\cos\left(\frac{\pi}{2} - i_p\right)} = \frac{\sin r}{\sin i_p} = \frac{1}{\mu}$$

As $i_p + r = \frac{\pi}{2}$ or $r = \frac{\pi}{2} - i_p$ or $\cos r = \cos\left(\frac{\pi}{2} - i_p\right) = \sin i_p$

$$\frac{1}{\mu} = \tan r \qquad \text{.......}\ (10.2)$$

Therefore, r is the angle of polarization for the lower surface of the slab. Hence the light reflected from lower surface of glass slab is completely plane polarized while that refracted into the air is partially plane polarized.

Example 10.1: A ray of light is incident on the surface of a glass plate at the polarizing angle. Calculate the angle of incidence and angle of refraction.(μ for glass plate=1.732)

Solution: According to Brewster's law $\mu = \tan i_p$

Given μ for glass plate = 1.732, hence 1.732 = $\tan i_p$

or $$i_p = \tan^{-1} 1.732 = \tan^{-1}\sqrt{3} = \frac{\pi}{3}$$

Now if r is the angle of refraction we know that $i_p + r = \frac{\pi}{2}$

or $$r = \frac{\pi}{2} - i_p = \frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6} = 30^0$$

Example 10.2: The polarizing angle of a piece of glass for green light is 60^0. Calculate the angle of minimum deviation for a prism made of the same glass? (given angle of prism = 60^0)

Solution: According to Brewster's law $\mu = \tan i_p$

Given $i_p = 60^0$ hence from Brewster's law, refractive index of glass is

$$\mu = \tan 60^0 = 1.732$$

Now we know that $$\mu = \frac{\sin\frac{A+\delta_m}{2}}{\sin\frac{A}{2}}$$

Given angle of prism A = 60^0, hence $1.732 = \frac{\sin\frac{60^0+\delta_m}{2}}{\sin\frac{60^0}{2}} = \frac{\sin\frac{60^0+\delta_m}{2}}{0.5}$

or $\sin\frac{60^0+\delta_m}{2} = 1.732 \times 0.5 = 0.8660$

or $\frac{60^0+\delta_m}{2} = \sin^{-1} 0.8660 = 60^0$

or $\delta_m = 120 - 60 = 60$

10.11 PLANE POLARIZED LIGHT BY REFRACTION (PILES OF PLATES METHOD)

As stated before when light wave is incident on a boundary between two a dielectric material, part of it is reflected and part of it is transmitted. When polarized light is incident at Brewster angle on a smooth glass surface, the reflected light is totally polarized while the refracted light is partially polarized. If unpolarized light is transmitted through a single plate, the transmitted beam is only partially polarized contains vibrations parallel as well as perpendicular to plane of incidence.

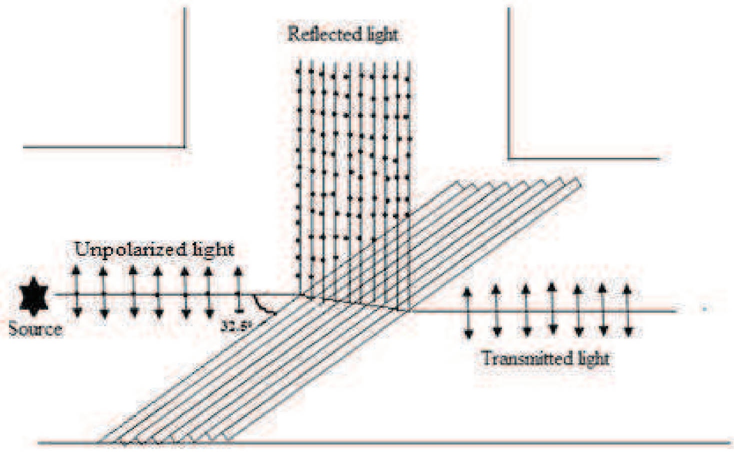


Figure 10.9

By increasing the number of plates more and more vibration perpendicular to the plane of incidence are reflected from successive surfaces resulting the filtering of from transmitted ray (figure 10.9). Consequently the refracted or transmitted beam gets richer and richer in the percentage of and ultimately the transmitted light is free from perpendicular vibration and consists of plane polarized vibrations parallel to plane of incidence only. The piles of plates consists of a number of thin glass plates supported in a tube of suitable size and inclined at an angle of about 33^0 to the axis of the tube. A beam of monochromatic light is incident at polarizing angle. The reflected as well as the transmitted beams of light are plane polarized parallel to the plane of incidence.

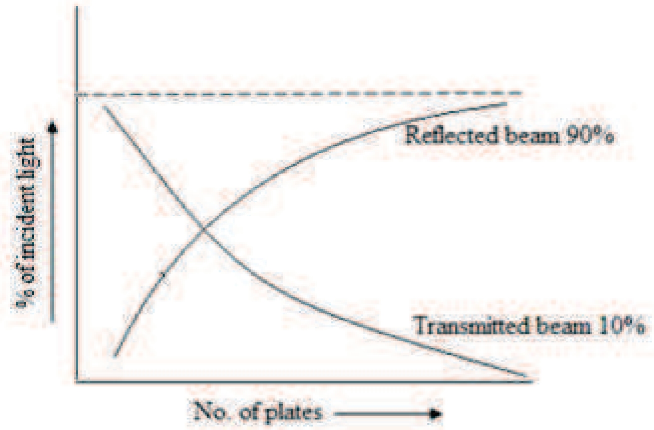


Figure 10.10

The graph shows the increase in intensity of the reflected light with increased number of plates (figure 10.10). However it serves no purpose to increase the number of plates after nearly 90% of the incident light is reflected.

If I_p and I_d denote the intensities of components with the vibrations parallel and perpendicular to the plane of incidence respectively in the transmitted light, then degree of polarization or proportion of polarization is given by

$$P = \frac{I_p - I_d}{I_p + I_d}$$

and % polarization is given by $\frac{I_p - I_d}{I_p + I_d} X\ 100$

For plane polarized light $I_d = 0$ hence P=1 and % polarization is 100%. For unpolarized light $I_p = I_d$ hence P = 0 and % polarization is 0%. As worked out by Provastaye and Desains

$$P = \frac{n}{n + \left(\frac{2\mu}{1-\mu^2}\right)^2}$$

Where n is number of plates and μ is the refractive index of the material.

However the concept of the degree of polarization cannot be applied to elliptically and circularly polarized light.

Example 10.3: A beam of light is passing through a pile of plate consisting of 12 plates of glass. Refractive index for the glass is 1.54. Find out the percentage of polarization.

Solution: According to Provastaye and Desains,

$$P = \frac{n}{n + \left(\frac{2\mu}{1-\mu^2}\right)^2}$$

Given n = 12 and $\mu = 1.54$, hence $P = \frac{12}{12 + \left(\frac{2 \times 1.54}{1-(1.54)^2}\right)^2} = .7041$

or $\% P = 70.41\%$

10.12 MALUS LAW

It is observed that whenever a plane polarized light falls on a rotating analyzer the intensity of light coming out from analyzer changes. Malus explains it by stating that "when plane polarized light falls on an analyzer the intensity of light coming out from the analyzer is proportional to the square of the cosine of the angle between the directions of transmission of the analyzer and the direction of vibration of electric vector in incident light. Mathematically Malus law can be expressed as

$$I = I_0 \cos^2\theta$$

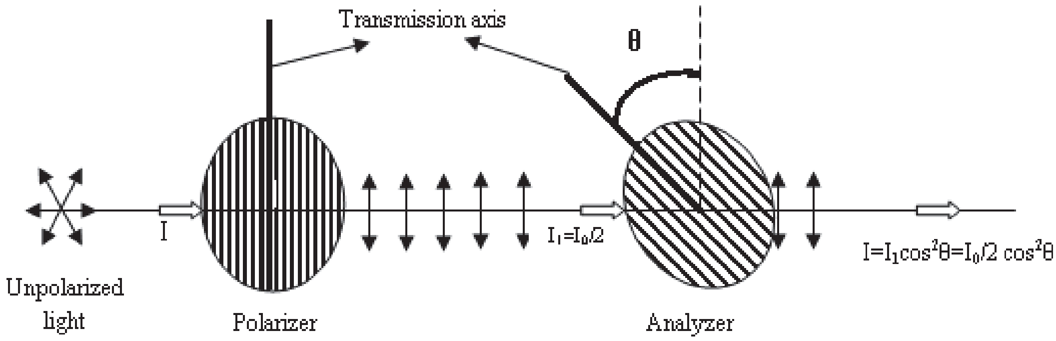


Fig 10.11

Where I_0 is the intensity of incident polarized light, θ is the angle between the direction of transmission of analyzer and polarizer.

Proof: Let A be the amplitude of the plane polarized light incident on analyzer and θ be angle between the plane of polarizer and analyzer. The amplitude A may be resolved into two components A $\cos\theta$ & A $\sin\theta$ which are parallel and perpendicular to the plane of transmission of analyzer respectively (Fig.11.12). As A $\cos\theta$ is parallel to the plane of transmission of analyzer it will be transmitted while component A $\sin\theta$ will be blocked.

Therefore intensity of transmitted beam $I = (A\cos\theta)^2 = A^2\cos^2\theta = I_0\cos^2\theta$ Where $I_0 = A^2$ is the intensity of incident plane polarized light. This proves the Malus law.

However Malus law doesn't hold good for unpolarized light as angle θ made by electric field vector with the plane of transmission is not constant for unpolarized light. As in unpolarized light the light vector vibrates in all possible direction in a plane perpendicular to direction by propagation. We have to put average value of $\cos^2\theta$ over all possible values of θ. Thus the intensity of the beam transmitted from the polarizing sheet is given by

$$I = I_0 \cos^2\theta = \frac{1}{2} I_0$$

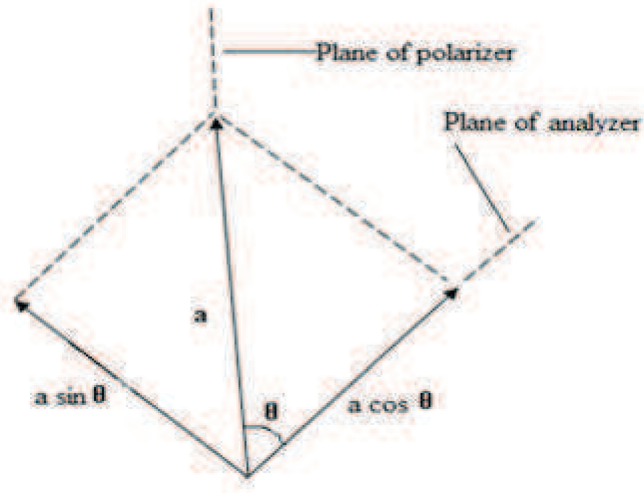


Fig.10.12

As the average value of $\cos^2\theta$ over all possible values of θ is ½ hence only 50% of incident unpolarized light is transmitted as plane polarized light by ideal polarizer.

When planes of polarizer and analyzer are parallel to each other the transmitted light is maximum. As in this case $\theta = 0^0$ hence $I = I_0 \cos^2 \theta = I_0 (\cos 0)^2 = I_0$

Also when planes of polarizer and analyzer are perpendicular to each other the transmitted light is zero. As in this case $\theta=90^0$ hence $I= I_0 \cos^2 \theta = I_0 (\cos 90)^2 = 0$

Example 10.4: If the intensity of the transmitted light by analyzer is 25% of the incident polarized light. What will be the angle between plane of polarizer and analyzer?

Solution: According to malus law $I = I_0 \cos^2 \theta$

Given $I = 25\%$ of I_0 i.e. $I_0/4$

Hence $I_0/4= I_0 \cos^2 \theta$

or $\cos^2 \theta = 1/4$

or $\cos \theta = 1/2$ or $\theta = 60^0$

Example 10.5: What is the intensity of the resultant beam if a) unpolarized light of intensity I_0 is incident on a polarizer? b) Plane polarized light of intensity I_0 is incident on a polarizer with its electric field makes an angle 60^0 with the axis of the polarizer?.

Solution: a) If unpolarized light of intensity I_0 is incident on a polarizer then the intensity of resultant beam is $I_0/2$.

b) When Plane polarized light of intensity I_0 is incident on a polarizer then according to Malus law $I = I_0 \cos^2 \theta$. Given $\theta = 60^0$, hence $I = I_0 \cos^2 60 = I_o/4$

10.13 SUMMARY

Polarization is the property which shows that light is a transverse wave. It not only tells the character of light wave but also tells whether the vibrations are linear, circular or elliptical. Generally light coming from a common light source is upolarized however can be transformed into different types of polarization. Unpolarized light is symmetrical while in polarized light electric field vibration deviates from completely symmetry and vibrates in single direction. Superposition of two mutually perpendicular plane polarized wave of equal amplitude and phase difference of 90^0 gives rise to circularly polarized light while different amplitude and any phase difference give rise to elliptically polarized light.

Plane polarized light can be obtained by many different methods like reflection, refraction, double refraction, dichroism and scattering. Brewster showed that reflected light is completely polarized at particular angle of incidence is called angle of polarization. He also showed that tangent of angle of polarization is equal to the refractive index of the medium. On the other hand Malus explains changes in intensity of light coming out from analyzer by stating that intensity of light coming out from the analyzer is proportional to the square of the cosine of the angle between the directions of transmission of the analyzer and the direction of vibration of electric vector in incident light.

10.14 GLOSSARY

Longitudinal Wave: A wave in which particles of the medium oscillate to and fro, in the form of compression and rarefaction, along the direction of wave propagation.

Transverse Wave: A wave in which every particle of the medium oscillates up and down, in the form of trough and crest, at right angles to the direction of wave propagation.

Unpolarized Light: The light in which the light vector (Electric vector) vibrates in all possible direction i.e. symmetrical about direction of propagation.

Polarized Light: The light, which acquired the property of one sidedness i.e. departure from complete symmetry.

Plane Polarized Light: The light in which the light vector (Electric vector) vibrates along a fixed straight line in a plane perpendicular to the direction of propagation.

Plane of Vibration: The plane containing the direction of vibration and direction of propagation or the plane in which the vibration takes place.

Plane of Polarization: A plane perpendicular to plane of vibration. It can also be defined as the plane passing through the direction of propagation and containing no vibrations.

Angle of Polarization: It is the angle of incidence on the surface of a given transparent medium for which the reflected light is completely plane polarized.

Polarizer: An optical device used to convert unpolarized light into polarized light.

Analyzer: Optical device which is used to analyze polarized light is called analyzer.

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10.15 REFERENCE BOOK

1. Optics by Ajoy Ghatak

2. A textbook of Optics by Brij Lal and Dr. N. Subrahmnyam

3. Optics by Dr. S.P. Singh and Dr. J.P. Agarwal

10.16 SUGGESTED READINGS

1. Fundamental of Optics by F. A. Jenkins and H. E. White.

2. The Feynman Lectures on Physics by Richard Feynman

3. Optics by Eugene Hecht

10.17 TERMINAL QUESTIONS

10.17.1 Short Answer Type Questions

1. What is polarization? Why sound waves can't be polarized?

2. Explain planes of polarization and plane of vibration?

3. State Brewster and Malus law.

4. Distinguish between polarized and unpolarized light.

5. Discuss some of the application of Brewster's law.

10.17.2 Long Answer Type Questions

1. Explain the concept of polarization of mechanical waves in a string by a two slit analogy. Also explain it with equivalent optical experiment.

2. Describe how to produce polarized light by reflection. Explain construction and working of Biot's polariscope.

3. State and explain Brewster's law? Prove that at polarizing angle of incidence the reflected and refracted rays are at right angles.

4. Describe how one can produce plane polarized light by refraction through piles of plate method.

5. State and prove law of Malus.

10.17.3 Numerical Questions

1. If a beam of light is passing through a pile consists of 8 plates of glass; what will be the percentage of polarization. (given μ_{glass}=1.5) (Ans. 58.1%)

2. A glass plate is to be used as a polarizer .find the angle of polarizer for it. Also find the angle of refraction. Given μ for glass=1.5 (Ans. 50.31)

3. The critical angle of incidence for total refraction in some case is 45^0. Find out the refractive index, polarization angle and angle of refraction corresponding to the polarizing angle. (Hint if c is critical angle than μ=1/Sin C)

(Ans. refractive index = 0.707, polarization angle = 35.3^0 and angle of refraction = 54.7^0)

4. Find the polarizing angle for light incident from (i)air to glass (ii)glass to air (iii)water to glass (iv)glass to water (v)air to water (vi)water to air. (Given Refractive index of glass =1.5 and for water=1.33). (Ans. (i) 57^0 (ii) 33^0 (iii) $49^012'$ (iv) $40^049'$ (v) $53^04'$ (vi) $36^057'$)

5. What will be the angle between plane of polarizer and analyzer if the intensity of the transmitted light by analyzer is 50% of the incident polarized light? (Ans. 45^0)

10.17.4 Objective Questions

1. Transverse phenomenon of light wave is proved by the phenomenon of

a. Polarization b. Interference
c. Diffraction d. Reflection

Ans. Option **'a'**

2. Plane of polarization and plane of vibration are

a. Inclined with 45^0 with each other b. Parallel to each other
c. Perpendicular to each other d. None of above

Ans. Option **'c'**

3. Sound wave in air can be polarized. This statement is:

a. True as they are transverse b. False as they are transverse.
c. True as they are longitudinal d. False as they are longitudinal

Ans. Option **'d'**

4. Malus law is

a. $\mu = \tan i_p$ b. $I = I_0 \cos^2\theta$
c. $\mu = \mu_0 \cos^2\theta$ d. $I = I_0 \cos\theta$

Ans. Option **'b'**

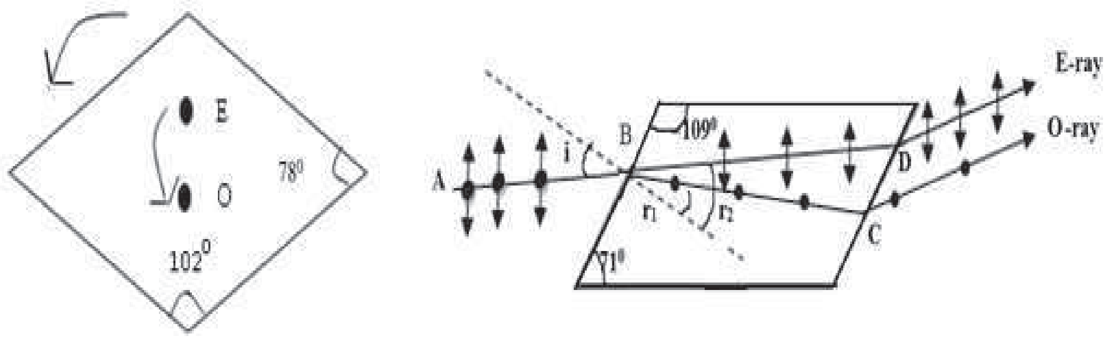
UNIT 11: DOUBLE REFRACTION

CONTANTS

11.1 INTRODUCTION

Splitting of a beam of unpolarised light into two refracted beam is called double refraction or birefringence. This phenomenon was first discovered by a Dutch philosopher E. Bartholinius in the year 1669.

Explanation: As shown in figure 11.1(a), if an image of ink dot on a sheet of paper is viewed through a calcite crystal we observe two images instead of usual one and if crystal is rotated slowly one of the two images rotates around the other stationary image.



a **b**

Figure 11.1

The refracted ray which produce stationary image is known as ordinary ray (O-ray) and the image as ordinary image. The refracted ray which produces the rotating image is called extraordinary ray (E-ray) and the image as extraordinary image. O-ray obeys the ordinary laws of refraction while E-ray doesn't.

Now let us explain this. As shown in Figure.11.1(b), if a narrow beam of light AB is incident on calcite crystal with angle of incidence i, it is split up into two rays (instead of one as usual) inside the crystal. The two images are obtained on the screen corresponding to O-ray and E-ray. O-ray travels along the side BC with angle of refraction r_1 while the E- ray travels along BD make angle of refraction r_2 and both the rays emerges out along CO and DE which are parallel to each other and also to incident beam.

By Snell's law $\mu = \frac{Sin\ i}{Sin\ r} = \frac{Velocity\ of\ light\ in\ vaccume}{Velocity\ of\ light\ in\ medium} = \frac{V_{vaccume}}{V_{medium}}$

Therefore, refractive index for O- ray and E- ray are respectively

$\mu_o = \frac{Sin\ i}{Sin\ r_1}$ and $\mu_e = \frac{Sin\ i}{Sin\ r_2}$ in case of crystal having $r_1 < r_2$ (e.g. calcite) $\mu_0 > \mu_e$

Hence $v_o < v_e$ i.e inside the calcite like crystal the velocity of E-ray is greater than O-ray or we can say that Inside the calcite crystal E-ray travels faster in comparison to O-ray.

It is also observed that for O-ray the refractive index is constant for all angle of incidence as it obeys ordinary law of refraction while for E-ray the refractive index is not constant but varies with angle of incidence. Therefore it can be easily concluded by this observation that the O-ray travels with the same speed in all the directions within the crystal while the E-ray travels with the different speeds in the different directions. Also both E-ray and O-ray shows variations in intensity when passes through rotating tourmaline crystal prove that both ordinary and extraordinary ray obtained by double refraction are plane polarized. It is also observed that their plane of polarization are at right angles to each other with O-ray vibrations perpendicular to the plane of paper while vibrations of E-ray is parallel to the plane of paper.

11.2 OBJECTIVES

After study of this unit the student will be able

- To understand the concept of double refraction or birefringence.
- To explain different terms e.g. optic axis, principal section and principal plane, positive and negative crystals.
- To understand Huygen's explanation of double refraction in uniaxial crystal.
- To describe construction and working of Nicol prism, double image prisms.
- To explain Huygen's construction for double refraction in uniaxial crystal.
- To explain polaroids and their uses.

11.3 GEOMETRY OF CALCITE CRYSTAL

Calcite crystal also known as Iceland spar (as its fine specimen are generally found in Iceland) is a colourless transparent crystal belongs to hexagonal system. Chemically it is hydrated calcium carbonate ($CaCO_3$). A piece of Iceland spar can be reduced to simple rhombohedron (as shown in figure 11.2) by cleavage or breakage. It is bounded by six faces each of which is a parallelogram with angles roughly equal to 102^0 and 78^0 (more accurately these are equal to $101^0 55'$and $78^0 5'$). The rhombohedron has only two diametrically opposite corners C and F where all the face angles are obtuse 102^0 ($101^0 55'$). These two corners termed as the blunt corners of the crystal. Out of the rest six corners, two angles are acute and the remaining one is obtuse

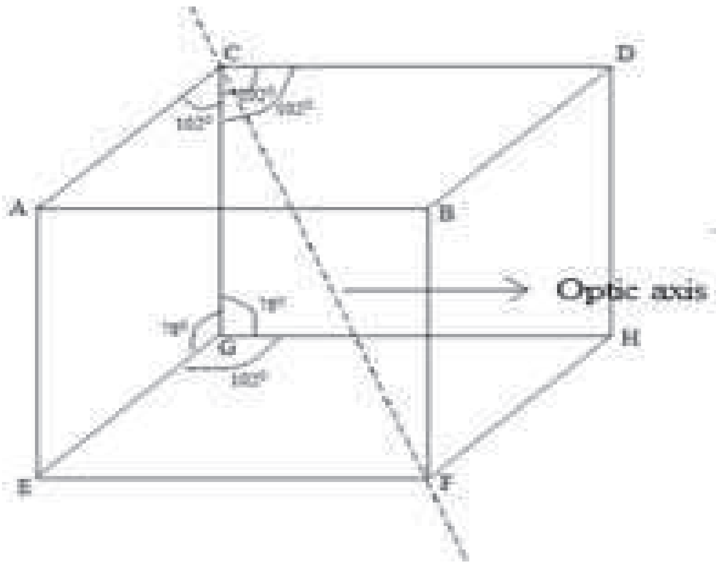


Figure 11.2

11.4 OPTIC AXIS, PRINCIPAL SECTION AND PRINCIPAL PLANE

Let us discuss about optic axis, principal section and principal plane one by one

11.4.1 Optic Axis

It is a direction along which or parallel to it a ray does not exhibit double refraction i.e. E-ray and O-ray travel with the same speed and in the same direction along the optic axis. Generally a line passing through any of the blunt corner and equally inclined with the three faces which meets at the corner, gives the direction of the optic axis of the crystal. We have to keep in our mind that optic axis is a direction not a particular line. Any line parallel to optic axis is also an optic axis. Crystal which has only one optic axis is known as uniaxial crystal e.g. calcite, quartz, tourmaline While in Borax, mica, aragonite & selenite there are two optic axes. Such crystals are known as biaxial crystal. However in case of biaxial crystal both the refracted rays are extraordinary.

Special cases: It has been observed that

a) A ray of light incident along the optic axis or in a direction parallel to optic axis the ray is not split up into ordinary and extraordinary components as E-ray and O-ray travel with the same speed and in the same direction along the optic axis.

b) A ray of light incident perpendicular to the optic axis the ray is not split up into ordinary and extraordinary components but in this case E-ray and O-ray travel with the same direction but with different speed.

11.4.2 Principal Section and Principal Plane

A plane containing the optic axis and perpendicular to the two opposite faces of crystal is called the principal section of the crystal for that pair of plane. For every point inside the crystal there are three principal sections, one for each pair of opposite crystal faces. For the calcite crystal principal section is a parallelogram having angles of 71^0 and 109^0.Since the opposite faces of a calcite crystals are always parallel the O-ray and E-ray emerges out parallel to each other and also parallel to incident ray.

To understand the directions of vibrations for the E-ray and O-ray we also have to define principal plane or O-ray and E-ray respectively. The principal plane of the ordinary ray is as the plane containing optic axis and O-ray and the plane containing optic axis and E-ray is called principal plane for E-ray. Except the case plane of incidence is the principal section of the crystal these two principal planes generally don't coincide.

11.5 HUYGEN'S EXPLANATION OF DOUBLE REFRACTION IN UNIAXIAL CRYSTAL

In order to explain the double refraction exhibited by uniaxial crystal Huygen's extended his wave theory of secondary wavelets. Huygen's postulated that

1. When light is incident on a doubly refracting crystal, every point of it becomes source of secondary wavelets and excites two separate wavelets within the crystals; one spherical wavelet associated with O-ray and other elliptical with E-ray. For O-ray, the crystal is isotropic and homogeneous, hence O-ray travels with the same velocity in all directions and therefore, the wavefront corresponding to it is spherical. For E-ray the crystal is anisotropic (different properties in different directions) hence its velocities varies with the directions i.e. wavefront can't be spherical it is ellipsoid.

2. The two wave fronts corresponding to E-ray and O-ray touch each other at the two points. The direction of the line joining these two points is called optic axis. As light advances through the crystal the two wave surfaces travel in different direction in the crystal therefore two refracted rays corresponding to E–ray and O-ray respectively emerges out of crystal.

11.5.1 Positive and Negative Crystals

Because of the two different types of wavefronts, two possibilities are there

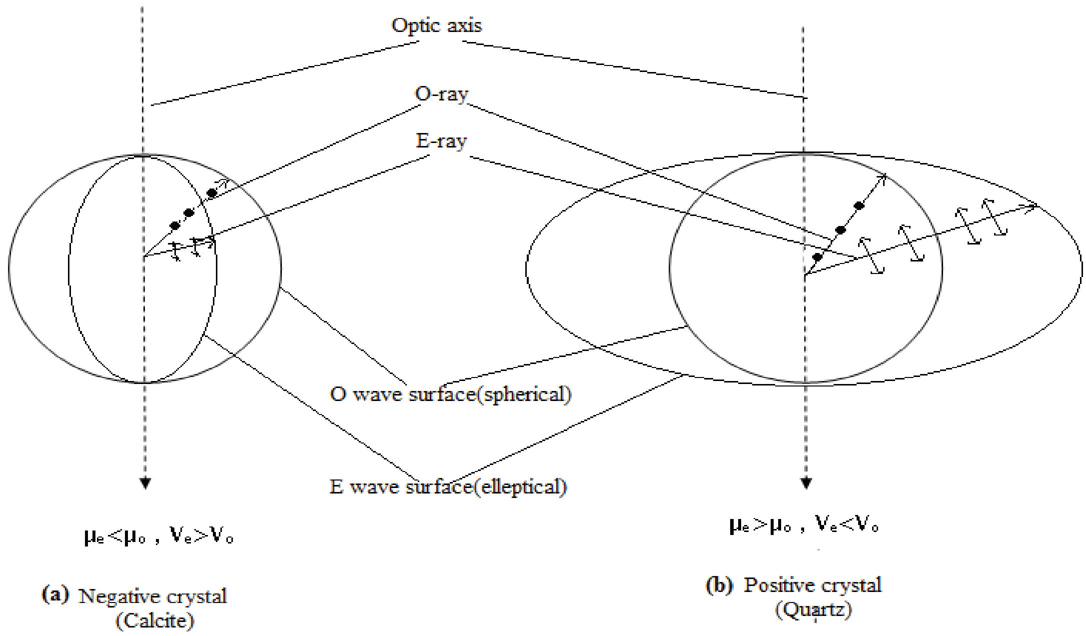


Figure 11.3

Case 1: The spherical wavefront of O-ray is enclosed by the ellipsoidal wavefront of E-ray. Such crystals are called negative crystals e.g. calcite. Obviously the diameter of the sphere is equal to the minor axis of the ellipsoid (figure 11.3(a)). For such crystals the velocity of ordinary ray is constant in all directions while velocity of extraordinary ray varies as the radius vector of ellipsoid. The velocity of E-ray is minimum and equal to O-ray along optic axis and a maximum in a direction perpendicular to direction of optic axis.

i.e., $v_e = v_o$ parallel to optic axis

$v_e > v_o$ in other direction

In this case, the refractive index for O-ray is greater than the refractive index for the E-ray

i.e., $\mu_e < \mu_o$

We can see that in negative crystals, extra ordinary wave surface behaves as if it were repelled away from the optic axis.

Case 2: The ellipsoidal wavefront of E-ray is enclosed by the spherical wavefront of O-ray. Such crystals are called positive crystals e.g., quartz. Obviously the diameter of the sphere is equal to the major axis of the ellipsoid (figure 11.3 (b)). For such crystals the velocity of ordinary ray is constant in all directions while velocity of extraordinary ray varies as the radius vector of ellipsoid. The velocity of E-ray is maximum and equal to O-ray along optic axis and a minimum in a direction perpendicular to direction of optic axis.

i.e., $v_e = v_o$ parallel to optic axis

$v_e < v_o$ in other direction

In this case, the refractive index for the E-ray is greater than the refractive index for the O-ray

i.e., $\mu_e > \mu_o$

In contrast to negative crystals we can see that in positive crystals extra ordinary wave surface behaves as if it were attracted towards the optic axis.

11.6 NICOL PRISM

Nicol prism is an optical device fabricated from calcite crystal for producing and analyzing plane polarized light named after its inventor William Nicol who designed it in 1820. Its action is based on phenomenon of double refraction. It's constructed in such a way that O-ray is eliminated by total internal reflection and we get only the plane polarized E-ray coming out of the Nicol.

11.6.1 Construction

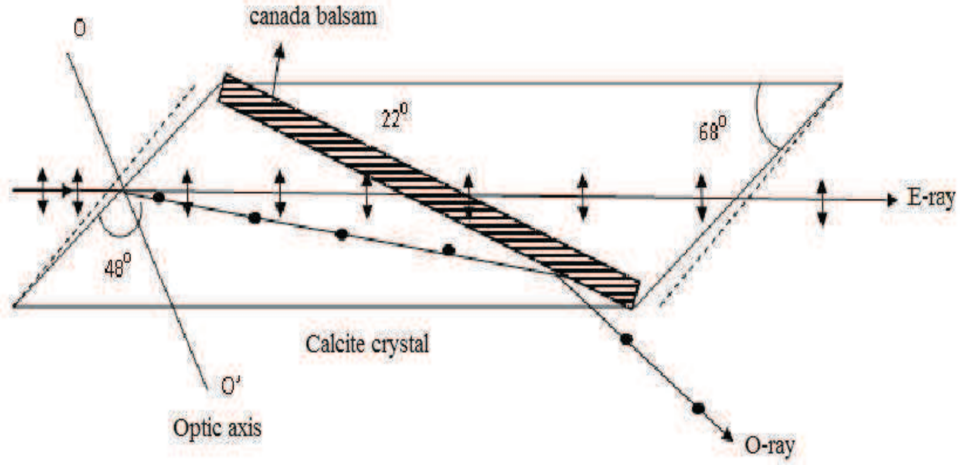


Figure 11.4

A rhomb of calcite crystal with length AB three times as long as its breadth CD is obtained by cleavage from the original crystal. The ends faces are grounded until they make an angle of 68^0 instead of 71^0 in natural crystal. Then the crystal is cut into two parts along a plane perpendicular to the principal axis as well as to the new end surfaces AB and CD (figure 11.4). Thus the two parts are grounded and polished optically flat than again cemented together with a layer of Canada balsam. Canada balsam is a clear transparent material whose refractive index (μ_{cb} = 1.55) lies between the refractive index for calcite for the O-ray (μ_o = 1.66) and E-ray (μ_e = 1.486) for sodium yellow light of mean wavelength λ = 5893 Å.

11.6.2 Action of Nicol Prism

When unpolarised light is incident on the prism parallel to AB. it suffers double refraction and splits into O-ray and E-ray as shown in figure. When these ray strikes at Canada balsam layer (which is denser than calcite for E-ray and less dense for O-ray) O-ray travels from an optically denser to a rarer medium it is totally reflected in case the angle of incidence is greater than a certain critical value (critical angle). This reflected ray is completely absorbed as the tube containing the crystal is coated black. E–ray is not reflected as it travels from an optically rarer to a denser medium hence E-ray is transmitted through the prism. Thus Nicol prism acts as a polarizer.

Now the reason behind natural angle 71^0 is reduced to 68^0 and choosing length three times to its width enables the O-ray to fall at the Canada balsam layer at an angle greater the critical angle.

Let θ is the critical angle for the O-ray. Than refractive index of O-ray with respect to Canada balsam layer is $\frac{\mu_o}{\mu_{cb}} = \frac{1.658}{1.55}$.

The condition for total internal reflection is sin θ = $\frac{1}{\mu} = \frac{1.55}{1.658} = 0.935$

Hence $$\theta = \sin^{-1}(0.935) = 69^0$$

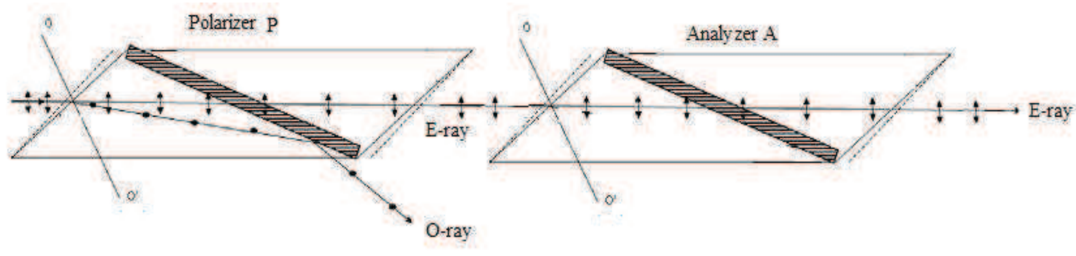
Thus, if the angle of incidence for the O-ray is greater than 69^0 it is totally internal reflected and absorbed by blackened wall.

11.6.3 Nicol Prism as Polarizer and Analyzer

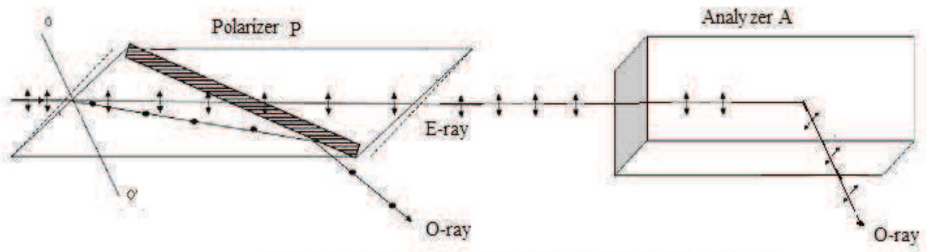
When two nicol prisms P and A are arranged coaxially adjacent to each other. First Nicol P acts as a polarizer and other acts as analyzer. Such a combination of polarizer and analyzer is called polariscope.

If unpolarized ray of light is incident on the first Nicol P, E-ray is transmitted with its vibration directly lying in the principal section of P. The state of the polarization of the light can be analyzed by another Nicol called analyzer. If principal section of analyzer A is parallel to principal section of polarizer P, E-ray is transmitted through analyzer A without any hindrance. In the case of Parallel Nicols the intensity of emergent E-ray is maximum (figure 11.5(a)). If the analyzer A is gradually rotated the intensity of E-ray decreases in accordance with Malus law.

In case the angle between principal sections of two Nicol is 90^0 they are called crossed Nicol. In the case E-ray when it enters the analyzer acts as O-ray inside the prism hence totally reflected at balsam layer hence no light comes out of the analyzer A (figure 11.5(b)). If A is further rotated the intensity of emergent light from A will go on increasing and is becomes maximum when its principal section is again parallel to that of P.



(a) Polariser P and analyser A are parrellel to each other



(b) Polarizer P and analyzer A are perpendicular to each other

Figure 11.5

In this way the Nicol prism A is used to analyze the plane polarized light produced by Nicol prism P hence named Analyzer and polarizer respectively.

11.6.4 Limitation of Nicol Prism

A Nicol prism cannot be used for highly convergent and divergent beams. It is found that the angle of incidence is limited for14^0 above which the O ray is also transmitted. It is also found that E- ray also totally reflected if the angle is greater than 14^0. Thus to avoid the transmission of O-ray and total internal reflection of E-ray the angle between the extreme rays of the incoming beam is limited to 2x14 = 28^0.

11.7 ACTION OF POLARIZER ON LIGHT OF DIFFERENT TYPE OF POLARIZATIONS

Whenever light of different types of polarizations is incident on polarizer its response is as follows

a) In case of unpolarized light, polarizer transmits half the intensity of light incident on it and shows no variation in intensity on rotation of polarizer.

b) In case of partially polarized light, intensity of transmitted light depends on the direction of transmission axis of the polarizer. It also shows variation in intensity on rotation of polarizer from maximum to minimum non zero value.

c) In case of plane polarized light, intensity of transmitted light varies two times from maximum to zero in one full rotation of the polarizer.

d) In case of circularly polarized light, intensity of transmitted light remains constant on rotation of polarizer like unpolarised light. This can be explained as follows: the circular vibrations can be resolved into two mutually perpendicular linear variations of equal amplitude .on incidence on polarizer the vibration parallel to its transmission axis passes through the polarizer while perpendicular component is blocked. It happens in all positions of the Polarizer in its rotation therefore transmitted light shows no variation in intensity on rotation of polarizer.

e) In case of elliptically polarized light, intensity of transmitted light varies two times from maximum to non zero minimum in one full rotation of the polarizer like partially polarized light. A maximum occurs when the transmission axis coincides with the semi major axis and minima when it coincides with the semi minor axis of the ellipse.

Example 11.1: Calculate the velocities of E-ray and O-ray in calcite (i) In a plane perpendicular to the optic axis (ii) along the optic axis. (Given $\mu_e = 1.486$, $\mu_o = 1.658$ and $C = 3\times10^8$)

Solution: (i) velocity of E-ray in calcite in a plane perpendicular to the optic axis

$$v_e = \frac{C}{\mu_e} = \frac{3 \times 10^8}{1.486} = 2.02 \times 10^8\ m/sec$$

Similarly, velocity of O-ray in calcite in a plane perpendicular to the optic axis

$$v_o = \frac{C}{\mu_0} = \frac{3\times10^8}{1.658} = 1.8 \times 10^8\ m/sec$$

Example 11.2: Two Nicols are first crossed and then one of them rotated through 45^0. Calculate what percentage of incident light will be transmitted.

Solution: Let the intensity of incident unpolarized light on first Nicol be I_0.On entering the first nicol it is broken up into O-ray and E-ray each of intensity $I_0/2$. First Nicol only transmits E-ray while O-ray is lost by total internal reflection. Therefore, $I_0/2$ will be the intensity of light incident on second Nicol.

Initially when two Nicols are in crossed positions, the angle between their principal planes is 90^0. If one of them is rotated through 45^0 from the crossed positions, the angle between the principal planes of transmission is = 90 ± 45 i.e. 135^0 or 45^0.

According to Malus law intensity of emerging beam from second Nicol is

$$I_1 = I\ Cos^2\theta = I_0/2Cos^2 135 \text{ or } I_0/2Cos^2 45$$

(As $I_0/2$ is the intensity of light incident on second Nicol)

Now $\quad Cos\ 135^0 = Cos\ 45^0 = 1/\sqrt{2}$

Hence $\quad I_1 = I_0/2\times (1/\sqrt{2})^2 = I_0/4$

∴ Percentage of light transmitted from the system = $\frac{emergent\ light\ from\ second\ nicol}{Incident\ light\ on\ the\ system} \times 100$

$$= \frac{I_0/4}{I_0} \times 100 = 25\ \%$$

11.8 HUYGEN'S CONSTRUCTION FOR DOUBLE REFRACTION IN UNIAXIAL CRYSTAL

Let us explain the phenomenon of double refraction in uniaxial crystal using Huygens theory. According to Huygen's theory O-ray and E-ray gives rise to spherical and ellipsoidal wavefront respectively. Except along optic axis (along which E- and O-ray travels with same velocity) E-wave travels with different velocity and the difference is maximum for direction perpendicular to the optic axis. The extraordinary wavefront depends upon how the surface of the crystal is cut relative to the optic axis. A thinner rectangular cross section can be cut from a bigger crystal in such a way that the optic axis lies

1. Inclined to the refracting edge.
2. Parallel to the refracting edge.
3. Perpendicular to the refracting edge.

Let us explain all possible 3 cases for the purpose of tracing the paths of O-ray and E-ray inside a negative uniaxial crystal.

11.8.1 When Optic Axis is Inclined to the Refracting Edge of Calcite Crystal

Let ABCD represent the calcite (negative crystal). Let PQ represent the incident plane wavefront incident normal to the crystal surface and crystal is so cut that the optic axis lies in the plane of incidence. According to Huygens, The Phenomenon of double refraction involves splitting of unpolarised wavefront to two wavefronts; one O-wave having spherical wavefront and other E-wave having ellipsoidal wavefront. Two wavefronts touch each other along the optic axis of the crystal. In negative crystal O-wave surface (sphere) lies within the E-wave (ellipsoid) (figure 11.6).

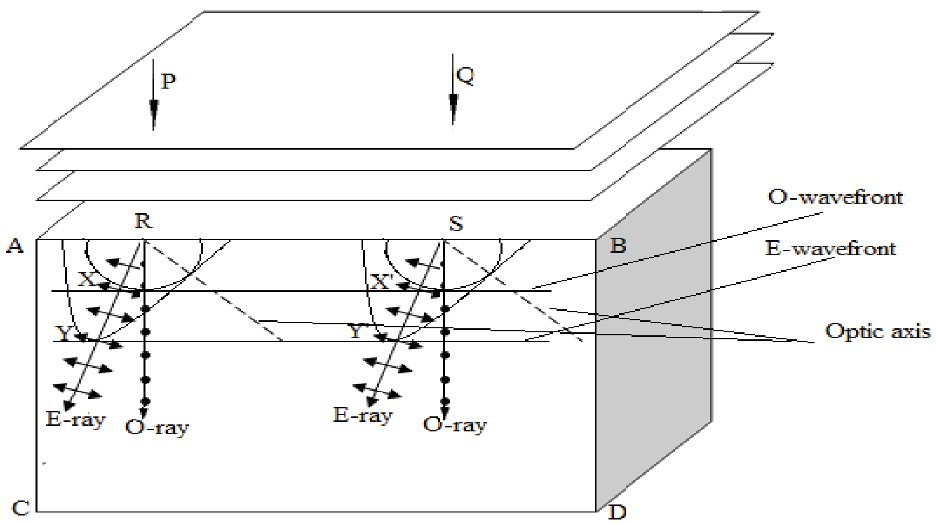


Fig. 11.6

Let us consider that parallel beam of light PQ falls normally in the surface of negative crystal. According to Huygen's theory as soon as the parallel beam strikes the crystal boundary, each point on the wavefront becomes a source of secondary disturbance and point R and S produce elliptical and spherical wavelets. After time T the new position of wavefront can be determined as follows. A circle of radius $v_o t$ is drawn taking R and S as centre. In negative crystal the elliptical wavefront can be drawn by taking major axis $2v_e t$ and minor axis $2v_o t$. Therefore, circular and elliptical wavefront touch each other along optic axis as required. If we now draw the common tangents to the secondary wavelets they represent the plane wavefront corresponding to the two rays thus XX' is the tangent to spherical wavefront and YY' is the tangent to the ellipsoidal wavefront.

Now the line joining the point of origin of wavelets to the point of tangency, give the direction of propagation of O- and E-ray. Therefore, O-ray travels along RX and SX' direction while E-ray travels along RY and SY' direction. This shows in this case O-ray and E–ray travels along different direction with different velocities.

11.8.2 (A) When Optic Axis is in the Plane of Incidence and Parallel to the Refracting Edge of Calcite Crystal

Let ABCD represent the calcite (negative crystal). Let PQ represent the incident plane wavefront incident normal to the crystal surface and crystal is so cut that the optic axis is parallel to the refracting edge CD and lies in the plane of incidence (figure11.7 (a)).

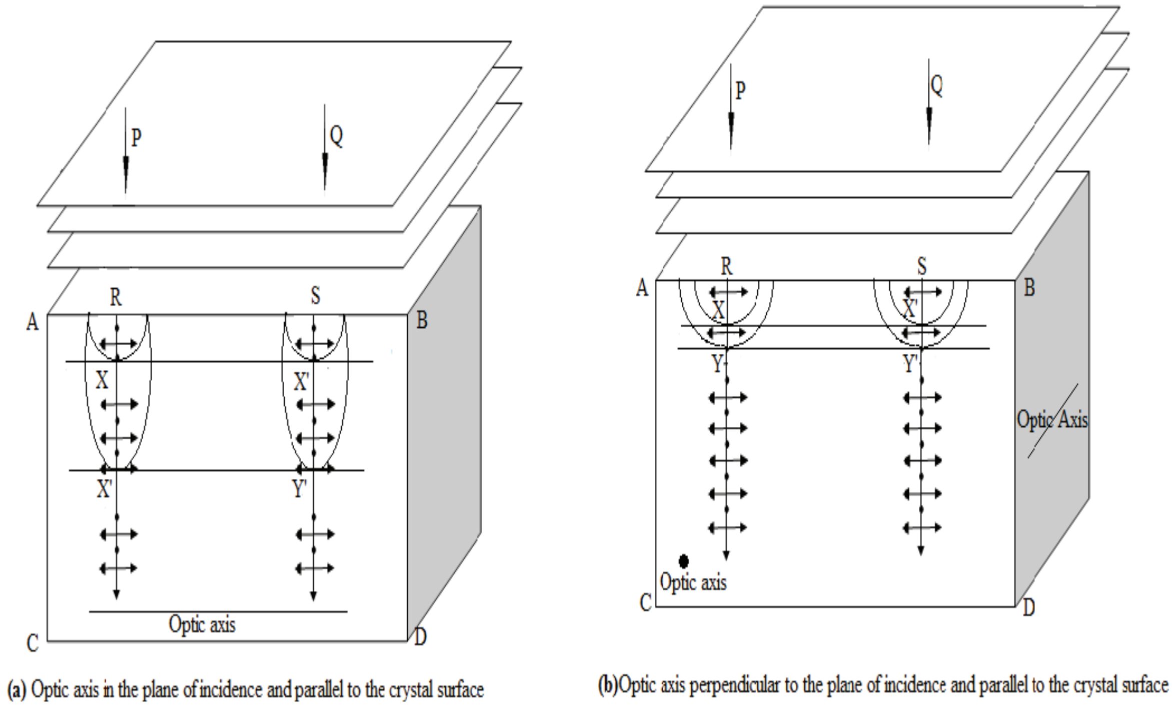


Fig. 11.7

According to Huygens, because of double refraction the unpolarized wavefront splits into two wavefront; one O-wave having spherical wavefront and other E-wave having ellipsoidal wavefront. Two wavefronts touch each other along the optic axis of the crystal. In negative crystal O-wave surface (sphere) lies within the E-wave (ellipsoid).

Let us consider that parallel beam of light PQ falls normally in the surface of calcite crystal. According to Huygen's theory as soon as the parallel beam strikes the crystal boundary, each point on the wavefront becomes a source of secondary disturbance and point R and S produce elliptical and spherical wavelets. After time t the new position of wavefront can be determined as follows. A circle of radius $v_o t$ is drawn taking A and B as centre. In negative crystal the elliptical wavefront can be drawn by taking major axis 2 $v_e t$ and minor axis $2v_o t$. Therefore, circular and elliptical wavefront touch each other along optic axis as required.

If we now draw the common tangents to the secondary wavelets they represent the plane wavefront corresponding to the two ray thus XX' is the tangent to spherical wavefront and YY' is the tangent to the ellipsoidal wavefront. Now the line joining the point of origin of wavelets to the point of tangency, give the direction of propagation of O- and E-ray. Therefore, O-ray travels along RX and SX' direction while E-ray travels along RY and SY' direction. This shows in this case although the O- and E-rays are not separated and travel along same direction, yet there is a double refraction as O-ray and E–ray travels with different velocities. As the E-ray travels faster than O-ray inside the crystal these ray comes out with a certain path difference. This property is used in the construction of quarter wave and half wave plate.

11.8.2(b) When Optic Axis is Perpendicular to the Plane of Incidence and Parallel to the Refracting Edge of Calcite Crystal

As shown in (figure 11.7 (b)) let ABCD represent the calcite (negative crystal) and PQ represent the incident plane wavefront incident normal to the crystal surface and crystal is so cut that the optic axis is parallel to the refracting edge CD but perpendicular to the plane of incidence. According to Huygens, because of double refraction unpolarised wavefront splits into two wavefront; one O wave having spherical wavefront and other E-wave having ellipsoidal wavefront. Two wavefronts touch each other along the optic axis of the crystal. In negative crystal O-wave surface (sphere) lies within the E wave (ellipsoid).

Let us consider that parallel beam of light PQ falls normally in the surface of calcite crystal. According to Huygen's theory as soon as the parallel beam strikes the crystal boundary, each point on the wavefront becomes a source of secondary disturbance and point R and S produce elliptical and spherical wavelets. As optic axis is perpendicular to the plane of incidence it means the spherical and ellipsoidal wavelets touch each other along a line perpendicular to the plane of incidence therefore in the plane of incidence both the wavelets appears spherical. After time t the new position of wavefront can be determined as follows.

For spherical wavefront A circle of radius $v_o t$ is drawn taking R and S as centre. In negative crystal the elliptical wavefront can be drawn by taking major axis 2 $v_e t$ and minor axis $2v_o t$.Therefore circular and elliptical wavefront touch each other along optic axis as required which in this case is perpendicular to the surface AB. If we now draw the common tangents to the secondary wavelets they represent the plane wavefront corresponding to the two ray thus XX' is the tangent to spherical wavefront and YY' is the tangent to the ellipsoidal wavefront. Now the line joining the point of origin of wavelets to the point of tangency, give the direction of propagation of O- and E-ray. Therefore O-ray travels along RX and SX' direction while E-ray travels along RY and SY' direction. This shows in this case although the O- and E- rays are not separated and travel along same direction, but with different velocities.

11.8.3 When Optic Axis is Perpendicular to the Refracting Edge and Lying in the Plane of Incidence of Calcite Crystal

As in previous case let ABCD represent the calcite (negative crystal) and PQ represent the incident plane wavefront incident normal to the crystal surface and crystal is so cut that the optic axis is parallel to the refracting edge CD but perpendicular to the plane of incidence (figure 11.8). According to Huygens, because of double refraction unpolarised wavefront splits into two wavefronts; one O wave having spherical wavefront and other E-wave having ellipsoidal wavefront. Two wavefront touches each other along the optic axis of the crystal. In negative crystal O-wave surface (sphere) lies within the E-wave (ellipsoid).

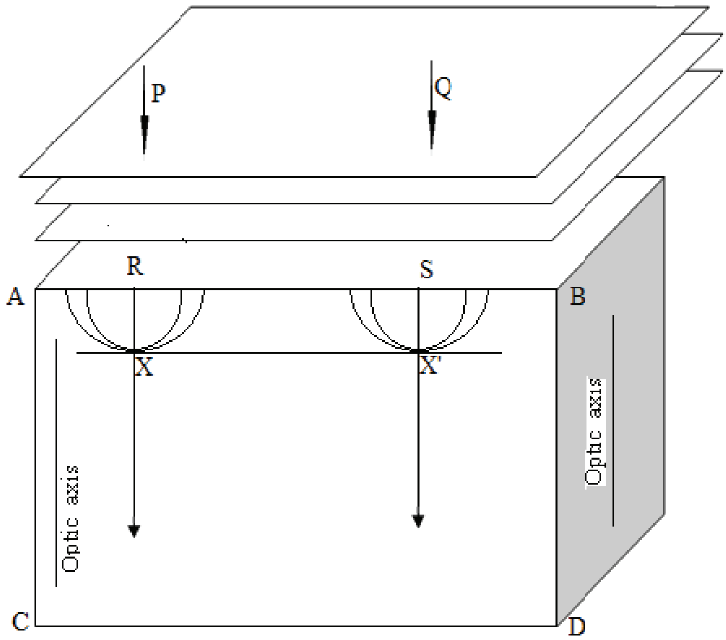


Figure 11.8

Let us consider that parallel beam of light PQ falls normally in the surface of calcite crystal. According to Huygen's theory, as soon as the parallel beam strikes the crystal boundary, each point on the wavefront becomes a source of secondary disturbance and point R and S produce elliptical and spherical wavelets. As the spherical and ellipsoidal wavelets touch each other along optic axis they touch at point X and X'. It is also evident from figure that section of ellipsoid lies outside the section of the circle. After time t the new position of wavefront can be determined as follows. For spherical wavefront A circle of radius $v_o t$ is drawn taking R and S as centre.

In negative crystal the elliptical wavefront can be drawn by taking major axis 2 $v_e t$ and minor axis $2v_o t$. Therefore circular and elliptical wavefront touch each other along optic axis as required which in this case is perpendicular to the refracting edge AB. If we now draw the common tangents to the secondary wavelets they represent the plane wavefront corresponding to the two ray thus XX' is the common tangent to spherical wavefront and the ellipsoidal wavefront. Now the line joining the point of origin of wavelets to the point of tangency, give the direction of propagation of O-and E-ray. Therefore O-ray as well as E-ray travels along RX and SX'. This shows in this case the O- and E- rays are not separated and travel along same direction with same velocity.

Note: *All the above cases can also be applied to positive uniaxial crystal like quartz. For them $\mu_e > \mu_o$ and the spherical wavefront of ordinary wave lies outside the ellipsoidal wavefront of extraordinary wave.*

Example 11.3: A plane polarized light is incident on a plate of calcite crystal with its faces parallel to the optic axis. If the light is incident with vibrations at an angle 60^0 on the face of the crystal, calculate the ratio of the intensities of ordinary and extraordinary ray.

Solution: Let A be the amplitude of the incident vibrations making an angle of 60^0 face of the crystal with. As optic axis is parallel to the faces ordinary and extraordinary ray vibration will be perpendicular and parallel to the optic axis. Therefore amplitude of ordinary and extraordinary ray will be A Sinθ and A Cosθ and intensity will be A^2 Sin$^2\theta$ and A^2 Cos$^2\theta$ where θ is given 60^0.

Hence the ratio of the intensities of ordinary and extraordinary ray

$$= \frac{A^2 \operatorname{Sin}^2 60}{A^2 \operatorname{Cos}^2 60} = \frac{(\sqrt{\frac{3}{2}})^2}{(\frac{1}{2})^2} = 3{:}1$$

Example 11.4: A 60^0 quartz prism is cut with its faces parallel to the optic axis. Calculate the angle of minimum deviation for yellow light for ordinary and extraordinary ray. (Given μ_o = 1.5422, μ_e = 1.5533, and λ = 5890 Å)

Solution: The refractive index of prism for ordinary and extraordinary ray is given by

$$\mu = \frac{\sin\frac{A+\delta_m}{2}}{\sin\frac{A}{2}}$$

where δ_m is angle of minimum deviation and A is the angle of prism = 60^0 (given)

(i) For ordinary ray μ_o = 1.5422

So $$\mu_0 = \frac{\sin\frac{A+\delta_{m0}}{2}}{\sin\frac{A}{2}} \quad \text{or} \quad 1.5422 = \frac{\sin\frac{60+\delta_{m0}}{2}}{\sin\frac{60}{2}}$$

or $$\sin\frac{60^0+\delta_{m0}}{2} = 1.5422\ \sin 30^0 = 1.5422\ \times \frac{1}{2} = 0.7711$$

or $$\sin\frac{60^0+\delta_{m0}}{2} = \sin(50^0 27')$$

or $$\frac{60^0+\delta_{m_0}}{2} = 50^0 27'$$

or $$\delta_{m_0} = 100^0 54' - 60^0 = 40^0 54'$$

(ii) For extraordinary ray $\mu_e = 1.5533$

So $$\mu_e = \frac{\sin\frac{A+\delta_{m_e}}{2}}{\sin\frac{A}{2}} \quad \text{or} \quad 1.5533 = \frac{\sin\frac{60+\delta_{m_e}}{2}}{\sin\frac{60}{2}}$$

or $$\sin\frac{60^0+\delta_{m_e}}{2} = 1.5533\ \sin 30^0 = 0.7766$$

or $$\sin\frac{60^0+\delta_{m_e}}{2} = \sin(50^0 57')$$

or $$\delta_{m_e} = 101^0 54' - 60^0 = 41^0 54'$$

11.9 DOUBLE IMAGES POLARIZING PRISMS

In Nicol prism O-ray is completely absorbed by total internal reflection and only the E-ray is transmitted. Also Nicol prism cannot be used in ultra violet light as Canada balsam layer absorbs these radiations. Double image prisms are optical devices used if two widely separated images due to both the ray on the emergent side of the prism are desirable. The Rochon prism and the Wollaston prism are two such double image prisms made by double refracting crystals (either quartz or calcite).

11.9.1 Rochon Prism

Rochon prism consists of two right angled prism ABC and BCD made of quartz or calcite and cemented together with glycerin or castor oil to enable them to be used with ultraviolet light so as to form a rectangular block with their refracting angles equal (figure 11.9 (a)). The prism ABC is cut such that the optic axis is parallel to the base AB and the incident ray (i.e. parallel to the plane of the paper and perpendicular to the refracting edge). While the other prism BCD has its optic axis perpendicular to the plane of incidence (i.e. perpendicular to the plane of the paper and parallel to the refracting edge). Light ray incident normally on the face AC of the prism passes without deviation up to boundary BC (O-ray and E-ray are not separated till the ray reach the boundary BC as optic axis is parallel to the plane of the paper.

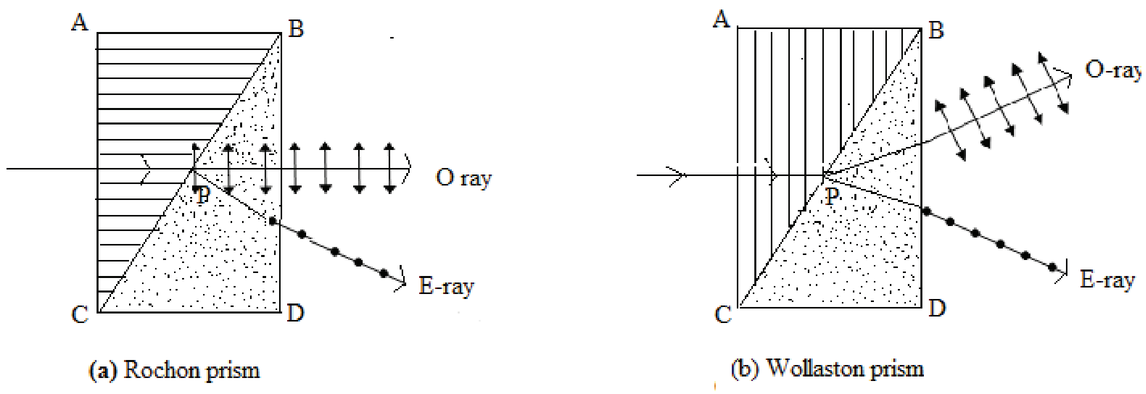


Fig 11.9

On interring the prism BCD at P it is split up into O-ray and E-ray and O-ray passes without deviation while the E-ray is deviated as well as dispersed. If the prism is made of Quartz the E-ray is deviated towards the base of the prism BCD because for quartz $\mu_0 < \mu_e$. In case of calcite the E-ray is deviated to the other side as for calcite $\mu_0 > \mu_e$. Here the emergent E-ray is chromatic while O-ray is achromatic.

11.9.2 Wollaston Prism

In construction like rochon prism Wollaston prism is also made of two right angled prism ABC and BCD made of quartz or calcite and cemented together with glycerin or castor oil to enable them to be used with ultra violet light so as to form a rectangular block with their refracting angles equal but here the optic axis of first prism ABC is perpendicular to base AB (figure 11.9 (b)). If a ray of light is made to incident normally on the face AC, then O-ray and E-ray travel along the same direction but with different speeds. After passing BC they exchange their nature i.e. ordinary ray behaves as the extraordinary and extraordinary as ordinary wave since the optic axis here is at right angles to that in ABC. Both rays are deflected in opposite direction as refractive index for one ray increases and for other ray decreases. In this way they form two widely separated plane polarized beams with mutually perpendicular plane of vibration. This prism is particularly useful in determining the percentage of polarization in a partially polarized beam.

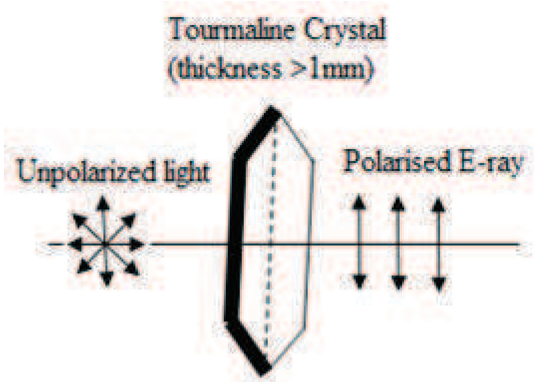


Fig. 11.10

Double image prisms are mostly used in telescopes for finding the angular diameter of planets, spectrophotometers and pyrometers.

11.10 DICHROISM OR SELECTIVE ABSORPTION

As explained earlier certain crystal like calcite, tourmaline etc. shows double refraction in which beam of unpolarised light splits up into E-ray and O-ray having vibration in the plane of incidence and perpendicular to plane of incidence. Some of the doubly refracting crystals or minerals absorbs either of E-ray or O-ray to a greater extent than the other. Tourmaline is one of such crystal which shows the property of selective absorption and absorbs O-ray to a greater extent than E-ray. Even a few mm thick crystal of tourmaline absorbs the O-ray completely and transmits only E-ray which is completely plane polarized with its vibration in the plane of incidence which is completely plane polarized. The phenomenon of selective absorption of one of the either ray (O-ray or E-ray) is called dichroism and crystals showing this property are called dichroic crystals. However the use of dichroic tourmaline crystal as a polarizer is limited as it also absorbs E-ray partially and the absorption of E-ray varies with wavelength. Therefore, whenever a white unpolarised light is incident on dichroic tourmaline crystal, emergent light is strongly chromatic.

11.11 POLAROIDS

In 1928, E.H. Land utilized the phenomenon of selective absorption in the construction of polaroids. Polaroids are basically large sized polarizing films mounting between two thin glass sheets. As these are inexpensive therefore widely used as a polarizer for commercial purposes.

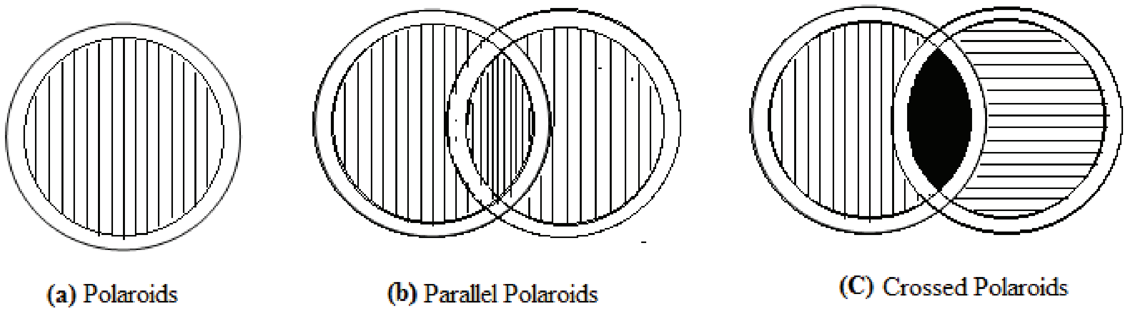


Figure 11.11

The Polaroid sheets can be fabricated in different manner.

1. Ultramicroscopic crystals of herpathite (an organic compound of iodine and quinine sulphate) are embedded in a thin sheet of nitrocellulose in such a manner that optic axis of all of them are parallel so that they function as single crystal of large dimensions. These herphatite crystals exhibits dichroism and even a very small thickness (≈ 0.005 inch) can absorb one of the doubly refracting ray completely.

2. H Polaroid: A sheet of polyvinyl alcohol (PVA) is heated and then stretched 3 to 8 times at its original length. During the stretch process the molecule of PVA are aligned along the direction of the stress and the material under stress becomes doubly refracting. It is than exposed to iodine vapor. The iodine atoms attach themselves to the straight long chain of

PVA molecule and consequently form long parallel conducting chain. Electrons from iodine can move easily along the aligned chain but not perpendicular to them. When unpolarised light is incident on the Polaroid sheet it absorbs E-ray because of the dissipative effects of the electron motion in the chain and O-ray is completely transmitted.

3. K Polaroid: If instead of iodine vapor the stretched film of PVA is heated in the presence of dehydrating agent it becomes strongly dichroic and very stable too.

For the protection of the Polaroid sheets these are generally mounted between plates of glass.

11.11.1 Uses of Polaroids

1. Polaroids are used to produce and analyze plane polarized light in modern polarizing instruments as they are relatively lower cost than Nicol.

2. They are widely used as polarized sun glasses in goggles to cut off glare produced by light reflected from polished and shining surface just as road and car hoods etc. to reach the eyes. The light reflected from such types of surfaces is partially plane polarized by reflection having more horizontal vibration than vertical. Polaroids with a vertical transmission plane avoids these horizontal vibrations and reduce glare considerably.

3. They are also used to control the intensity of light in aeroplane and train. For this one Polaroid is fixed outside the window and other is rotable and fitted inside. By rotation of inner Polaroid the intensity of light entering inside the train or aeroplane can be adjusted.

4. Polaroid glasses are used to produce 3D moving pictures.

5. They are also used improve the color contrast in old oil painting as well as in photography.

11.12 POLARIZATION BY SCATTERING

If a narrow beam of natural unpolarized light passes through a transparent medium containing a suspension of ultramicroscopic particle, the scattered light is found to be partially polarized. When incident light passes through the scattering medium, Electrons in the scattering medium starts to vibrate and emits most light in a direction perpendicular to its vibration. No light is emitted in a direction parallel to its vibration. Hence light scattered through about 90^0 to the incident direction is strongly polarized. The direction of vibration of light vector will be perpendicular to the plane of the paper. When ordinary unpolarized light passes through the atmosphere containing extremely small suspended particles of dust, smoke etc. it is scattered in all directions. The light is partially polarized. Blue colour of sky and red colour of sunrises and sunsets is due to the scattering of light ray from small atmospheric particles having dimensions less wavelength of light.

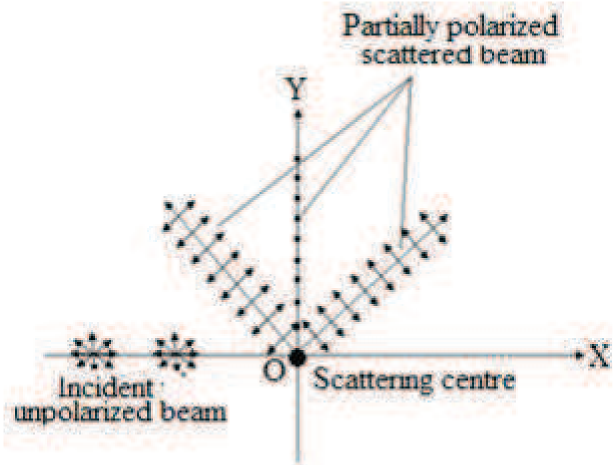


Figure 11.12

11.13 SUMMARY

Splitting of a beam of unpolarized light into two refracted beam named ordinary ray (O-ray) and extraordinary ray (E-ray), when falls upon calcite or quartz like crystals, is called double refraction or birefrigerence. Both E-ray and O-ray are plane polarized with their plane of polarization are at right angles to each other with O-ray vibrations perpendicular to the plane of paper while vibrations of E-ray is parallel to the plane of paper. E-ray and O-ray travel with the same speed and in the same direction along the optic axis. Crystal which has only one optic axis is known as uniaxial crystal and those two optic axes are known as biaxial crystal. However in case of biaxial crystal both the refracted rays are extraordinary. In order to explain the double refraction exhibited by uniaxial crystal Huygens extended his wave theory of secondary wavelets to explain double refraction in uniaxial crystals.

Double refraction is used to construct an optical device Nicol prism fabricated from calcite crystal for producing and analyzing plane polarized light. It's constructed in such a way that O-ray is eliminated by total internal reflection and we get only the plane polarized E-ray coming out of the Nicol.

The main results of Huygens construction for double refraction into uniaxial crystal using Huygens theory can be summarized as under. When Optic axis is inclined to the refracting edge of calcite crystal, O-ray and E–ray travels along different direction with different velocities. In other case E-ray travels faster than O-ray inside the crystal. Thus these ray comes out with a certain path difference when optic axis in the plane of incidence and parallel to the refracting edge of calcite crystal.

If Optic axis is perpendicular to the plane of incidence and parallel to the refracting edge of calcite crystal although the O- and E-rays are not separated and travel along same direction, but with different velocities. However if Optic axis is perpendicular to the refracting edge and lying in the plane of incidence of calcite crystal O- and E-rays are not separated and travel along same direction with same velocity.

Double image prisms are optical devices used if two widely separated images due to both the ray on the emergent side of the prism is desirable. The Rochon prism and the Wollaston prism are two such double image prisms made by double refracting crystals (either quartz or calcite). .

Some of the doubly refracting crystals or minerals absorbs either of E-ray or O-ray to a greater extent than the other. Tourmaline is one of such crystal which shows the property of selective absorption and absorbs O ray to a greater extent than E-ray. Polaroids are basically large sized polarizing films mounting between two thin glass sheets, show the phenomenon of selective absorption and are of multiple uses.

Polarized light can also be obtained by scattering of unpolarized light through a transparent medium containing a suspension of ultramicroscopic particle, Blue colour of sky and red colour of sky in sunrises and sunset are natural examples of polarized light from scattering.

11.14 GLOSSARY

Double Refraction: Splitting of a beam of unpolarised light into two refracted beam is called double refraction or birefringence.

Ordinary and Extra-ordinary Ray: In double refraction the refracted ray which obeys the ordinary laws of refraction is called ordinary ray or O-ray while the refracted ray which do not obeys the ordinary laws of refraction is called extraordinary ray or E- ray.

Optic Axis: It is a direction along which or parallel to it a ray does not exhibit double refraction i.e. E-ray and O-ray travel with the same speed and in the same direction along the optic axis. Any line parallel to optic axis is also an optic axis,

Uniaxial and Biaxial Crystal: Crystal which has only one optic axis is known as uniaxial crystal e.g., calcite, quartz, tourmaline While Borax, mica, aragonite & selenite, there are two optic axes. Such crystals are known as biaxial crystal. However in case of biaxial crystal both the refracted rays are extraordinary.

Principal Section and Principal Plane: A plane containing the optic axis and perpendicular to the two opposite faces of crystal is called the principal section of the crystal for that pair of plane. The principal plane of the ordinary ray is as the plane containing optic axis and O-ray and the plane containing optic axis and E-ray is called principal plane for E-ray.

Positive and Negative Crystals: Because of splitting of ordinary ray into two different ray both have different wavefront. When the spherical wavefront of O-ray is enclosed by the ellipsoidal wavefront of E-ray, crystals are called negative crystals e.g., calcite. If the ellipsoidal wavefront of E-ray is enclosed by the spherical wavefront of O-ray, crystals are called positive crystals e.g., quartz.

Nicol Prism: Nicol prism is an optical device fabricated from calcite crystal and constructed in such a way that O-ray is eliminated by total internal reflection and we get only the plane polarized E-ray coming out of the Nicol.

Double Image Prisms: Double image prisms are optical devices used if two widely separated images due to both the ray on the emergent side of the prism is desirable. The Rochon prism and the Wollaston prism are two such double image prisms made by double refracting crystals (either quartz or calcite).

Dichroism: Some of the doubly refracting crystals or minerals absorbs either of E-ray or O-ray to a greater extent than the other. The phenomenon of selective absorption of one of the either ray is called dichroism and crystals showing this property are called dichroic crystals.

Polaroids: Polaroids are basically large sized polarizing films mounting between two thin glass sheets. Polaroids are widely used as polarizer and analyzer for plane polarized light in modern polarizing instruments as they are relatively lower cost than nicol. They are also used in polarized sun glasses in goggles, in aeroplane and train, to produce 3D moving pictures and photography etc.

11.15 REFERENCE BOOKS

1. Optics by Ajoy Ghatak
2. A textbook of Optics by Brij Lal and Dr. N. Subrahmnyam
3. Optics by Dr. S.P. Singh and Dr. J.P. Agarwal

11.16 SUGGESTED READINGS

1. Fundamental of Optics by F. A. Jenkins and H. E. White.
2. The Feynman Lectures on Physics by Richard Feynman
3. Optics by Eugene Hecht

11.17 TERMINAL QUESTIONS

11.17.1 Short Answer Type Questions

1. What is double refraction?
2. What are ordinary and extraordinary rays?
3. Distinguish between i) Positive and negative crystal. ii) Uniaxial and Biaxial crystals
4. Unpolarized light falls on two polarizing sheets in crossed position so no light is transmitted from the combination. Can any light be transmitted if a third polarizing sheet is placed between them? Explain.
5. Write short notes on

   i) Double refraction ii) Dichroism iii) Polarized light by scattering

   iv) Principal section and principal plane v) Optic axis

11.17.2 Long Answer Type Questions

1. What is double refraction? Give the Huygens theory of double refraction in uniaxial crystal?

2. Explain the construction and working of Nicol prism? How it can be used as polarizer and analyzer.
3. Explain the construction and working of double image prism. Compare its properties with a Nicol prism
4. Explain the propagation of ordinary and extraordinary wavefronts in a calcite crystal for normal incidence with optic axis
   (i) Parallel to refracting edge (ii) Normal to the refracting edge
   (iii) Inclined to the refracting edge
5. What is dichroism? Explain the construction and uses of Polaroids.

11.17.3 Numerical Questions

1. Two nicols have parallel setting hence intensity of transmitted light is maximum. Through what angle should either Nicol be rotated in order to drop the intensity to one half of its maximum value? **(Ans.± 45^0)**
2. 2. Two plane polarized adjacent light beam **A** and **B** having mutually perpendicular plane of vibration are analyzed by a Nicol prism. In one position of analyzer beam **A** shows zero intensity. From this position when Nicol is rotated through angle 30^0, intensity of both beams becomes same .Calculate the intensity ratio of both the beam. **(Ans. I_A/I_B=2:1)**
3. 3. Two Nicols are first crossed and then one of them rotated through 60^0.Calculate what percentage of incident light will be transmitted. **(Ans. 37.5 %)**
4. 4. A plane polarized light is incident on a plate of calcite crystal with its faces parallel to the optic axis. If the light is incident with vibrations at an angle 45^0 on the face of the crystal. Calculate the ratio of the intensities of ordinary and extraordinary ray. **(Ans. 1:1)**
5. A partially polarized light beam is passed through a Nicol. the intensity of beam changes to 60% of maximum intensity when nicol is rotated through 90^0. Find the degree of polarization. **(Ans. 25%)**

11.17.4 Objective Questions

1. Plane polarized light produced by Nicol prism is due to
   a. Reflection
   b. Total internal reflection
   c. Refraction
   d. None of the above

   Ans. Option **'b'**

2. Calcite crystal is an example of
   a. Positive uniaxial crystal
   b. Negative biaxial crystal
   c. Positive biaxial crystal
   d. Negative uniaxial crystal

   Ans. Option **'d'**

3. Phenomenon of selective absorption of either of E-ray or O-ray is called
   a. Double refraction
   b. Diffraction
   c. Dichroism
   d. Total internal reflection

   Ans. Option **'c'**

4. E-ray and O-ray travels with the same velocity along the
   a. Along the optic axis
   b. along all the direction

c. Perpendicular to optic axis d. none of the above

Ans. Option **'a'**

UNIT 12: PLANE, CIRCULARLY AND ELLIPTICALLY POLARIZED LIGHTS

CONTANTS

12.1 INTRODUCTION

As of now, you will be well familiarized with the phenomenon of birefringence (or double refraction) in birefringent crystals (or doubly refracting crystals). Birefringence is a phenomenon of the production of two refracted rays instead of one after a beam of ordinary unpolarized light is allowed to pass through the birefringent crystals or doubly refracting crystals. These two refracted rays are generally known as ordinary and extraordinary rays are plane polarized and having perpendicular plane of vibrations. The ordinary rays (O-rays) obey the ordinary laws of refraction while the extraordinary rays (E-rays) do not obey the laws of refraction. These two rays travel with different speed in the birefringent crystals. The difference between the speeds of these two light rays produces the difference in the refracting angle.

For negative birefringent crystals, the refractive index for ordinary light is greater than extraordinary light ($\mu_o > \mu_e$), in contrast to positive crystal where the refractive index for extraordinary light is greater to ordinary light ($\mu_o < \mu_e$). For a given thickness of the birefringent crystal, a path difference or phase difference has been produced between the ordinary rays (O-rays) and extraordinary rays (E-rays) due to the different speeds inside the crystal. In this unit, we will study the various applications of birefringence property of doubly refracting crystals to produce and analyze plane, circularly and elliptically polarized light.

12.2 OBJECTIVES

After studying this unit, you should be able to

- Understand the concept of retardation plates
- Solve problems on retardation plates
- Apply the properties of retardation plates in producing plane, circular and elliptically polarized waves.
- Analyze the plane, circular and elliptically polarized waves using retardation plates.
- Understand the concept of rotatory polarization.

12.3 RETARDATION PLATES

A retardation plate, also known as plate retarders or wave plates, is an optically transparent birefringent crystal which resolves a beam of unpolarized light into two orthogonal components (ordinary light rays and extra ordinary light rays); change the relative phase difference between the components; then recombines the components into a single beam with new polarization characteristics. These plates are very useful to produce different kind of polarized light and convert one type of polarized light to other.

A retardation plate is generally a plane-parallel plate of a birefringent crystal like quartz, mica, magnesium fluoride and sapphire, with the optic axis in the plane of the surface. It is oriented so that incident polarized light may be resolved into components projected along the optic axis and perpendicular to it. These two components will experience a relative phase shift (retardation) proportional to the thickness of the plate. When the fractional part of this retardation is a nonzero value, the waveplate modifies the polarization state of incident polarized light polarizations from one state to another. A waveplate does not polarize light, but modifies the state of polarized light. Further, a relative phase shift produced by the retardation plate is subject to the availability of both components (E- rays and O- rays) of incident light which are parallel and perpendicular to its optic axis.

The amount of retardation can be expressed as birefringence times thickness. Birefringence ($\mu_o - \mu_e$) varies with temperature and wavelength. For a given temperature and wavelength, one can form the retardation plate or the wave plate for producing a wanted phase shift between the two components (ordinary light rays and extra ordinary light rays) for specific purposes. The retardation plates are mainly of two types as follows.

12.3.1 Quarter Wave Plate

A retardation plate of such a thickness that it produce a path difference of λ/4 (quarter of the wavelength of incident light) or a phase difference π/2 between the two components (ordinary light rays and extra ordinary light rays) of incident light beam passing through it, is known as quarter wave plate. Hence, for a quarter wave plate

$$\text{Path difference} = (\mu_o \sim \mu_e)\ x \text{ thickness}$$

or $$\lambda/4 = (\mu_o \sim \mu_e)\ t\ ,$$

Where, t is the thickness of the doubly refracting crystal or birefringent crystal

$$t = \frac{\lambda}{4(\mu_o \sim \mu_e)} \qquad \text{.......} \quad (12.1)$$

12.3.2 Half Wave Plates

If the thickness of the retardation plate is such that it produce a path difference of λ/2 (half of the wavelength of incident light) or a phase difference π between the two components (E-ray and O-ray) of incident light beam passing through it, then this retardation plate is known as half wave plate. Hence, for a half wave plate

$$\text{Path difference} = (\mu_o \sim \mu_e)\ x \text{ thickness}$$

$$\lambda/2 = (\mu_o \sim \mu_e)\ t\ ,$$

Where$t$ is the thickness of the doubly refracting crystal or birefringent crystal

$$t = \frac{\lambda}{2(\mu_o \sim \mu_e)} \qquad \text{.......} \quad (12.2)$$

Example 12.1: Plane polarized light is incident on a piece of quartz cut parallel to the axis. Calculate the least thickness for which the ordinary ray and the extraordinary ray combine to form the pane polarized light. Given $\mu_o = 1.5442$, $\mu_e = 1.5533$, and $\lambda = 5\times10^{-5}$cm.

Solution: The ordinary ray and the extraordinary ray combine to form the plane polarized light on emergence if the plate introduce a phase difference of π or a path difference of λ/2 between the ordinary ray and the extraordinary ray. The plate, which introduces a phase difference of π or path difference of λ/2 between the ordinary ray and extraordinary ray, is the half wave plate.

The data given are μ_o=1.5442, μ_e=1.5533, $\lambda=5\times10^{-5}$cm

Here, $$t = \frac{1}{\mu_o - \mu_e}\left(\frac{\lambda}{2}\right)$$

is the least thickness of a half-wave plate in positive crystals

Therefore, $$t = \frac{1}{1.5442 - 1.5533} \times \left(\frac{5\times10^{-5}}{2}\right) \text{ cm} = 2.75\times10^{-3} \text{ cm.}$$

Example 12.2: Calculate the thickness of the mica sheet required to make a quarter-wave plate and a half-wave plate for λ=5460 Å. The indices of refraction for the ordinary and extraordinary waves in mica are 1.586 and 1.592 respectively.

Solution: The data given are μ_o=1.586, μ_e=1.592, $\lambda=5460\times 10^{-8}$cm.
Thickness for the quarter- wave plate in the positive crystals is given by

$$t_4 = \frac{1}{\mu_e - \mu_0}\left(\frac{\lambda}{4}\right)$$

or $$t_4 = \frac{1}{1.592 - 1.586}\left(\frac{5460\times10^{-8}}{4}\right) = 2.275\times10^{-3} \text{ cm.}$$

Thickness for the half-wave plate in positive crystals is

$$t_2 = \frac{1}{\mu_e - \mu_0}\left(\frac{\lambda}{2}\right)$$

$$t_2 = \frac{1}{1.592 - 1.586}\left(\frac{5460\times10^{-8}}{2}\right) = 4.55\times10^{-3} \text{ cm.}$$

12.4 APPLICATION OF RETARDATION PLATES: THEORY OF THE PRODUCTION OF POLARIZED LIGHT

You have well understood so far about a birefringent crystal and its property of birefringence (production of E-rays and O-rays with their plane of vibrations perpendicular to each other), when an unpolarized light passes through it. Now, consider a beam of plane polarized light having their vibrations of electric field along one axis only perpendicular to the direction of propagation of beam, falls normally on a birefringent crystal (like calcite crystal) cut with optic axis parallel to its faces. Let $A = XY$ be the maximum amplitude of incident light which makes angle θ with optic axis of crystal. The plane polarized light split up in two components ordinary light (O-ray) and extra ordinary light (E-ray) with amplitude $XY \cos\theta$ along XA and $XY \sin\theta$ along XB.

As discussed earlier, these two rays travel with different speeds but in same direction inside the crystal.

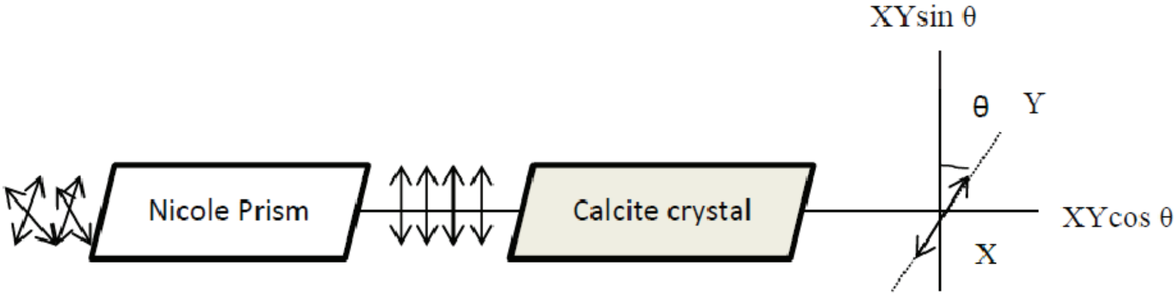


Figure 12.1

Due to this, the E-ray and O-ray develops a path difference or phase difference (δ) after emerging from the crystal. If A is the amplitude of the incident wave and plane polarized wave ($XY=A$ in figure 12.1), the E-rays and O–rays vibrates along perpendicular directions with amplitude $A\cos\theta$ and $A\sin\theta$ with a phase difference of δ depending upon the thickness of the crystal. The form of resultant vibration in the resultant vibration will be given by the resultant of these two components having simple harmonic vibrations in perpendicular directions with angular frequency ω and phase difference δ.

Let us consider the equations of these two components as

$$X = A\cos\theta\sin(\omega t + \delta) \qquad .... \ (12.3)$$

$$Y = A\sin\theta\sin\omega t \qquad .... \ (12.4)$$

Let $A\cos\theta = a$ and $A\sin\theta = b$, the above equation reduces to new form

$$X = a\sin(\omega t + \delta) \qquad .... \ (12.5)$$

$$Y = b\sin\omega t \qquad .... \ (12.6)$$

From equation (12.6), we have, $\sin\omega t = Y/b$ and $\cos\omega t = \sqrt{\left(1 - \frac{Y^2}{b^2}\right)}$

And from equation (12.5), $\frac{X}{a} = \sin\omega t\cos\delta + \cos\omega t\sin\delta$

$$\frac{X}{a} = \frac{Y}{b}\cos\delta + \sqrt{\left(1 - \frac{Y^2}{b^2}\right)}\sin\delta$$

or

$$\frac{X}{a} - \frac{Y}{b}\cos\delta = \sqrt{\left(1 - \frac{Y^2}{b^2}\right)}\sin\delta$$

Squaring on both sides, we get,

$$\frac{X^2}{a^2} + \frac{Y^2}{b^2}\cos^2\delta - 2\left(\frac{X}{a}\right).\left(\frac{Y}{b}\right)\cos\delta = \left(1 - \frac{Y^2}{b^2}\right)\sin^2\delta$$

On simplifying, we obtain,

$$\frac{X^2}{a^2} + \frac{Y^2}{b^2} - 2\left(\frac{XY}{ab}\right)\cos\delta = \sin^2\delta \qquad .... \ (12.7)$$

Thus the resultant wave equation is a general equation of ellipse. The polarization state of emerging light beam will depend on δ. We will discuss the special cases of above equation depending upon the phase difference (δ) between two components after emergence from the crystal:

12.4.1 Production of Plane Polarized Light

Case 1: When δ = 0, 2π, 4π, 6π . . . . . . , we have, sin δ = 0 and cos δ=1

From equation (12.7), we have,

$$\frac{X^2}{a^2}+\frac{Y^2}{b^2} - 2\left(\frac{XY}{ab}\right)= 0$$

$$\left(\frac{X}{a}-\frac{Y}{b}\right)^2 = 0$$

or
$$Y=\frac{b}{a}X \qquad \text{....... (12.8)}$$

This is an equation for straight line with slope $\frac{b}{a}$. Hence, the resultant light beam will be plane polarized with plane of vibration in a direction $\tan^{-1}(\frac{b}{a})$. If the amplitudes of both components are same ($a = b$), the resultant polarized light will vibrate at an angle of 45^0 with x axis.

Case 2: When δ = π, 3π, 5π, 7π . . . . . . then sin δ = 0 and cos δ= - 1

From equation (12.7), we have,

$$\frac{X^2}{a^2}+\frac{Y^2}{b^2} + 2\left(\frac{XY}{ab}\right)= 0$$

$$\left(\frac{X}{a}+\frac{Y}{b}\right)^2 = 0$$

$$\left(\frac{X}{a}+\frac{Y}{b}\right) = 0 \qquad \text{....... (12.9)}$$

Hence, the resultant light beam will be plane polarized with plane of vibration in a direction $tan^{-1}(-\frac{b}{a})$.If the amplitudes of both components are same ($a = b$), the resultant polarized light will vibrate at an angle of 135^0 with x axis.

12.4.2 Production of Elliptical and Circular Polarized Light

Case1: When δ = 3π/2, 7π/2, 11π/2, 15π/2 . . . . . ., we have, sin δ = -1 and cos δ=0.

From equation (12.7), we get,

$$\frac{X^2}{a^2}+\frac{Y^2}{b^2} = 1 \qquad \text{....... (12.10)}$$

This is an equation of symmetrical ellipse with major and minor axes as a and b. Hence, the resultant light beam will be elliptical polarized. From figure 12.2, we draw some position of the both of components (E-ray and O-ray) having phase differences of δ = 3π/2, 7π/2, 11π/2, 15π/2....... . When the displacement of one component (X= a sin (ωt + δ)) is maximum, the other has zero displacement (Y= b sin ωt = 0). As the displacement of the first component comes below its maximum displacement, an increase in the displacement of second component starts with phase differences of $\delta = 3\pi/2, 7\pi/2, 11\pi/2, 15\pi/2$. This process continues up to when the displacement of X- component becomes zero and the Y- component shows maximum displacement. The resultant of these intermediate displacements of both the component shows a clockwise or right handed rotation. Hence, it can be concluded that the resultant wave will be right handed elliptically polarized light.

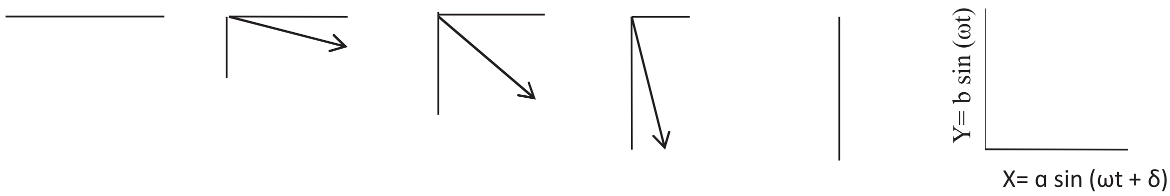


Figure 12.2

If the amplitudes of both the components (E–ray and O–ray) are same, i.e., a = b. The above equation of ellipse reduces to following form:

$$X^2 + Y^2 = a^2$$

This is an equation of circle. The emergent light will be right handed (clockwise) circularly polarized light if the of E-ray and O-ray has same amplitudes and maintains a phase difference of $\delta = 3\pi/2, 7\pi/2, 11\pi/2, 15\pi/2 \ldots$

Case2: When $\delta = \pi/2, 5\pi/2, 9\pi/2, 13\pi/2 \ldots\ldots$, we have, $\sin\delta = -1$ and $\cos\delta=0$

From equation (12.7), we obtain,

$$\frac{X^2}{a^2} + \frac{Y^2}{b^2} = 1$$

This is again an equation of symmetrical ellipse with major and minor axes are a and b, but the resultant is a combination of E- rays and O- rays with a phase differences which are different to Case 1. In this case, the resultant light beam will be left handed elliptical polarized. From figure 12.2, we can understand this situation. When the displacement of one component ($X= a \sin(\omega t + \delta)$) is maximum, the other has zero displacement ($Y= b \sin(\omega t) = 0$). As the displacement of the first component comes below its maximum displacement, an increase in the displacement of second component will appears with a phase difference of $\pi/2, 5\pi/2, 9\pi/2, 13\pi/2$...... .

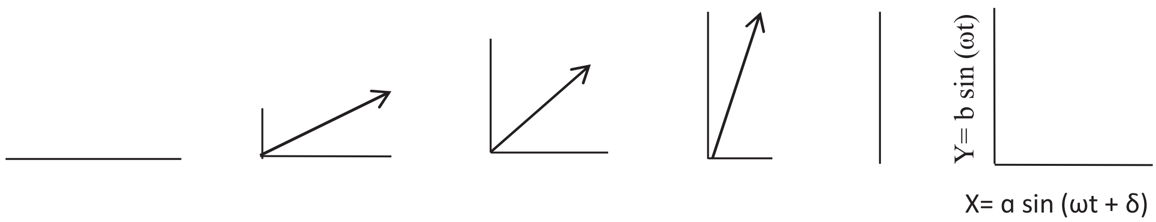


Figure 12.3

This process continues to the condition where the displacement of X-component becomes zero and the Y-axes has maximum displacement. The resultant of these intermediate displacements of both the component shows an anticlockwise or left handed rotation. Hence, it can be concluded that the resultant wave will be right handed elliptically polarized light. If the amplitudes of both the components (E– ray and O– ray) are the same, i.e., $a = b$ then above equation of ellipse reduces to following form

$$X^2 + Y^2 = a^2$$

This is an equation of circle. The emergent light will be left handed (anti clockwise) circularly polarized light if the of E- ray and O- ray has same amplitudes and maintains a phase difference of $\delta = \pi/2, 5\pi/2, 9\pi/2, 13\pi/2.....$.

12.5. EXPERIMENTAL ARRANGEMENTS FOR THE PRODUCTION OF POLARIZED LIGHT

1. Plane Polarized Light

When a beam of unpolarized monochromatic light is allowed to pass through a polarizer (Nicol Prism), it splits up into two components (O-rays and E-rays). Both rays are plane polarized with perpendicular plane of vibrations. The O-rays, follows the ordinary laws of refraction, total internally reflected and get absorbed inside the Nicol prism. The only ray coming out from the Nicol prism is E-rays, which is plane polarized. The Experimental arrangement to produce plane polarized light contains a Nicol prism which is placed after the unpolarized source light (figure 12.4).

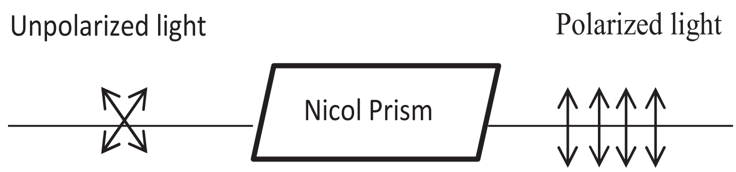


Figure 12.4

2. Circularly Polarized Light

Circularly polarized light is the resultant of two mutually perpendicular vibrations of equal amplitudes with a phase difference of $\pi/2$. To obtain circularly polarized light, the experimental arrangement shown in Figure 12.5.

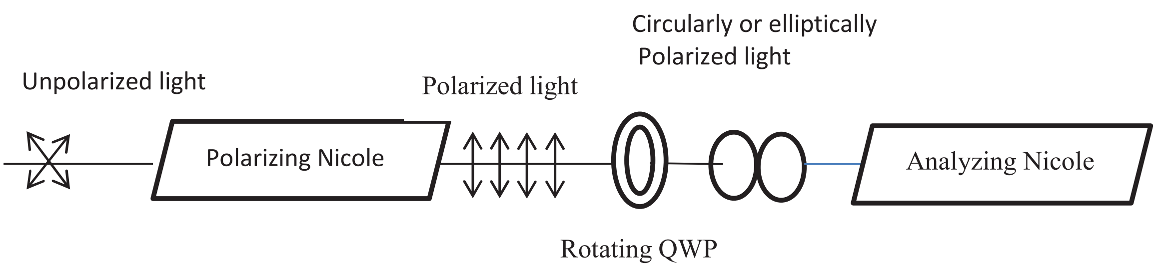


Figure 12.5

The unpolarized light is allowed to fall on a Nicol prism. The emergent light from Nicol is plane polarized light. Another Nicol prism (Analyzer) is placed at some distances from polarizing Nicol with crossed position. In this position no light is transmitted from second

Nicole (Analyzer), hence the field of view appears dark. A quarter wave plate (QWP) is then introduced between two Nicole in the path of plane polarized light such that polarized beam falls normally on the rotating quarter wave plate. The amplitude of polarized beam on entering the quarter wave plate is splits up into two mutually perpendicular components and receives a phase difference of π/2 after emergence. Now, the quarter wave plate rotated and the position of the quarter wave plate is such that the vibration in the incident polarized light make an angle of 45^0 with the optic axis of the plate. In this position the plane polarized light on entering the quarter wave plate is split up into two components (E-ray and O-ray) of equal amplitudes.

In this way the resultant of two perpendicular components with equal amplitudes having a phase difference of π/2 produces a circularly polarized light after emerging out from the quarter wave plate.

3. Elliptically Polarized Light

This is a more general case than circular polarization, in which there is a phase difference of π/2 between the two components. Elliptical polarization is the result when the components are equal with non-quarter-wave phase difference.

To obtain the elliptically polarized light, the experimental arrangement and undergoing processes is the same as shown in figure 12.5. The only change is the rotation of quarter wave plate should maintain such that the vibrations of light incident on it makes any angle other than 45^0. This makes two perpendicular components with unequal amplitudes having a phase difference of π/2 and the resultant wave produces an elliptically polarized light.

12.6 APPLICATION OF RETARDATION PLATES: ANALYSIS OF POLARIZED LIGHT

12.6.1 Analysis of Plane Polarized Light

When a given light is allowed to pass through a rotating Nicol prism and the intensity of emergent light varies from zero to maximum. The given light beam is plane polarized.

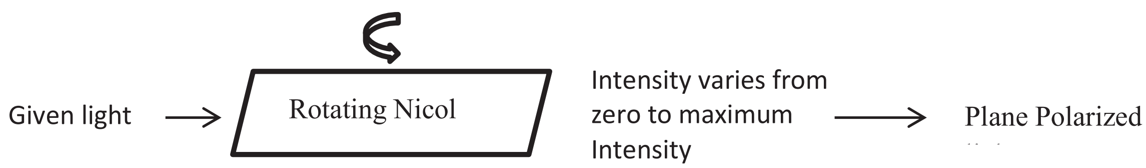


Figure 12.6

12.6.2 Analysis of Circularly Polarized Light

When a given light is allowed to pass through a rotating Nicol prism and the intensity of emergent light remains unchanged, this shows that incident light beam is either circularly polarized or un-polarized.

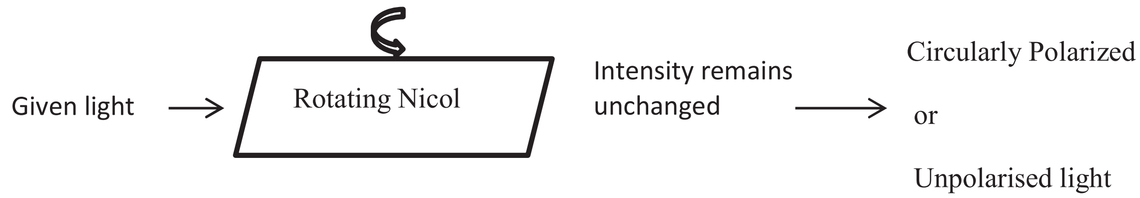


Figure 12.7

Now, given light is first passed through a quarter wave plate (QWP). If the given light is circularly polarized light, then it splits into two components (E-ray and O-ray) and because of QWP there is a phase difference of π/2 developed between them in addition to existing phase difference of π/2 which exists because of it being a circularly polarized light.

Therefore, the total phase difference between the E-rays and O-rays becomes 0 or π, which results in rectilinear vibrations and the emergent light is now plane polarized. If we pass this light again through a rotating Nicol prism, the intensity of emergent light will vary with zero to maximum intensity.

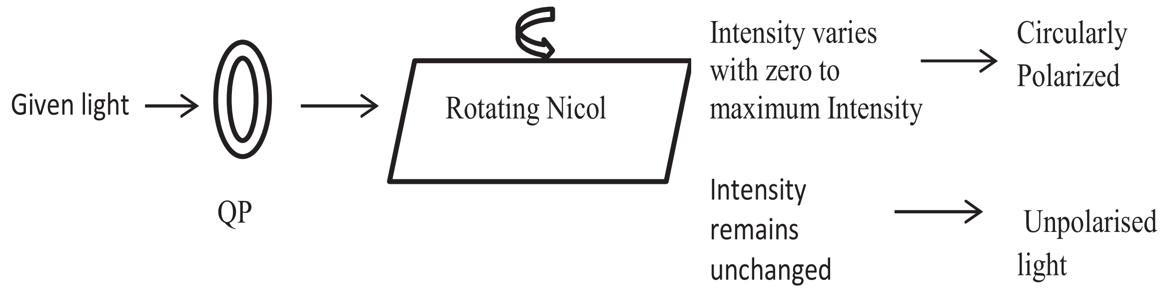


Figure 12.8

If the intensity of emergent light will remain unchanged even after passing a quarter wave plate and rotating Nicole, this shows that the incident light beam is unpolarised light.

12.6.3 Analysis of Elliptically Polarized Light

When beam of incident light is passed through the rotating Nicol prism and the intensity of emergent light varies from maximum to non-zero intensity, the incident light may be elliptically polarized or partially plane polarized light.

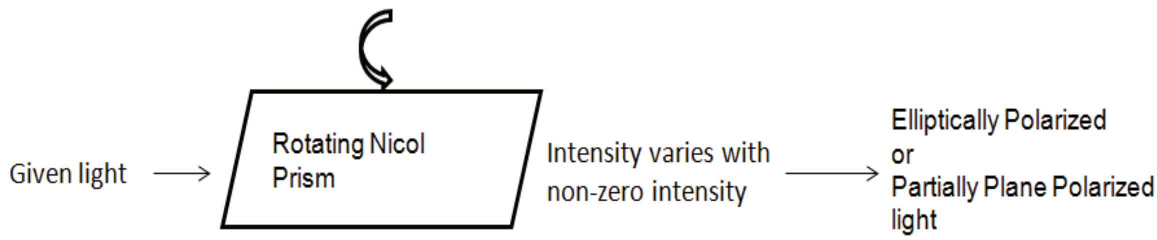


Figure 12.9

Now, given light is first passed through a quarter wave plate (QWP). If the given light is elliptically polarized light, then it splits into two components (E-ray and O-ray) and because of QWP there is a phase difference of π/2 developed between them in addition to existing phase difference of π/2 which exists because of it being an elliptically polarized light. Therefore, the total phase difference between the E- rays and O-rays becomes 0 or π, which results in rectilinear vibrations and the emergent light is now plane polarized.

If we pass this light again through a rotating Nicol prism, the intensity of emergent light will vary with zero to maximum intensity. On the other hand, if incident light is a partially plane polarized light, a mixture of plane polarized and unpolarized light, then after passing through a quarter wave plate it cannot become plane polarized and thus the intensity through the Nicol prism will change from maximum to non-zero.

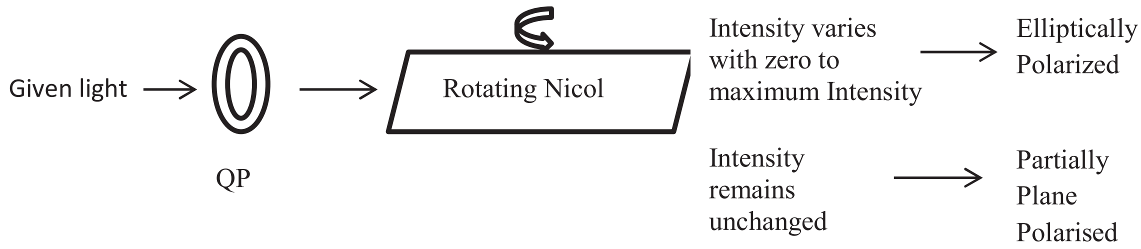


Figure 12.10

12.7 ROTATORY POLARIZATION

Rotatory polarization or optical activity is an ability of any substances to rotate the plane of incident linearly polarized light. The substances rotate the plane of vibration in clockwise direction are known as dextrorotatory substances and those substances produce a counterclockwise rotation are known as levorotatory substances. This property was discovered in quartz in 1811 by Arago. Two different crystalline structures of quartz produce d-rotatory and l-rotatory behavior. In the case of many naturally occurring organic compounds such as turpentine, sugar and, tartaric acid optical activity also shows rotatory polarization in the liquid state. This shows that the activity is associated with the individual molecules themselves. The detailed studies of rotatory polarization or optical activity are given in next unit (Unit 14).

Self Assessment Questions

1. How will you convert right circularly polarized light into left circularly polarized light?

2. How will you convert plane polarized light circularly polarized light into left circularly polarized light?

3. Fill in the blank

(i) When the emergent light is ……… ………polarized, the intensity varies from maximum to zero minimum after passing through rotating Nicol.

(ii) When a given light is allowed to pass through a rotating Nicol prism and the intensity of emergent light remains unchanged, this shows that incident light beam is either ………………… or ……………………...

12.8 SUMMARY

In this unit, you have studied about the application of birefringence or double refraction in birefringent crystals or doubly refracting crystals. To present the clear understanding of motion, some basic concepts like ordinary light and extraordinary light have been discussed. You have studied that the quarter wave plates and half wave plate's produces phase differences of π/2 and π respectively having numerous applications. These plates are formed by cutting the doubly refracting crystal for desired thickness. When a plane polarized light incident on a half wave plate, the emergent light is plane polarized. When a plane polarized light incident on a quarter wave plate such that E–rays and O–rays make an angle 45^0 with the optic axis, the emergent light is circularly polarized. You have also studied the theory behind the production of circularly and elliptical polarized lights. The phase difference between E-ray and O- ray is a deciding factor for clockwise circularly polarized light or counter clockwise circularly polarized light. By using quarter wave plate and Nicol prism, you can check about the polarization sate of any given light. You have also introduced about the rotatory polarization or optical activity. The substance showing this property is known as optically active substance. These are of two types: laevorotatory and dextrorotatory.

12.9 GLOSSARY

Birefringence: Double refraction

Ordinary light ray: A light ray obeys the ordinary laws of refraction (Snell' law)

Extraordinary light ray: A light ray does not obey the ordinary laws of refraction (Snell's law)

Limited: Restricted

Undergo: Suffer

Maintain: Sustain

Interactions: Exchanges

Resist: Refuses to go along with

Friction: Resistance

Retardation plates: The doubly refracting crystals cut along the thickness to produce a special phase difference or path difference

Assessment: Evaluation

12.10 REFERENCES

1. Optics, IGNOU, New Delhi
2. Engineering Physics, S.K. Gupta, Krishna Publication, Meerut.
3. Objective Physics, Satya Prakash, AS Prakashan, Meerut.

4. Optics by E. Hecht, 4th edition, Pearson Education Inc., New Delhi.
5. Physics, Part I, S. L. Gupta, Shubham Publication, Delhi.

12.11 SUGGESTED READINGS

1. Optics, Ajoy Ghatak, Tata McGraw Hill
2. Introduction to Electrodynamics by D.J. Griffith, Pearson Publication.
3. Waves by F.S. Crawford Jr., Berkeley Physics Course Vol. 3.
4. Optics by M.V. Klein and T.E. Furtak, Wiley.

12.12 TERMINAL QUESTIONS

12.12.1ObjectiveType Questions

1. When a plane polarized light is incident on a quarter wave plate with its vibration making an angle of 45^0 with the optic axis, the emergent light is

   (a) elliptically polarized

   (b) plane polarized

   (c) a mixture of elliptically and circularly polarized

   (d) circularly polarized

2. When a plane polarized light is incident on a quarter wave plate with its vibration making an angle other than 45^0 with the optic axis, the emergent light is

   (a) elliptically polarized

   (b) plane polarized

   (c) a mixture of elliptically and circularly polarised

   (d) circularly polarized

3. When a plane polarized light is passed through a half wave plate, the emergent light is

   (a) elliptically polarized

   (b) plane polarized

   (c) a mixture of elliptically and circularly polarized

   (d) circularly polarized

4. When a plane polarized light is passed through a quarter wave plate, the emergent light is

   (a) elliptically polarized

   (b) plane polarized

   (c) either elliptically or circularly polarized depending on the angle of incidence

   (d) circularly polarized

5. Circularly polarized light is produced if the amplitudes of the ordinary and extraordinary rays are equal and there is a phase difference of

(a) π
(b) $\pi/2$
(c) $\pi/4$
(d) 0

6. Elliptically polarized light is produced if the amplitudes of the ordinary and extraordinary rays are unequal and there is a phase difference of

(a) π
(b) $\pi/2$
(c) $\pi/4$
(d) 0

7. When elliptically polarized light, after passing through quarter wave plate, is observed through a rotating Nicol, the emergent light would have shown

(a) the variation of intensity with minimum not zero

(b) no variation in intensity

(c) the variation of intensity with minimum zero

(d) the intensity of incident and emergent light is same

8. When elliptically polarized light is observed through a rotating Nicol, the emergent light would have shown

(a) the variation of intensity with minimum not zero

(b) no variation in intensity

(c) the variation of intensity with minimum zero

(d) the intensity of incident and emergent light is same

9. When a circularly polarized light, after passing through quarter wave plate, is observed through a rotating Nicol, the emergent light would have shown

(a) the variation of intensity with minimum not zero

(b) no variation in intensity

(c) the variation of intensity with minimum zero

(d) the intensity of incident and emergent light is same

10. In an elliptically polarized light

(a) amplitude of vibration changes in direction only

(b) amplitude of vibration changes in magnitude only

(c) amplitude of vibration changes in magnitude and direction both

(d) none of above

12.12.2 Short Answer Type Question

1. What is the polarization state of emergent light if a plane polarized light is incident on a half wave plate ?
2. A plane polarized light is incident on a quarter wave plate such that O- rays and E-rays make an angle 45^0 with the optic axis. What will be polarization state of the emergent light?
3. A plane polarized light is incident on a quarter wave plate such that O- rays and E-rays make an angle other than 45^0 with the optic axis. What will be polarization state of the emergent light is?
4. What do you mean by rotatory polarization?

12.12.3 Long Answer Type Questions

1. Give the construction and theory of quarter wave plate and half wave plate.
2. Describe how, with the help of Nicole prism and quarter wave plate, plane polarized, circularly polarized and elliptically polarized lights are produced and detected.
3. What is meant by plane-polarized circularly polarized and elliptically polarized light? Show that the plane polarized and circularly polarized lights are special case of elliptically polarized light.
4. How will you find whether a given beam of light is ordinary, plane polarized, circularly polarized or elliptically polarized?

12.12.4 Numerical Answer Type Questions

1. Calculate the thickness of a calcite plate which would convert plane polarized light into circularly polarized light. The principal refractive indices are $\mu_0 = 1.658$ and $\mu_e = 1.486$ at the wavelength of light used as 5890 Å.
2. Calculate the thickness of mica sheet required for making a quarter wave plate for λ = 5460 Å. The indices of refraction for the ordinary and extraordinary rays in mica are 1.586 and 1.592.
3. Calculate the thickness of a double refracting plate capable of producing a path difference of $\lambda/4$ between ordinary and extraordinary waves. (λ = 5890 A, μ_o = 1.53 and μ_E = 1 .54).
4. Calculate thickness of quarter wave plate for light of wavelength 5000 Å. Given μ_o = 1.54 and ratio of velocity extraordinary to ordinary wave is 1.006.
5. We have a QWP of calcite corresponding to 4046 Å. A left circularly polarized light λ= 7065Å is incident on the plate. Obtain the polarization of the emergent beam. Use the information λ = 4046 Å, 7065 Å, μ_o = 1.68134, 1.65207, μ_E = 1.49694, 1.48359.
6. Quartz plate has μ_o = 1.54 and μ_e = 1.55 for sodium light. What minimum thickness of the quartz plate between the crossed polarizer and analyzer will produce annulment of the light, while the plate is cut parallel to optic axis?
7. (a) A left circularly polarized beam (λ = 5893 Å) is incident normally on a calcite crystal (with its optic axis cut parallel to the surface) of thickness 0.0005141mm. Find the state of polarization of the emergent beam. Given μ_o = 1.65836 and μ_e = 1.48641.
8. Repeat the above problem for quartz plate where all parameters are same except that μ_e = 1.54425 and μ_e = 1.55356.

9. A quarter waveplate is meant for wavelength 5893 Å. How much phase retardation it will show for wavelength 4300 Å?

12.13 ANSWERS

12.13.1 Self Assessment Questions

1. A right circularly polarized light is a combination of two plane polarized light with equal amplitudes and having a phase difference of $3\pi/2$, $7\pi/2$, $11\pi/2$; while a left circularly polarized light is a combination of two plane polarized light with equal amplitudes and having a phase difference of $\pi/2$, $5\pi/2$, $9\pi/2$. Hence, if a right circularly polarized light is allowed to pass through a half wave plate, which provide an additional phase difference of π between the two plane polarized light, the emergent light will be left circularly polarized light.

2. After passing through a quarter wave plate, the plane polarized light is converted to circularly polarized light.

3. (i) plane

(ii) Circularly polarized, unpolarized

12.13.2 Terminal Questions

Objective Type Question

1. (d) , **2.** (a) , **3.** (b) , **4.** (c), **5.** (b) , **6.** (b) , **7.** (c) , **8.** (a) , **9.** (c) , **10.** (c)

Short Answer Type Questions

1. Polarization state of emergent light will be plane polarized light.

2. Polarization state of emergent light will be circularly polarized light.

3. Polarization state of emergent light will be elliptically polarized light.

4. Rotatory polarization or optical activity is an ability of any substances to rotate the plane of incident linearly polarized light. The substances rotate the plane of vibration in clockwise direction are known as dextrorotatory substances and those substances produce a counterclockwise rotation are known as levorotatory substances

Numerical Type Questions

1. Thickness = $\lambda / [4\,(\mu_0 - \mu_e)] = 0.856\ \mu m$

2. $t = \lambda / 4\,(\mu_e - \mu_o) = 5460 \times 10^{-8} / [\,4\,(1.592 - 1.586)\,] = 2.275 \times 10^{-2}$ cm

3. $t = \lambda / 4\,(\mu_e - \mu_O) = 5.890 \times 10^{-5} / [4\,(1.54 - 1.53)\,] = 1.47 \times 10^{-3}$ cm

4. $t = \lambda / 4(\mu_e - \mu_O) = 5000 \times 10^{-10} / [4\,(1.54 - 1.53\,)] = 1.25 \times 10^{-5}$ cm

5. $t = \lambda / 4(\mu_e - \mu_O) = 4046 \times 10^{-10} / [4\,(1.68134 - 1.49694\,)] = 4046 \times 10^{-10} / [0.7376] =$ $0.54\ \mu m$

For λ = 7065 Å, birefringence = $(\mu_e - \mu_O)$ t = (1.65207 – 1.48359) 0.54 x 10^{-6} = 90980 Å

& the path difference for λ = 7065 Å is λ / 0.0775

6. $t = \lambda / 2(\mu_e - \mu_O)$ = 5893 x 10^{-10}/ [2 (1.55 - 1.54)] = 5893 x 10^{-10}/ [0.02] = 29.5 μm

7. Birefringence = $(\mu_e - \mu_O)$ t = (1.65836 – 1.48641) 5141 x 10^{-10} = 883.99 Å

& the path difference for λ = 5893 Å is λ /6.67

8. Birefringence = $(\mu_e - \mu_O)$ t = (1.55356 – 1.54425) 5141 x 10^{-10} = 47.86 Å

& the path difference for λ = 5893 Å is λ /123.12

9. $t = \lambda / 4(\mu_e - \mu_O)$ or $t (\mu_e - \mu_O) = \lambda / 4$ = 5893 Å / 4 = 1473 Å

For λ = 4300 Å, λ/ [t $(\mu_e - \mu_O)$] =4300 / 1473 = 2.91.

Hence a path difference between E-ray and O-ray is λ/2.91 or a phase change of

$(2\pi/\lambda)(\lambda/2.91) = 2\pi / 2.91 = 123.71^0$

UNIT 13: OPTICAL ACTIVITY

CONTANTS

13.1 INTRODUCTION

When two Nicol prisms (polarizer and analyser) are placed in the line of vision in the crossed position, no light gets through and the field of view is dark. Now, a crystal of quartz or a tube of sugar solution is placed between the crossed polarizer and analyzer, the light reappears. This shows that the quartz or sugar solution has rotated the plane of polarization of emergent plane polarized light after the first Nicole prism. In this unit, you will study about this property and its types. The angle through which the plane of polarization is rotated and the direction of rotation will also be discussed.

13.2 OBJECTIVES

After studying this unit, you should be able to

- Understand the concept of rotatory polarization or optical activity
- Apply the properties of optical activity to measure specific rotation.
- Solve problems on specific rotation
- Understand different kinds of polarimeters.

13.3 ROTATORY POLARIZATION AND OPTICAL ACTIVITY

When a plane polarized light is allowed to pass through certain substance, it is found that the plane of vibration of the emergent light is not the same as that of the incident light and it is rotated through a certain angle. This property of rotating the plane of vibration and plane of polarization is called rotatory polarization or optical activity and the substances which rotates plane of polarized light are called optically active substances, e.g., sugar solution, turpentine, sodium chlorate, cinnabar etc. Compounds that are optically active contain molecules that are chiral. Chirality is a property of a molecule that results from its structure. Optical activity is a macroscopic property of a collection of these molecules that arises from the way they interact with light.

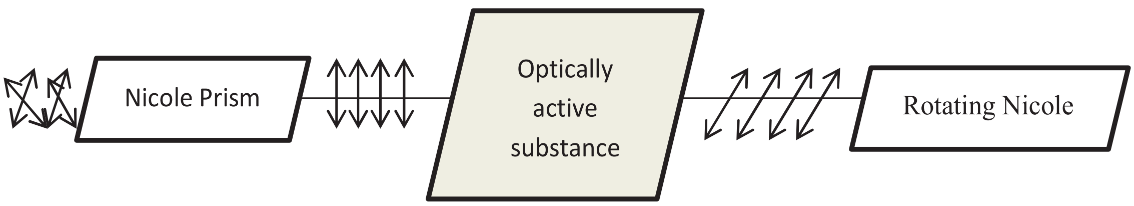


Figure 13.1

This ability of rotating the plane of polarization in certain substances was discovered by a French Scientist Arago in 1811. In figure 13.1, a Nicol prism (polarizer) is placed with monochromatic source of light. The emergent light after passing through Nicol prism is plane polarized, which is again passing through an optically active substance like turpentine. It is clearly observable from figure 13.1 that the plane of polarization and plane of vibration of emergent plane polarized light are different from the incident plane polarized light. This can be verified by using a second Nicol prism as an analyzer.

The rotation of polarized light by optically active substances has massive applications in different field of science. It provided insight into the physics of light, the structure of molecules, and the nature of life etc. The optically active substances are of two types

(i) Dextra rotatory or right handed.

(ii) Laevo rotatory or left handed.

13.3.1 Dextro Rotatory

The substance which rotates plane of polarization of light towards right or in clockwise Direction are called dextro rotatory or right handed.

13.3.2 Laevo Rotatory

The substance which rotates plane of polarization of light towards left or in anti-clockwise directions are called laevo rotatory or left handed.

13.4 FRESNEL'S THEORY OF OPTICAL ROTATION

Fresnel's theory of optical rotation is based on the fact that a linearly plane polarized light consists of resultant of two circularly polarized vibrations rotating in opposite directions with same angular velocities. Fresnel's made the following assumptions

(1) When a beam of plane polarized light enters in a crystal along the optic axis, it is broken up into two circularly polarized vibrations, one right handed and the other left handed.

(2) When a plane polarized light enters a crystal of an optically inactive substance (like calcite) along optical axis, it breaks up into two circularly polarized vibrations rotating in opposite direction with same angular frequency or velocity such that resultant of these two vibrations at all point of time is along the optic axis. The Vibrations of clockwise direction rotation in (in figure 13.2 (a)), are represented by OR and vibration rotating in counterclockwise direction are represented by OL. The resultant OR and OL at all the point of time will be along AB.

(3) In case of an optically active substance (like Quartz), a linearly polarized light on entering the crystal is resolved into two circularly polarized vibrations rotating in opposite direction with different angular velocity or frequency. In case of left handed optically active quartz crystal, anticlockwise vibrations travel faster, while in case of right handed optically active, quartz crystal, clockwise vibrations travel faster.

Consider a right handed quartz crystal in which clockwise component travels faster than left handed component. Suppose at any instant of time, right handed component traverse angle δ greater than left handed component as shown in figure 13.2 (b). The new position of resultant of L and OR will be along CD i.e., plane of vibration of light has been rotated through angle δ/2 towards right after passing through quartz crystal. The angle δ/2 depends upon thickness of crystal.

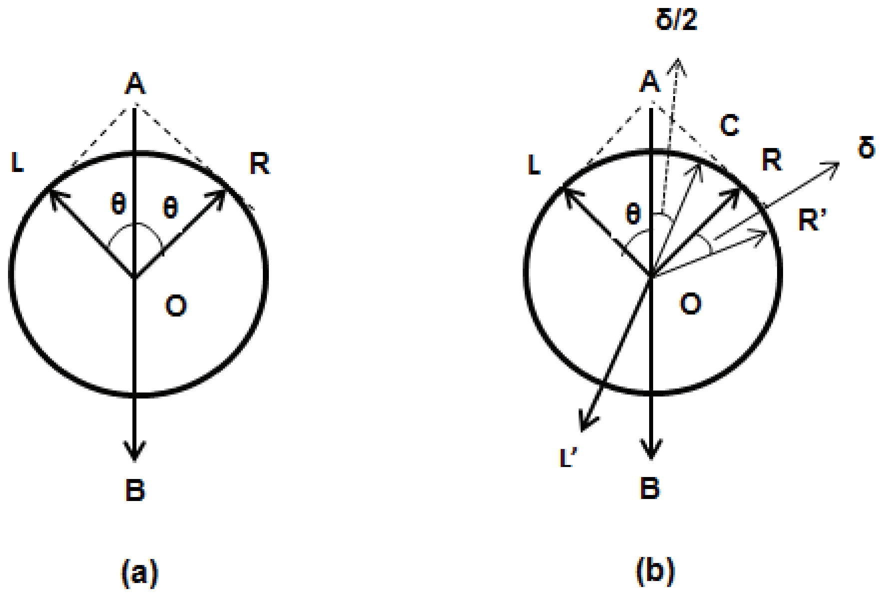


Figure 13.2

(a) For Optically Inactive Crystals: when linearly plane polarized light enters a calcite crystal it gets resolved into two circularly polarized vibrations. One is moving anticlockwise with same angular frequency or velocity. As each circularly polarized vibration further consist of two rectangular components having phase differences π/2, so for clockwise circular vibration

$$x_1 = a\, sin\theta = a\, sin\, \omega t$$

$$y_1 = a\, cos\, \theta = a\, cos\omega t$$

For anticlockwise circular vibration

$$x_2 = -a\, sin\, \theta = -a\, sin\, \omega t$$

$$y_2 = a\, cos\, \theta = a\, cos\, \omega t$$

From above, the resultant displacement of vibrations along x-axis and y-axis respectively are given as,

$$x = x_1 + x_2 = a\, sin\, \theta - a\, sin\, \theta = 0$$

$$y = y_1 + y_2 = a\, cos\, \omega t + a\, cos\, \omega t = 2a\, cos\, \omega t$$

Hence resultant vibration has amplitude 2a and is plane y-axis, i.e., along original direction AB. Hence two oppositely circularly polarized vibrations give rise to a plane polarized vibrations.

(b) For Optically Active Crystal (Quartz): When linearly plane polarized light enters quartz crystal (right handed), it gets resolved into circularly polarized vibrations moving in opposite direction with different angular frequency or velocity. In case of right handed crystal clockwise vibrations travel faster than anticlockwise vibrations. Let at any instant of time anticlockwise vibrations has traversed angle e and clockwise vibrations has traversed angle$(\theta + \delta)$. Therefore, component of two circular vibrations at that instant of time will be, for clockwise vibration

$$x_1 = a \sin(\omega t + \delta)$$

$$y_1 = a \cos(\omega t + \delta)$$

For anticlockwise circular vibration

$$x_2 = - a \sin \omega t$$

$$y_2 = a \cos \omega t$$

From resultant displacement of vibrations along x-axis and y-axis respectively are given as

$$X = x_1 + x_2 = a \sin(\omega t + \delta) - a \sin \omega t = 2a \sin \delta/2 \cos(wt + \delta/2)$$

$$Y = y_1 + y_2 = a \cos(\omega t + \delta) + a \cos\omega t = 2a \cos \delta/2 \cos(\omega t + \delta/2)$$

The resultant vibration along x-axis and y-axis are in same phase, so resultant of these vibrations is plane polarized and makes an angle of δ/2 with original direction AB. Thus, plane of vibrations get rotated through angle ő/2 towards right after passing through a right handed quartz crystal. From above, we get

$$\tan \delta/2 = y/x \qquad \text{....... (13.1)}$$

Angle of Rotation

If μ_R and μ_L be the refractive indices of quartz crystal for right handed and left handed vibrations respectively then optical path difference on passing through a quartz crystal slab of thickness t is given as,

$$\text{Path difference} = (\mu_L - \mu_R)\, t$$

If λ be the wavelength of light used, then phase difference,

$$\delta = 2\pi/\lambda\, (\mu_L - \mu_R)\, t.$$

Angle of rotation

$$\theta = \delta/2 = \pi/\lambda\, (\mu_L - \mu_R)\, t \qquad \text{....... (13.2)}$$

If v_L and v_R be the velocities of the left handed and right handed circular vibrations respectively, then, $\mu_L = c/v_L$ and $\mu_R = c/v_R$ where c is the velocity of light.

$$\theta = \frac{\pi t}{\lambda}\left(\frac{c}{v_L} \sim \frac{c}{v_R}\right)$$

Or

$$\theta = \frac{\pi c t}{\lambda}\left(\frac{1}{v_L} \sim \frac{1}{v_R}\right)$$

If T be the time period, then $c = \lambda/T$ or $1/T = c/\lambda$

$$\theta = \frac{\pi t}{T}\left(\frac{1}{v_L} \sim \frac{1}{v_R}\right)$$

In case of left handed substances (case of dextrorotatory substance), $v_R > v_L$

$$\theta_L = \frac{\pi t}{T}\left(\frac{1}{v_L} - \frac{1}{v_R}\right) \qquad \text{....... (13.3)}$$

In case of left handed substances (case of laevorotatory substance), $v_L > v_R$

$$\theta_R = \frac{\pi t}{T}\left(\frac{1}{v_R} - \frac{1}{v_L}\right) \qquad \text{....... (13.4)}$$

In case of non-optically active substances, $\theta = 0^0$. Hence the direction of vibrations remains unchanged. Such substance does not exhibit the phenomenon of optical rotation.

Experimental Verification

Fresnel's verified his hypothesis by arranging a number of right handed and left handed prism of quartz to form a parallelepiped. Axes of the prism are parallel to the bases. When a plane polarized light is incident normally on one face, the beam splits into two circularly polarized beams which are widely separated. These are analyzed by a rotating Nicol. It was observed that the intensity of emergent beam is constant. This establishes the fact that both beams are circularly polarized as assumed by Fresnel.

Example 13.1 The refractive indices for right- and left-handed vibrations are 1.5580 and 1.55821 respectively for quartz for sodium light of wavelength 5890 Å. Find the optical rotation for the same light by quartz of thickness 1.00 mm when its faces are cut perpendicular to the optic axis.

Solution: The angle of rotation of the plane of vibration is given as

$$\theta = \frac{\pi t}{\lambda}(\mu_R - \mu_L)$$

The data given are

t = 1.00 mm = 0.100 cm, λ = 5890 Å = 5890×10^{-8} cm, μ_R = 1.55810, μ_L = 1.55821

Putting these data into the aforementioned equation, we get the angle of rotation as

$$\theta = \frac{\pi \times 0.1}{5890 \times 10^{-8}}(1.55821 - 1.55810)$$

$$= 0.5870 \text{ radian} = 33.63^0$$

13.5 SPECIFIC ROTATION

When a linearly plane polarized light is passed through an optically active medium/ substance, the plane of linearly polarized light gets rotated through certain angle either towards left or right. The angle through which plane polarized light get rotated depends upon

(i) thickness of the medium

(ii) density of active substance or concentration of solution

(iii) wavelength of light

(iv) temperature.

Hence, mathematically $\theta \propto L$,

$\theta \propto C$,

$\theta \propto \lambda$,

and $\theta \propto t$

$\therefore$ $\theta \propto L\,C\,\lambda\,t$

or $\theta = \mathbf{S}\,L\,C\,\lambda\,t$

or $S = \theta / (L\,C\,\lambda\,t)$

Where S is proportionality constant and is known as specific rotation. θ = rotation in degree, L = length of tube in decimeter, C = concentration of solution in gm/ cc, t = temperature and λ. = wavelength of linearly polarized light

For given wavelength and temperature,

$$S = \theta / (LC) \qquad \text{(when L is in decimeter)} \qquad .... \ (13.5)$$

or

$$S = 10\theta / (L\,C) \qquad \text{(when L is in centimeter)} \qquad .... \ (13.6)$$

Hence, specific rotation is defined as the rotation produced by 1 decimeter long solution of concentration 1 g/cc at given temperature for given wavelength. The rotation produced by optically active/medium/substance can be measured by polarimeter.

Example 13.2: The plane of polarization of a plane polarized light is turned through an angle 12.6^0 passing through a 10% sugar solution of length 20 cm. Calculate the specific rotation.

Solution- The data given are $\theta = 12.6^0$, L = 20 cm, C = 10% = 0.1 gm/cm^3

The specific rotation S is given by $S = \frac{10\theta}{LC} = \frac{10\times12.6}{20\times0.1} = 63^0$ cm^2/gm

13.6 POLARIMETER

Polarimeters are the optical instruments designed to measure the angle of rotation of plane polarized light after passing through an optically active substance. When used for finding the optical rotation of sugar it is called a Saccharimeter. If the specific rotation of sugar is known, the concentration of sugar solution can be determined. Generally, there are two types of polarimeters used

1. Laurent's half-shade polarimeter

2. Bi-quartz polarimeter.

13.6.1 Laurent's Half-Shade Polarimeter

Construction: The optical parts of a Laurent's half shade polarimeter are shown in figure 13.3. A source of monochromatic light (usually a sodium lamp) is placed at the focus of a convex lens, which makes the emerging light rays parallel. This parallel light beam is allowed to fall on a polarizer (usually Nicol prism P) such that the emerging light become plane polarized light. This plane polarized light is then passing through the half shade device H (called Laurent's), a glass tube T containing the solution of optically substance and an analyzer (Nicol prism A). The light is viewed through a telescope E. The analyzing Nicol A can be rotated about the axis of the tube and its rotation can be measured with the help of a vernier scale on the graduated circular scale C divided in degrees and mounted on telescope.

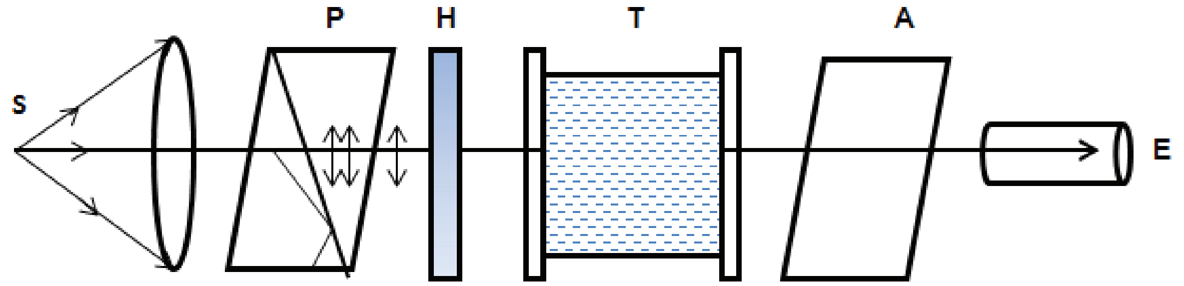


Figure 13.3

Working: The Laureate half shade device is a combination of two semicircular plates of glass (ADB) and quartz (ACB) shown in figure 13.4. The quartz plate is cut such that its optic axis becomes parallel to the line of separation AOB. The thickness of the quartz plate is chosen such that it works as half wave plate and offers a phase difference of π between the O-rays and E-rays. The thickness of the glass plate is such that it absorbs the equal amount of light that by the quartz half wave plate.

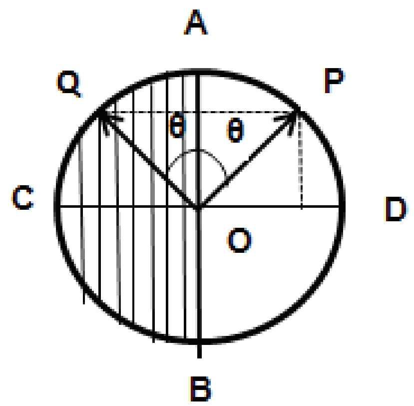


Figure 13.4

The source light is first allowed to pass through the polarizer P and emergent plane polarized light with vibrations along OP is incident normally on the half-shade plate (figure 13.4). On passing through the glass half (right half) the vibrations will remain along OP, but on passing through the quartz half (left half), the vibrations along OP split up into two perpendicular components (E -rays and O-rays) along OQ (in left half or through quartz half) and perpendicular to OQ (in right half or glass half). The analysis of the emergent light after Laurent half shade can be done with the help of another Nicol as follows

(1) If the analyzing Nicol is fixed with its principal plane parallel to OP, the plane polarized light through glass half will pass and hence it will appear brighter than the quartz half from which light is partially obstructed.

(2) If the principal plane of the Nicol is parallel to OQ the quartz will appear brighter than the glass half due to the same reason.

(3) When the principal plane of the analyzing Nicol is parallel to AOB, the two halves will appear equally bright. It is because the vibrations emerging out of the two halves are equally inclined to its principal plane and hence two components will have equal intensity.

(4) When the principal plane of the analyzer is at right angle to AOB again the components of OP and OQ are equal. The two halves are again equally illuminated, but as the intensity of the components passing through is small as compared to that in the previous case, the two halves are said to be equally dark.

The eye can easily detect a slight change when the two halves are equally dark. The readings are, therefore, taken for this position.

Application: The Laurent half shade polarimeter can be used to measure the concentration of sugar solution. For this purpose, the following steps to be followed:

(1) Fill the polarimeter tube with water and find the reading on a circular scale corresponding to equally dark position of the half shade device.

(2) Now fill the tube completely with the given sugar solution and again find the reading on the circular scale for equally dark positions of the half shade device.

(3) The difference between the scale readings gives the optical rotation θ produced by the given length l in decimeters of the sugar solution.

(4) If S is the specific rotation of sugar for the same wavelength and at the same temperature, then concentration

$$C = \theta/(LS) \text{ g/cc} \qquad \ldots\ldots\ (13.7)$$

Polarimeter is of great importance in the industries for estimating the quantity of sugar in the presence of an optically inactive impurity. A polarimeter calibrated to read directly the percentage of cane-sugar in the solution is called as saccharimeter.

Merits and Demerits of Laurent's Half Shade Polarimeter

- This instrument is suitable for monochromatic light source. It is usually constructed for sodium source.
- When position is adjusted for equally dark halves, it gives fairly accurate observation as slight rotation of the analyzer changes the intensity of two halves.

13.6.2BiquartzPolarimeter

It is also an instrument used for finding the optical rotation of certain optical active solutions. The arrangement of bi-quartz polarimeter is similar to that of a Laurent's half-shade polarimeter except a white light is used instead of monochromatic light.

Construction: The experimental arrangement for a bi-quartz polarimeter is shown in figure13.5. According to the figure 13.5, the light from a source S (white light) which is just an ordinary bulb after passing through a slit is incident on a convex lens L which converts the emergent light into a parallel beam. This parallel beam of light falls on a NicolP (polarizer) and the emergent light from P become plane polarized light.

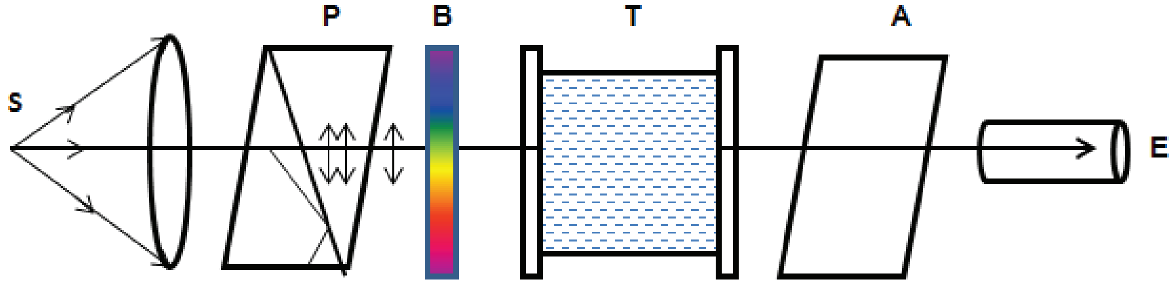


Figure 13.5

This plane polarized light is then passed through the Biquartz plate B, a glass tube T containing the solution of optically active substance and an analyzer (Nicol prism A). The light is viewed through a telescope E. The analyzing Nicol A can be rotated about the axis of the tube and its rotation can be measured with the help of a vernier scale on the graduated circular scale C divided in degrees and mounted on telescope E.

Working: A bi-quartz plate is made up of two semicircular plates out of which one is of left handed quartz and other of right handed quartz. The quartz plates are cut perpendicular to their optic axes and cemented along a diameter (YZ) as shown in figure 13.6. In order to remove double refraction, the optic axes of both semicircular quartz plates are made perpendicular to their planes. The thickness of each plate is nearly 4mm which is so much that each plate can rotate the plane of polarization of yellow light through 90°. One clockwise rotation is through right handed quartz and other anticlockwise rotation is through left handed quartz.

The working of biquartz polarimeter may be divided into following points

(1) A white light is incident on Nicol P and the emergent light becomes plane polarized. This plane polarized light is then allowed to fall on the bi-quartz plate B normally and it travels in the direction of optic axis. In the plate the rotatory dispersion of every color takes place in opposite direction. ($\because \theta \; \alpha \; 1/\lambda^2$).

(2) If the vibrations of incident plane polarized light after passing through polarizer are along YZ direction (figure 13.7), then the emergent light from the bi-quartz plate for yellow

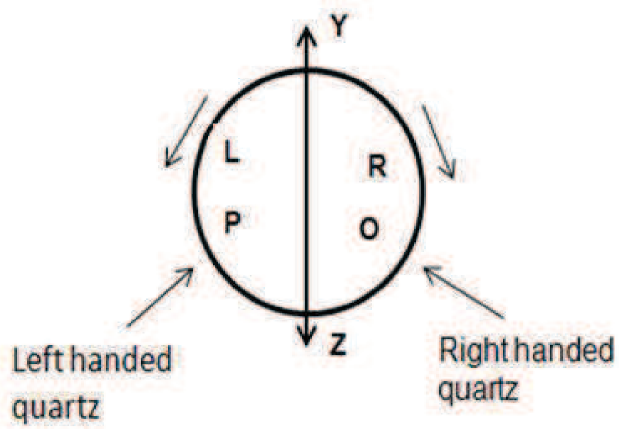
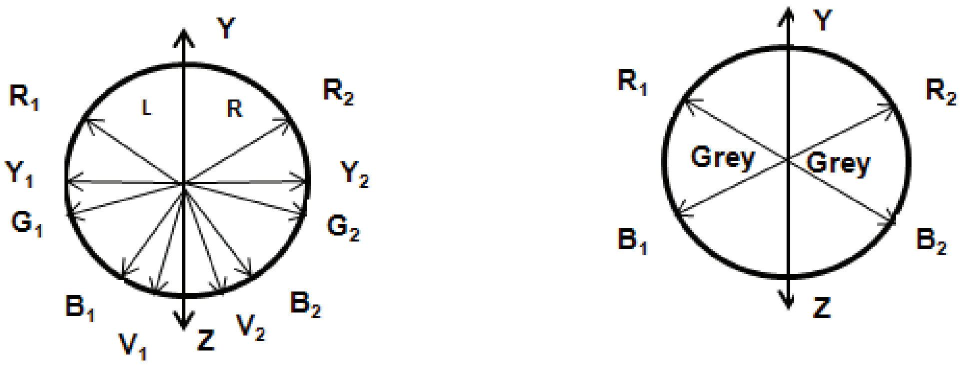


Figure 13.6

color the vibrations will be perpendicular to YZ because the vibrations are rotated through 90° by bi-quartz, whereas the red light is rotated through the smallest amount and that of violet light is rotated through the largest angle.



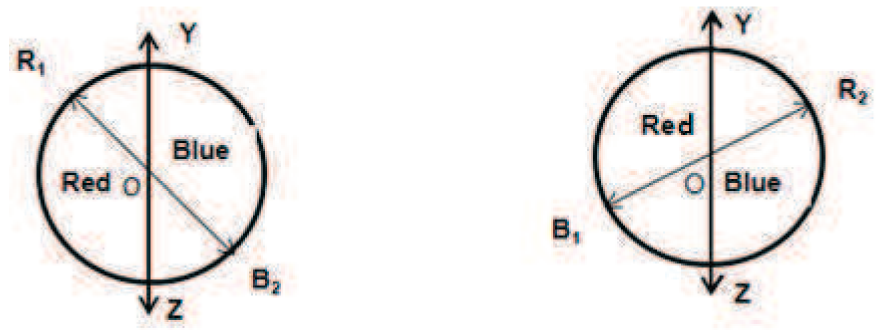
(a) **(b)**

Figure 13.7

(3) If the principal section of is placed parallel to YOZ, the yellow colour vibrations terminates and due to the mixture of red and blue, both the halves appear to the similar and has grayish violet tint, called the tint of passage [figure13.7(b)].

(4) Now if the principal section of A is rotated through a small angle from his position as shown R_1B_2 in figure13.8(a), then right half appears more bluish and left more red. If the principal section of Nicol N_2 is rotated in opposite direction R_2B_1 [figure13.8 (b)] we see an observation just opposite to the above.

The tint is very sensitive and depends on position. Hence, it can be used for accurate determination of optical rotation.



(a) (b)

Figure 13.8

Application: Determination of Specific Rotation of Sugar Solution

(1) The position of analyzer (A) is adjusted to (a sensitive tint) without using optically active solution. The position of the analyzer is recorded using a mounted circular scale on the telescope E.

(2) Now place the optically active solution (sugar solution) in the tube T and the position of A is again adjusted so as to get again sensitive tint (grayish shade) and its position noted again. The difference between the two positions of A gives the optical rotation produced by the optically active solution.

(3) The length L of solution is found in decimeter and the concentration C of the solution is found in g/cc by using the formula S = θ/(LC), specific rotation of optically active solution, such as sugar, can be determined.

Merits of Bi-quartz Polarimeter

- As the transition from red to blue is very rapid, the zero position can be obtained very accurately.
- For this instrument, white light can also be used.
- It is highly sensitive device for measuring optical rotation.

Demerits of Bi-quartz Polarimeter

- This instrument does not give accurate result for colorless optically active substance.
- It is not possible for a color blind person to use this instrument.

Self Assessment Question

1. Specific rotation is measured using _____
2. Two unknown sample liquids are measured in a polarimeter. They are found to have the same observed rotation. What is the relationship of these two liquids?
3. For which type of molecules can you measure specific rotation?
4. What is saccharimeter?
5. The substances that rotate the plane of vibration in the clockwise direction is known as

13.7 SUMMARY

In this unit, you have studied about the substances that rotate the plane of polarization of light which is passed through them. These substances are called optically active substances and the property is called as optical activity. So, this was discussed along with the concept of specific rotation. To present the clear understanding of optical activity and specific rotation, Fresnel's theory was discussed which explains the reason of rotatory polarization and its dependency of the speed of left handed and right handed components of plane polarized light. The polarimeters are used to produce polarized light and also to determine the rotation of its plane of polarization when passed through an optically active substance e. g. sugar solution. Specifically Laurent half shade polarimeter and biquartz polarimeter with their working and principal were discussed. The concept of polarimeter may be extended to Saccharimeter, an instrument to determine the concentration of sugar solution based on the measurement of angle of rotation of the plane of polarization of light. Many solved examples are given in the unit to make the concepts clear. To check your progress, self assessment questions (SAQs) are given place to place.

13.8 GLOSSARY

Optical rotation: rotation in plane polarized light when passing through optically active substances

Ordinary light ray: A light ray obeys the ordinary laws of refraction (Snell' law)

Extraordinary light ray: A light ray does not obey the ordinary laws of refraction (Snell's law)

Limited: restricted

Undergo: suffer

Maintain: sustain

Polarimeter: device to measure optical rotation

Assessment: evaluation

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13.10 SUGGESTED READINGS

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2. Introduction to Electrodynamics by D.J. Griffith, Pearson Publ

3. Waves by F.S. Crawford Jr., Berkeley Physics Course Vol. 3

4. Optics by M.V. Klein and T.E. Furtak Wiley

13.11 TERMINAL QUESTIONS

13.11.1 Objective type

11. The substance that show the phenomenon of optical rotation are said to be

(e) optically active (b) optically inactive

(c) crystals (d) polaroids

12. In optically inactive substances, the relation between the velocities of right handed and left handed circularly polarized lights is

(a) $v_R > v_L$ (b) $v_R < v_L$

(c) $v_R = v_L$ (d) none of these

13. In a dextro-rotatary substances, the relation between the refractive indices of right handed and left handed circularly polarized lights is

(a) $\mu_R > \mu_L$ (b) $\mu_R < \mu_L$

(c) $\mu_R = \mu_L$ (d) none of these

14. In a leavo-rorotatary substances, the relation between the refractive indices of right handed and left handed circularly polarized lights is

(a) $\mu_R > \mu_L$ (b) $\mu_R < \mu_L$

(c) $\mu_R = \mu_L$ (d) none of these

15. Choose the incorrect statement. The amount of rotation of the plane of polarization is

(e) Directly proportional to the concentration of the solution

(f) inversely proportional to the concentration of the solution

(g) least for violet and greatest for red

(h) directly proportional to the length of optically active substance

16. The device which measures the angle through which the plane of polarization of a plane polarized beam is rotated by a given medium is called

(e) Babinet compensator (b) Polarimeter

(c) Refractometer (d) Polaroid

17. The device based on the measurement of angle of rotation of the plane of polarization of light and measures the concentration of sugar solution is called

(a) Babinet compensator (b) Saccharimeter

(c) Refractometer (d) Polaroid

18. Dextro rotatory optically active substance rotates the plane of vibrations

(a) In clockwise direction (b) In anticlockwise direction

(c) By 180^0 (d) None of these

19. Laevo rotatory optically active substance rotates the plane of vibrations

(a) In clockwise direction (b) In anticlockwise direction

(c) By 180^0 (d) None

20. Polarimeter is the instrument designed for

(a) measures the angle of rotation produce by a substance

(b) to study nature of the polarized light

(c) to study the phase difference between circularly polarized light and elliptically polarized light.

(d) none of these

21. A tube 2 decimeter long is filled with a cane sugar solution 20 gm in 100 cm^3. The specific rotation of cane sugar is 60^0. The angle of rotation of plane polarized light passing through it is

(a) 15^0 (b) 20^0

(c) 24^0 (d) 30^0

22. A sugar solution in a tube of length 20 cm produces optical rotation of 13^0. The solution is then diluted to one third of the previous concentration. The optical rotation produced by a 30 cm long tube containing the diluted solution is

(a) 5.5^0 (b) 6.5^0

(c) 7.5^0 (c) 8.5^0

13.11.2 Short Answer Type Questions

1. What do you mean by optical rotations?

2. Define plane of vibration and plane of polarization

3. For which type of molecules can you measure specific rotation?

4. Which device is used to measure the optical rotation?

5. Which device is used to measure the concentration of sugar solution?

13.11.3 Long Answer Type Questions

1. Give two differences between Laurent's half shade polarimeter and biquartz polarimeter.

2. Define specific rotation. Describe the construction and working of Laurent's half shade polarimeter. Discuss the relative merit of biquartz polarimeter and half shade polarimeter.

3. What is optical activity? Describe the construction, theory and working of biquartz polarimeter to find the optical rotation of a solution and also discuss the action of biquartz plate in it.

4. What do you understand by specific rotation? Describe the construction and working of biquartz polarimeter to find the specific rotation of sugar solution and discuss the utility of biquartz plate in it.

13.11.4 Numerical Answer Type Questions

1. The plane of polarization of plane polarised light is rotated through 6.5^0 in passing through a length of 2.0 decimeter of sugar solution of 5 % concentration. Calculate the specific rotation of sugar solution. A quarter wave plate is meant for wavelength 5893 Å. How much phase retardation it will show for wavelength 4300 Å?
2. A tube of sugar solution 20 cm long is placed between crossed Nicols and illuminated with light of wavelength 6 x 10^{-5} cm. If the optical rotation produced is 13^0 and the specific rotation is 65^0/dm/gm/cm^3, determine the strength of the solution.
3. Determine the specific rotation of a given sample of sugar solution if the plane of polarization is turned through 13.2°. The length of the tube containing 10% sugar solution is 20 cm.
4. On introducing a polarimeter tube 25 cm long and containing sugar solution of known strength, it is found that the plane of polarization is rotated through 10°. Find the strength of sugar solution in g/cm^3. Given that specific rotation of sugar solution is 60° per decimeter per unit concentration.
5. A tube of 20 cm long with solution of 15 gm of cane sugar in 100 cc of water is placed in the path of polarization if the specific rotation of cane sugar is 66°.
6. 80 gm of impure sugar when dissolved in a litre of water gives an optical rotation of 9.9^0. When placed in a tube of length 20 cm. If the specific rotation of sugar is 66^0, find the percentage purity of the sugar solution.
7. Calculate the specific rotation if the plane of polarization is turned through 26.4^0, traversing 20 cm length of 20 % sugar solution.
8. A Sugar solution in a tube of length 20 cm produces optical rotation of 13^0. The solution is then diluted to one-third of its previous concentration. Find the optical rotation produced by 30 cm.

13.12 ANSWERS

13.12.1 Self Assessment Questions

1. Polarimeter

2. They are the same compound.

3. Chiral molecules or optically active substances

4. A polarimeter calibrated to read directly the % age of cane sugar in the solution is called saccharimeter.

5. Right handed or dextro-rotatary

13.12.2 Objective Type Questions

1. (a), **2.** (c), **3.** (b), **4.** (a), **5.** (c), **6.** (b), **7.** (b), **8.** (a), **9.** (b), **10.** (a), **11.** (c), **12.** (b)

13.12.3 Short Answer Type Questions

1. The rotations produced by a decimeter long column of the liquid containing 1 gm of active substance in 1 cc of the solution.

2. The plane of polarization is that plane in which no vibrations occur and the plane in which vibrations occur known as plane of vibration. The vibrations occur at the right angle to the plane of polarization.

3. Optically active substances e.g. sugar solution.

4. Polarimeter

5. Saccharimeter

13.12.4 Numerical Type Questions

1. $S = \theta/ (LC) = 65/(2 \times 0.05) = 650\ (dm)^{-1}\ (g/cc)^{-1}$

2. $S = \theta/ (LC)$, $C = \theta/(LS) = 13/(2 \times 65) = 0.1$ g / cc = 10 % .

3. $S = 10\theta /L .C = 10 \times 13.2/20 \times 0.1 = 660(dm)^{-1}\ (g/cc)^{-1}$

4. $S = 10\theta/ L.C = 60°$; $C =10\theta /L.C =10 \times 10 / 25 \times 60 =1/15 = 0.0679\ g/m^3$.

5. $S = 10\theta/L.C$; $\theta = SLC/L.C = 66 \times 20 \times 15 /10 \times 100 = 19.8^0$

6. $S = \theta/ (LC)$, $C = \theta/(LS) = 9.9/(2 \times 66) =0.0751$ g / cc = 75 g/L and purity % = 93.75 %

7. 20 % = 0.2 g/cc, $S = \theta/(LC) =26.4/(2 \times 0.2) = 66\ (dm)^{-1}\ (gm/cc)^{-1}$

8. $S = 10\ \theta/(LC) = 10\ \theta'/(L'C')$, $\theta'= 6.5^0$

UNIT 14: OPTICAL INSTRUMENTS

Structure

14.1 INTRODUCTION

An eye is one of the most important organ of our body which works as an optical instrument and enables us to view all things around us. Optical instruments process light wave so that image of object is enhanced as well as clearer. Generally optical instrument are used to look things bigger and to see the fine detail of any object. Optical instruments can be broadly divided into two categories. One forms real images of an object which is projected onto a screen. Image can be viewed simultaneously by many observers, e.g., projectors.

Other kind of optical instruments forms virtual images of an object and only one observer can see the image. The virtual image formed by the instrument is transformed by the eye into a real image, e.g., spectrometer, microscopes, telescopes.

14.2 OBJECTIVES

After study of unit the student will be able

- To understand the concept of optical instruments
- To explain working of human eye
- To explain different types of eye piece
- To compare different types of eye pieces
- To describe about spectrometer and electron microscopes

14.3 HUMAN EYE

Our eyes are not only vital for seeing the world around us but also an essential part of all optical instruments. The eye is a slightly asymmetrical sphere in shape and about one inch (2.5 cm) in diameter. The detailed structure of front part of the eye (the part we see in the mirror) is discussed below.

14.3.1 Sectional View and Different Parts of Human Eye

The Sclera: It is the white part of the eye which is opaque, fibrous, tough, protective outer layer of the eye that is directly continuous with the cornea in front and with the sheath covering the optic nerve behind. It is lined inside with vascular tissue which consists of blood vessel feeding the eye.

The Iris: It is the pigmented muscular ring and can be of different color contributes to eyes with different color, i.e., green, blue, brown or black.

The Cornea: It is a clear dome over the iris which acts as an entrance lens for the eye. It is the clear, transparent front part of the eye that covers the iris, pupil and anterior chamber and provides most of an eye's optical power (if too flat = hyperopia/far sightedness; if too steep = myopia/near sightedness). It needs to be smooth, round, clear, and tough. It is like a protective window. The function of the cornea is to let light rays enter the eye and converge the light rays.

The Pupil: It is the black circular opening with variable diameter in the iris, which lets light in and opens and closes to adapt to changing light intensity.

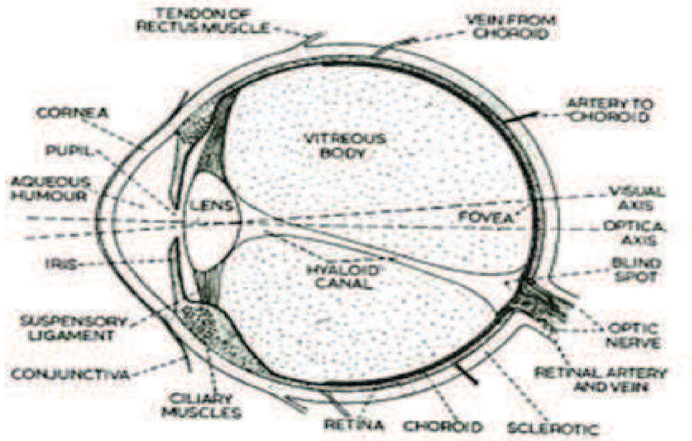


Figure 14.1

The Conjunctiva: It is an invisible, clear layer of tissue covering the front of the eye, except the cornea.

Crystalline Lens: The lens is the natural biconvex lens of the eye behind the iris. Transparent, biconvex intraocular tissue that helps brings rays of light to focus on the retina (It bends light, but not as much as the cornea). The lens is held in place by fine ligaments (zonules) attached between ciliary processes. The lens contains a fibrous jelly, hard at center and progressively softer at the outer portions.

Anterior Chamber: anterior chamber is in front of the lens and behind the cornea filled with Aqueous Humor. The eye receives oxygen through the aqueous. Its function is to nourish the cornea, iris, and lens by carrying nutrients; it removes waste products excreted from the lens, and maintains intraocular pressure and thus maintains the shape of the eye.

Vitreous Chamber: It is the transparent, colorless gelatinous mass that fills rear two-thirds of the eyeball, between the lens and the retina. It has to be clear so light can pass through it and it has to be there or eye would collapse.

Retina: Vitreous Chamber is lined with a sensory layer called retina. The retina is the light sensitive nerve tissue in the eye that converts images from the eye's optical system into electrical impulses that are sent along the optic nerve to the brain, to interpret as vision. The retina has the shape of hemisphere consists of layers that include two types of light receptor cells called rods and cones. The human eye has a total of 125 million rod and 6.5 million cones.

Cones: The cones are the light-sensitive retinal receptor cell that provides the sharp visual acuity (detail vision) and color discrimination; most numerous in macular area. Function under bright lighting.

Rods: The light-sensitive, specialized retinal receptor cell that works at low light levels (night vision). The rods function with movement and provide light/dark contrast. It makes up peripheral vision.

Optic Nerve: The optic nerve is the largest sensory nerve of the eye. It carries impulses for sight from the retina to the brain. Composed of retinal nerve fibers that exit the eyeball through the optic disc, traverse the orbit, pass through the optic foramen into the cranial cavity, where they meet fibers from the other optic nerve at the optic chiasm.

Blind Spot: This is the sightless area within the visual field of a normal eye. It is caused by absence of light sensitive photoreceptors where the optic nerve enters the eye.

Fovea: Contrary to blind spot the fovea is the central pit in the macula that produces the sharpest vision. It contains a high concentration of cones within the macula and no retinal blood vessels. The remaining area of the retina is occupied mainly by rods.

Macula: It is the yellow spot in the small (3°) central area of the retina surrounding the fovea. It is the area of acute central vision (used for reading and discriminating fine detail and color). Within this area is the largest concentration of cones.

Choroid: The vascular (major blood vessel), central layer of the eye lying between the retina and sclera. Its function is to provide nourishment to the outer layers of the retina through blood vessels. It is part of the uveal tract.

Extra Ocular Muscles: There are six extra ocular muscles in each eye.

Rectus Muscles: There are four Rectus muscles that are responsible for straight movements: Superior (upward), Inferior (lower), Lateral (toward the outside, or away from the nose), and Medial (toward the inside, or toward the nose).

Oblique Muscles: There are two Oblique muscles that are responsible for angled movements. The superior oblique muscles control angled movements upward toward the right or left. Inferior oblique muscles control angled movements downward toward the right or left.

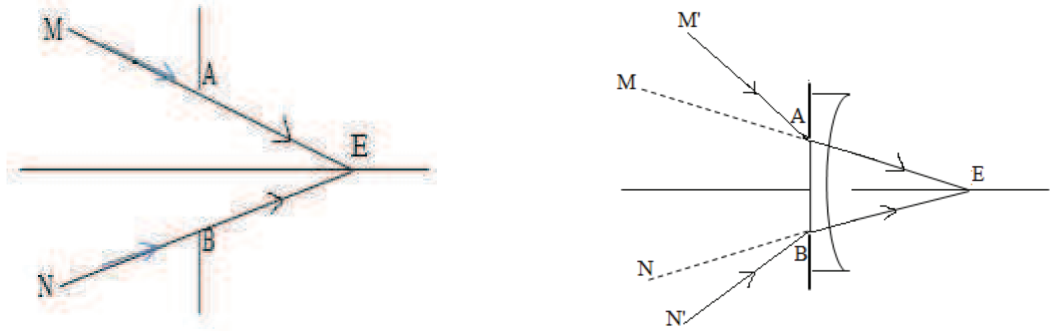
14.3.2 Action of Human Eye

The refractive index of both the aqueous humor and vitreous humor are about 1.336. The crystalline lens has an average index of 1437. Most of the refraction of light entering the eye occurs at the outer surface of the cornea. Refraction at the cornea and the surfaces of the lens produces a real and inverted image of the object on the retina. The optic nerve sends a signal to the brain.

The image must be formed exactly at the location of retina for the object to be seen clearly. The eye adjust different object distances by changing the focal length f of its lens by varying its radii of curvature with the help of ciliary muscle without changing lens to retina distance. The process of adjustment is called accommodation. However, it is the ability of human eye to accommodate in terms of the far point and the near point of the eye. For the normal eye the far point of the eye is infinity and the near point of the eye is 25 cm [normal distance of distinct vision (NDDV)]. Range of accommodation diminishes with age as the crystalline lens grows through a person's life and the ciliary muscle is less able to distort a larger lens.

14.4 FIELD OF VIEW

Field of view of an optical instrument may be defined as the extent or range of its visual area. For an eyepiece it is the widest dimension of an object which is visible through the eyepiece. Generally it is expressed as the width in feet at 1000 yards or in degree of field.



(a) **(b)**

Figure 14.2

The field of view is shown without a lens (figure 14.2 (a)) and with a lens (figure 14.2 (b)) When the field of view is expressed in feet, it is called the linear field of view. When it is shown in degree, it is known as angular field of view. We can convert one to other by using a formula

1 angular degree of view = 52.5 feet.

As shown in figure light is passing through the aperture AB and reaching the eye E. The eye can view objects in the angle range MEN. The widest possible angle MEN which we can see, is called field of view. If a concave lens is added in plane AB from a wider field M'EN' can reach the eye. We say the field of view is enlarged. On the other hand convex lens narrow the field of view.

14.5 NEED OF MULTIPLE LENS EYEPIECES

The basic purpose of optical instrument is to produce a magnified image free from aberrations and enlarged field of view. A single lens fails to fulfill both the requirements as the image formed by single lens as it suffers from chromatic and spherical aberration. Also the field of view is small and with the increase in magnification of optical instruments it became even smaller as the image formed by marginal rays refracted through the peripheral portion of eye lens can't simultaneously enter the small aperture of the pupil of eye placed near the eye lens. Hence only the part of image nearer to the axis will be seen and final image will cover a small field of view.

Field of view is also decreased with the increase in distance between the objective and eye lens. As the magnifying power depends on distance so field of view is also decreased as the instrument is set for higher magnifying power by adjusting the distance between objective lens and eye lens. Now to overcome the shortcomings of a single lens the eye lens is replaced by multiple lens eyepiece in optical instruments. In general, in an eyepiece there are two lens made by suitable material and type separated by a suitable distance. The extra lens facing the objective of instruments is known as field lens. Field lens and eye lens are made and kept in such a way that their combination minimized chromatic and spherical aberrations. Also the field lens gathers more of the rays from the objective towards the axis of the eyepiece and covers all the rays from the image to enter the eye lens. In this way it increases the field of view.

14.5.1 Positive and Negative Eyepiece

Eyepiece in which the first focal plane of an eyepiece lies in the object space outside the eyepiece is called positive eyepiece. In such eyepiece a real object can be placed on first focal plane to be in focus with final image, i.e., Ramsden's eyepiece. Eyepiece in which the first focal plane of an eyepiece lies within eyepiece i.e. between eye lens and field lens where no real object can be place is called negative eyepiece, i.e., Huygen's eyepiece.

14.6 RAMSDEN'S EYEPIECE

14.6.1 Construction

Ramsden's Eyepiece consists of two planoconvex lenses made of same material with their curved (convex) side facing each other inwards (figure 14.3). In this type of eyepiece focal length of field lens and eye lens is same and distance between field lens and eye lens is $\frac{2}{3}f$ where f is focal length of eye lens. The field lens is a little larger rand is placed close to intermediate image to allow maximum possible light to pass through it. The eye lens has a smaller diameter but caries out the actual magnification.

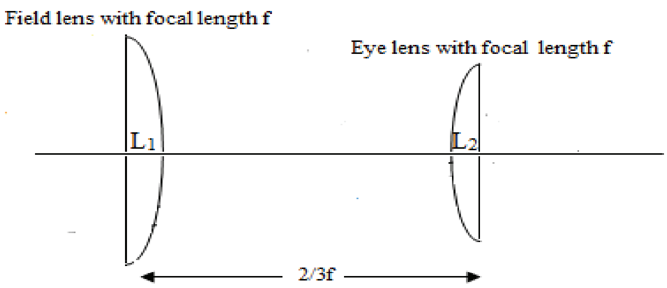


Figure 14.3

Condition of Achromatism

For achromatism the distance d between two lenses should be

$$d = \frac{f_1 + f_2}{2}$$

In case of Ramsden's eyepiece $f_1 = f \; and \; f_2 = f$

$\therefore \qquad d = f$

But the distance between field lens and eye lens in Ramsden's eyepiece is kept $\frac{2}{3}f$ which is slightly less than *f,* therefore, not completely free from chromatic aberration. If the distance between field lens and eye lens is kept *f* the field lens will be at the focal plane of eye lens. In this position any dust particle or scratch would be magnified and final image will not be clear.

Condition for Minimum Spherical Aberration: The distance between the two lenses for minimum spherical aberration should be

$$d = f_1 - f_2$$

In case of Ramsden's eyepiece $f_1 = f \; and \; f_2 = f$

$\therefore \qquad d = f - f = 0$

It means that it doesn't satisfy condition for minimum spherical aberration. However to reduce spherical aberration in Ramsden's eyepiece two plano convex lenses with their convex lens facing each other are used.

14.6.2 Working

When eye piece is adjusted for normal vision, the final image formed by it is at infinity. For this the image formed, the field lens should lie in the first focal plane of the eye lens. The objective forms the real inverted image I of a distant object. This acts as an object for the field lens. This gives rise to a virtual image I_1. I_1 in turns serves as an object for the eye lens therefore it must be at distance equal to focal length f from eye lens to make final image at infinity. Hence image I_1 lies at a distance f/3 to the left of the field lens.

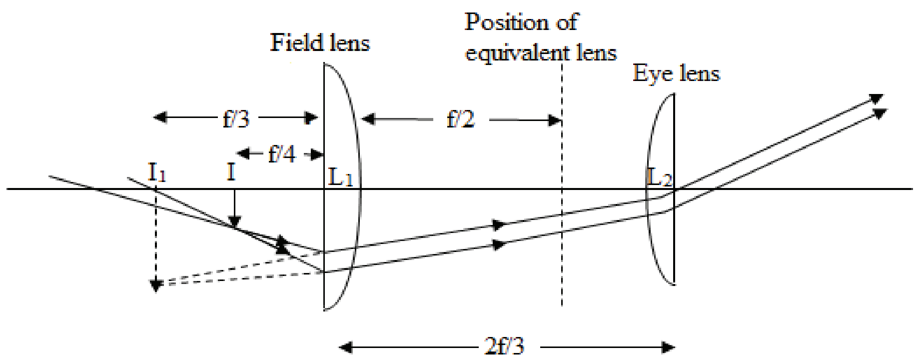


Figure 14.4

For field lenses the image I_1 formed by the objective of the instrument (in which eye-piece is used) acts as an object for eye lens.

If u is the distance of I_1 from the field lens, than from lens formula

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

Since here $v = \frac{-f}{3}$ and $f = f$, we have,

$$\frac{1}{-\frac{f}{3}}-\frac{1}{u}=\frac{1}{f} \qquad \text{or} \qquad \frac{1}{u}=-\frac{3}{f}-\frac{1}{f}=-\frac{4}{f}\text{, Hence } u=-\frac{f}{4}$$

That is, the image I formed by objective lies to the left of the field lens. The rays coming from I, after emerging from the field lens appear to come from I_1 at a distance f/3 left to field lens. These rays emerge from the eye lens as a parallel beam.

Equivalent Focal Length

Equivalent focal length of Ramsden's eyepiece is given by

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{d}{f_1 f_2}$$

$$\frac{1}{F}-\frac{1}{f}+\frac{1}{f}-\frac{\frac{2}{3}f}{f^2}$$

$$\frac{1}{F}=\frac{2}{f}-\frac{2}{3f} \qquad \text{or} \qquad \frac{1}{F}=\frac{4}{3f}$$

$$F=\frac{3f}{4}$$

When eye piece is adjusted for normal vision, the final image formed by it should be at infinity. For this the image formed by the equivalent lens must be at infinity. For this I_1 should lies in the first focal plane of the equivalent lens.

14.6.3 Cardinal Points of Ramsden's Eyepiece

Position of Focal Points

The distance of first focal point F_1 from the field lens L_1 is given by

$$\beta_1=-F\left(1-\frac{d}{f_2}\right)=-\left(\frac{3}{4}f\right)\left(1-\frac{\left(\frac{2}{3}\right)f}{f}\right)=-\frac{3}{4}f\times\frac{1}{3}=-\frac{f}{4}$$

Where negative sign indicates that first focal point F_1 lies at a distance of f/4 to the left of field lens L_1. The distance of second focal point F_2 from the eye lens L_2 is given by

$$\beta_2=F\left(1-\frac{d}{f_1}\right)=\left(\frac{3}{4}f\right)\left(1-\frac{\left(\frac{2}{3}\right)f}{f}\right)=\frac{3}{4}f\times\frac{1}{3}=+\frac{f}{4}$$

Where positive sign indicates that second focal point F_2 lies at a distance of f/4 to the right of eye lens L_2.

Position of Principal Points

The distance of first principal point H_1 from the field lens L_1 is given by

$$\alpha_1 = F\frac{d}{f_2} = \frac{\left(\frac{3}{4}f\right)\left(\frac{2}{3}\right)f}{f} = +\frac{f}{2}$$

Where positive sign indicates that the first principal point H_1 lies at a distance of f/2 to the right of field lens L_1. The distance of second principal point H_2 from the eye lens L_2 is given by

$$\alpha_2 = -F\frac{d}{f_2} = -\frac{\left(\frac{3}{4}f\right)\left(\frac{2}{3}\right)f}{f} = -\frac{f}{2}$$

Where negative sign indicates that the second principal point H_2 lies at a distance of f/2 to the left of eye lens L_2.

Position of Nodal Points

As the medium on either side of the eye piece is same (air), the nodal points N_1, N2 coincide with the principal points H_1, H_2 respectively.

The position of cardinal points for Ramsden's eyepiece can be plotted as:

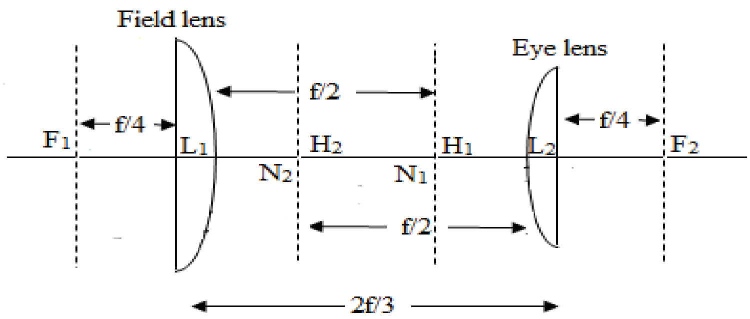


Figure 14.5

Position of Cross Wires

The cross wires, if used, must be placed at the position where the image due to objective is formed so that they would be in focus with the final image and magnified by both the lens of eyepiece. The cross wire placed in first focal plane satisfies both the conditions. Therefore cross wire must be placed at a distance of f/4 in front of field lens. As it is outside both the lenses in object space and is real hence it is used to examine a real object or real image hence it is called a positive eyepiece. Therefore this eyepiece is used in optical instruments where accurate quantitative measurements of distances and angles are made.

Example 14.1: In Ramsden's eyepiece focal length of eye lens is 6 cm. Locate the position of cardinal points in a diagram.

Solution: In a Ramsden's eyepiece field lens and eye lens having same focal length f are separated by a distance (2/3) f.

Given focal length of eye lens is f = 6 cm. So focal length of field lens is also 6 cm and separation between lens is, d = (2/3) f = (2/3) 6 = 4 cm.

The equivalent focal length of Huygen's eyepiece is

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

Here $f_1 = 6\ cm.,\ f_2 = 6\ cm.\ and\ d = 4\ cm.$

$$\therefore\ \frac{1}{F} = \frac{1}{6} + \frac{1}{6} - \frac{4}{36}$$

$$\frac{1}{F} = \frac{6+6-4}{36} \qquad \text{or} \qquad \frac{1}{F} = \frac{8}{36} = \frac{2}{9}\text{cm.}$$

$$F - 4.5\ cm.$$

Position of Focal Points

The distance of first focal point F_1 from the field lens L_1 is given by

$$\beta_1 = L_1F_1 = -F\left(1 - \frac{d}{f_2}\right) = -4.5\left(1 - \frac{4}{6}\right) = -1.5\ cm.$$

Negative sign indicates that first focal point F_1 lies at a distance 1.5 cm to the left of field lens L_1. The distance of second focal point F_2 from the eye lens L_2 is given by

$$\beta_2 = L_2F_2 = F\left(1 - \frac{d}{f_1}\right) == 4.5\left(1 - \frac{4}{6}\right) = 1.5\ cm.$$

Positive sign indicates that second focal point F_2 lies at a distance of 1.5 cm. to the right of eye lens L_2.

Position of Principal Points

The distance of first principal point H_1 from the field lens L_1 is given by

$$\alpha_1 = L_1H_1 = F\frac{d}{f_2} = 4.5\frac{4}{6} = 3\ cm.$$

Positive sign indicates that first principal point H_1 lies at a distance of 3 cm to the right of field lens L_1. The distance of second principal point H_2 from the eye lens L_2 is given by

$$\alpha_2 = L_2H_2 = -F\frac{d}{f_1} = -4.5\frac{4}{6} = -3$$

Negative sign indicates that second principal point H_2 lies at a distance of 3 cm. to the left of eye lens L_2.

Position of Nodal Points

As the medium on either side of the eye piece is same (air), the nodal points N_1, N2 coincide with the principal points H_1, H_2 respectively.

The position of cardinal points for Ramsden's eyepiece can be plotted as:

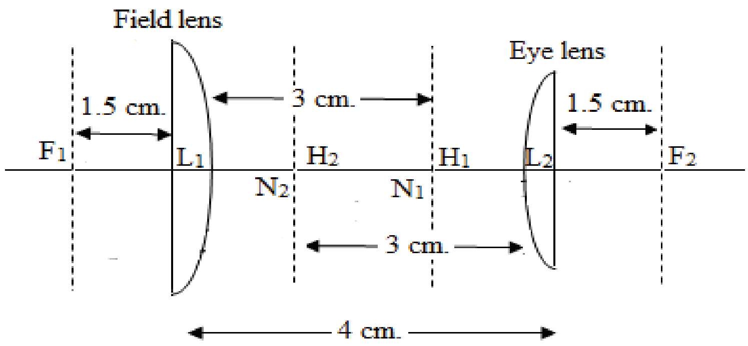


Figure 14.6

$\therefore\ L_1F_1 = -1.5\ cm.,\ L_2F_2 = 1.5\ cm.,\ L_1H_1 = L_1N_1 = 3\ cm.,\ L_2H_2 = L_2N_2 = -3\ cm.$

Example 14.2: In Ramsden's eyepiece equivalent focal length is 9 cm. Calculate the position of cardinal points. If an object is situated at 9 cm. in front of the field lens, find the position of the image formed by the eyepiece.

Solution: In a Ramsden's eyepiece field lens and eye lens having same focal length f are separated by a distance (2/3) f. Given equivalent focal length of eyepiece is F = 9 cm.

The equivalent focal length of Huygen's eyepiece is

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

$$\frac{1}{9} = \frac{1}{f} + \frac{1}{f} - \frac{(2/3)f}{f.f} = \frac{4}{3f}$$

Hence, $f = \frac{4}{3} \times 9\ cm. = 12\ cm.$

So focal length of eye lens is f= 12 cm, focal length of field lens is f = 12 cm and separation between lens is d = (2/3) f i.e. d = (2/3) x 12 = 8 cm.

$$f_1 = 12\ cm.,\ f_2 = 12\ cm.,\ d = 8\ cm.,\ F = 9\ \ cm.$$

Position of Focal Points

The distance of first focal point F_1 from the field lens L_1 is given by

$$\beta_1 = L_1F_1 = -F\left(1 - \frac{d}{f_2}\right) = -9\left(1 - \frac{8}{12}\right) = -3\ cm.$$

Negative sign indicates that first focal point F_1 lies at a distance 3 cm to the left of field lens L_1. The distance of second focal point F_2 from the eye lens L_2 is given by

$$\beta_2 = L_2F_2 = F\left(1 - \frac{d}{f_1}\right) == 9\left(1 - \frac{8}{12}\right) = 3\ cm.$$

Positive sign indicates that second focal point F_2 lies at a distance of 3 cm to the right of eye lens L_2.

Position of Principal Points

The distance of first principal point H_1 from the field lens L_1 is given by

$$\alpha_1 = L_1H_1 = F\frac{d}{f_2} = 9\frac{8}{12} = 6cm.$$

Positive sign indicates that first principal point H_1 lies at a distance of 6 cm to the right of field lens L_1.

The distance of second principal point H_2 from the eye lens L_2 is given by

$$\alpha_2 = L_2H_2 = -F\frac{d}{f_1} = -9\frac{8}{12} = -6$$

Negative sign indicates that second principal point H_2 lies at a distance of 6 cm to the left of eye lens L_2.

Position of Nodal Points:

As the medium on either side of the eye piece is same (air), the nodal points N_1, N2 coincide with the principal points H_1, H_2 respectively.

The position of cardinal points for Huygen's eyepiece can be plotted as:

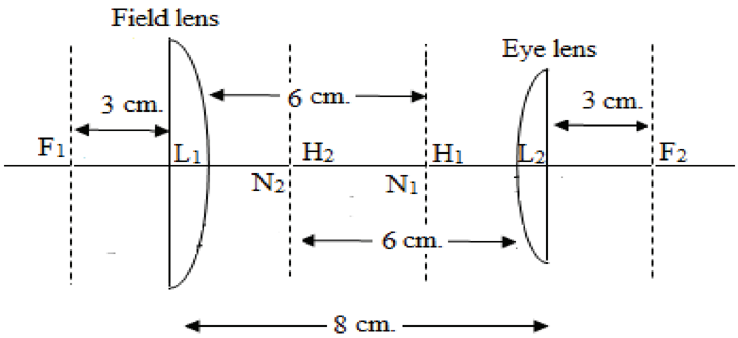


Figure 14.7

Position of Image: Given that object is situated at a distance 9 cm. in front of the field lens. If u and v are the distance of the object O and image I from the first and second principal point H_1 and H_2 .

Then u= - H_1O= - (H_1 L_1+ L_1O) = - (6+9) = -15 cm., F = 9 cm.

If u is the distance of I_1 from the field lens, than from lens formula

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{F}$$

$$\frac{1}{9} = \frac{1}{v} - \frac{1}{(-15)} \quad or\ \frac{1}{v} = \frac{1}{9} - \frac{1}{15} = \frac{5-3}{45} \quad or\ v = \frac{45}{2} = 22.5\ cm.$$

Hence, $L_2I = H_2I - H_2L_2 = 22.5 - 6 = 16.5$ cm.

Thus the image lies at a distance of 16.5 cm. to the right of eye lens L_2.

$$L_1F_1 = -3\,cm.,\ L_2F_2 = 3\,cm.,\ L_1H_1 = L_1N_1 = 6\ cm.,\ L_2H_2 = L_2N_2 = -6\ cm.,$$

Also, position of image formed by eyepiece, lies at a distance of 16.5 cm to the right of eye lens L_2.

14.7 HUYGEN'S EYEPIECE

14.7.1 Construction

Huygens's eyepiece consist of a combination of two coaxial plano convex lenses having focal length in the ratio 3:1 separated by the distance between them is equal to the difference in their focal length. The focal length and the positions of the two lenses are such that the system is free from chromatic as well as spherical aberrations. The field and eye lenses are placed with their convex surface towards the incident ray.

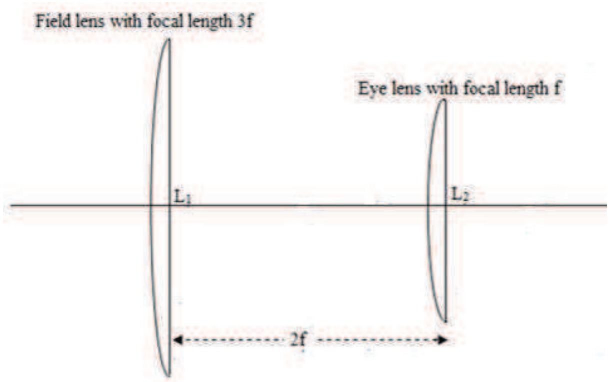


Figure 14.8

Condition of Achromatism: For achromatism the distance d between two lenses should be

$$d = \frac{f_1 + f_2}{2}$$

In case of Huygen's eyepiece $f_1 = 3f \; and \; f_2 = f$

$\therefore$ $$d = \frac{3f + f}{2} = 2f$$

This is the distance between field lens and eye lens. Hence Huygen's eyepiece is free from chromatic aberration

Condition for Minimum Spherical Aberration: The distance between the two lenses for minimum spherical aberration should be

$$d = f_1 - f_2$$

In case of Huygen's eyepiece $f_1 = 3f \; and \; f_2 = f$

$\therefore$ $$d = 3f - f = 2f$$

This is the distance between field lens and eye lens in Huygen's eyepiece hence Huygen's eyepiece is free from spherical aberration.

14.7.2 Working

Like Ramsden's eyepiece when eye piece is adjusted for normal vision, the final image formed by it is at infinity. For this the image formed by the field lens should lies in the first focal plane of the eye lens i.e. at a distance f to the left of eye lens or at a distance f to the rigt of field lens as the distance between eye and field lens is 2f. The inverted image I_1 of a distant object, formed by objective, acts as an object for the field lens. This gives rise to a virtual image I_2. I_2 in turns serves as an object for the eye lens therefore it must be at distance equal to focal length f from eye lens to make final image at infinity.

For field lens if u is the distance of I_1 from the field lens, than from lens formula

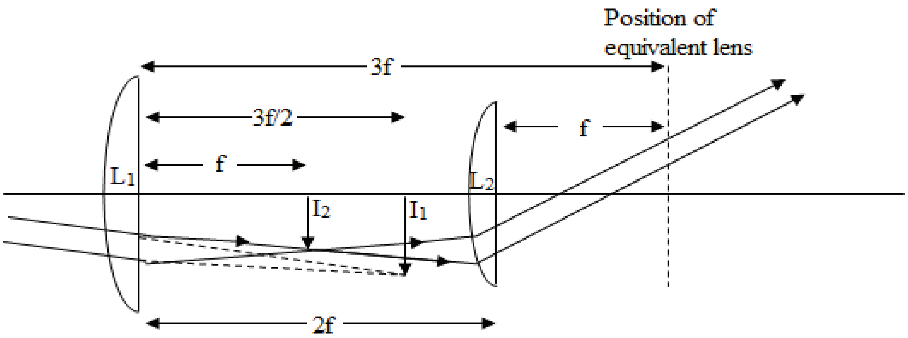


Figure 14.9

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

Since here $v = f$ and $f = 3f$ we have

$$\frac{1}{f} - \frac{1}{u} = \frac{1}{3f} \quad or \quad \frac{1}{u} = \frac{1}{f} - \frac{1}{3f} = \frac{2}{3f} \quad or \quad u = \frac{3}{2}f$$

The positive sign indicates that image I_1 formed by field lens as well as image I_2 formed by objective lies on the same side, i.e., the field lens focused the rays at I_2 which otherwise would be focused at I_1 by objective. The rays coming from I_2 emerge from the eye lens as a parallel beam.

Equivalent Focal Length

Equivalent focal length of Huygen's eyepiece is given by:

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

Here $f_1 = 3f$, $f_2 = f$ $and\ d = 2f$

$$\frac{1}{F} = \frac{1}{3f} + \frac{1}{f} - \frac{2f}{3f^2}$$

$$\frac{1}{F} = \frac{4}{3f} - \frac{2}{3f} \qquad \text{or} \qquad \frac{1}{F} = \frac{2}{3f}$$

$$F = \frac{3f}{2}$$

When eye piece is adjusted for normal vision, the final image formed by it should be at infinity. For this, the image formed by the equivalent lens must be at infinity. .For this equivalent lens must be placed at a distance 3f/2 to the right of I_1 or at a distance f to the right of eye lens.

14.7.3 Cardinal Points of Huygen's Eyepiece

Position of Focal Points:

The distance of first focal point F_1 from the field lens L_1 is given by

$$\beta_1 = -F\left(1-\frac{d}{f_2}\right) = -\left(\frac{3}{2}f\right)\left(1-\frac{2f}{f}\right) = -\frac{3}{2}f \times (-1) = +\frac{3f}{2}$$

Positive sign indicates that first focal point F_1 lies at a distance of 3f/2 to the right of field lens L_1. The distance of second focal point F_2 from the eye lens L_2 is given by

$$\beta_2 = F\left(1-\frac{d}{f_1}\right) = \left(\frac{3}{2}f\right)\left(1-\frac{2f}{3f}\right) = \frac{3}{2}f \times \frac{1}{3} = +\frac{f}{2}$$

Positive sign indicates that second focal point F_2 lies at a distance of f/2 to the right of eye lens L_2.

Position of Principal Points

The distance of first principal point H_1 from the field lens L_1 is given by

$$\alpha_1 = F\frac{d}{f_2} = \frac{\left(\frac{3}{2}f\right)2f}{f} = +3f$$

Positive sign indicates that first principal point H_1 lies at a distance of 3f to the right of field lens L_1. The distance of second principal point H_2 from the eye lens L_2 is given by

$$\alpha_2 = -F\frac{d}{f_1} = -\frac{\left(\frac{3}{2}f\right)2f}{3f} = -f$$

Negative sign indicates that second principal point H_2 lies at a distance of f to the left of eye lens L_2.

Position of Nodal Points

As the medium on either side of the eye piece is same (air), the nodal points N_1, N2 coincide with the principal points H_1, H_2 respectively.

The position of cardinal points for Huygen's eyepiece can be plotted as:

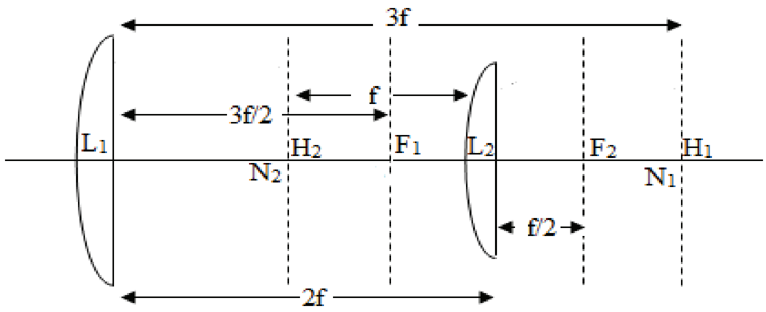


Figure 14.10

Position of Cross Wires

The cross wires if used must be placed at the position where the image due to objective is formed so that they would be in focus with the final image and magnified by both the lens of eyepiece. The cross wire placed in first focal plane satisfies both the conditions. However in Huygen's eyepiece first focal plane lies between the lenses of eyepiece and is virtual. Therefore, crosswire must be placed at a distance of 3f/2 in right side of field lens and f/2 left to eye lens, i.e., in between the lenses of eyepiece. So it is magnified by eye lens only while the image is magnified by both the lenses. It is called a negative eyepiece as the first focal plane lies within the eyepiece where no real objects can be placed. Therefore this eyepiece is used to examine a virtual image like in microscope. Therefore generally eyepiece is not used in this type of eyepiece.

Example 14.3: In Huygen's eyepiece focal length of field lens is 6 cm. Locate the position of cardinal points in a diagram.

Solution: In a Huygen's eyepiece field lens and eye lens having focal length 3f and f are separated by a distance 2f.

Given focal length of field lens is = 2 cm, i.e., 3f = 6cm, hence f = 6/3 = 2 cm.

So focal length of eye lens is = 2 cm and separation between lens is = 2f, i.e., d = 2x2 = 4 cm.

The equivalent focal length of Huygen's eyepiece is

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

Here $f_1 = 6\ cm.,\ f_2 = 2\ cm.\ and\ d = 4\ cm.$

$$\frac{1}{F} = \frac{1}{6} + \frac{1}{2} - \frac{4}{12}$$

$$\frac{1}{F} = \frac{2+6-4}{12} \quad \text{or} \quad \frac{1}{F} = \frac{4}{12}$$

or F = 3 cm

Position of Focal Points

The distance of first focal point F_1 from the field lens L_1 is given by

$$\beta_1 = L_1F_1 = -F\left(1-\frac{d}{f_2}\right) = -3\left(1-\frac{4}{2}\right) = 3\ cm.$$

Positive sign indicates that first focal point F_1 lies at a distance 3 cm. to the right of field lens L_1. The distance of second focal point F_2 from the eye lens L_2 is given by

$$\beta_2 = L_2F_2 = F\left(1-\frac{d}{f_1}\right) == 3\left(1-\frac{4}{6}\right) = 1\ cm.$$

Positive sign indicates that second focal point F_2 lies at a distance of 1cm to the right of eye lens L_2.

Position of Principal Points

The distance of first principal point H_1 from the field lens L_1 is given by

$$\alpha_1 = L_1H_1 = F\frac{d}{f_2} = 3\frac{4}{2} = 6\ cm.$$

Positive sign indicates that first principal point H_1 lies at a distance of 6 cm to the right of field lens L_1. The distance of second principal point H_2 from the eye lens L_2 is given by

$$\alpha_2 = L_2H_2 = -F\frac{d}{f_1} = -3\frac{4}{6} = -2$$

Positive sign indicates that second principal point H_2 lies at a distance of 2 cm to the left of eye lens L_2.

Position of Nodal Points

As the medium on either side of the eye piece is same (air), the nodal points N_1, N_2 coincide with the principal points H_1, H_2 respectively.

The position of cardinal points for Huygen's eyepiece can be plotted as

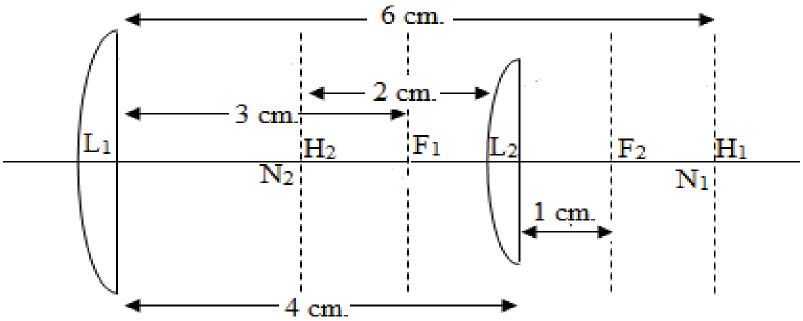


Figure 14.11

$\therefore$ $\quad L_1F_1 = 3\ cm,\ L_2F_2 = 1cm,\ L_1H_1 = L_1N_1 = 6\ cm,\ L_2H_2 = L_2N_2 = -2\ cm,$

Example 14.4: In Huygen's eyepiece equivalent focal length of Huygen's eyepiece is 9 cm. Calculate the position of cardinal points. If an object is situated at 9 cm. in front of the field lens, find the position of the image formed by the eyepiece.

Solution: In a Huygen's eyepiece field lens and eye lens having focal length 3f and f are separated by a distance 2f.

Given equivalent focal length of field lens is F= 9 cm

The equivalent focal length of Huygen's eyepiece is

$$\frac{1}{9} = \frac{1}{3f} + \frac{1}{f} - \frac{2f}{3f.2f}$$

Here, $F = 9\ cm.,$

$$\frac{1}{9} = \frac{2}{3f}$$

$f = 6\ cm.$ so 3f = 18 cm.

So focal length of eye lens is f = 6 cm, focal length of field lens is 3f = 18 cm and separation between lens is d = 2f = 12 cm.

$\therefore$ $here\ f_1 = 18\ cm., f_2 = 6\ cm., d = 12\ cm., F = 9\ \ cm.$

Position of Focal Points:

The distance of first focal point F_1 from the field lens L_1 is given by

$$\beta_1 = L_1F_1 = -F\left(1 - \frac{d}{f_2}\right) = -9\left(1 - \frac{12}{6}\right) = 9\ cm.$$

Positive sign indicates that first focal point F_1 lies at a distance 9 cm to the right of field lens L_1. The distance of second focal point F_2 from the eye lens L_2 is given by

$$\beta_2 = L_2F_2 = F\left(1 - \frac{d}{f_1}\right) == 9\left(1 - \frac{12}{18}\right) = 3\ cm.$$

Positive sign indicates that second focal point F_2 lies at a distance of 3 cm to the right of eye lens L_2.

Position of Principal Points

The distance of first principal point H_1 from the field lens L_1 is given by

$$\alpha_1 = L_1H_1 = F\frac{d}{f_2} = 9\frac{12}{6} = 18\ cm.$$

Positive sign indicates that first principal point H_1 lies at a distance of 18 cm to the right of field lens L_1. The distance of second principal point H_2 from the eye lens L_2 is given by

$$\alpha_2 = L_2H_2 = -F\frac{d}{f_1} = -9\frac{12}{18} = -6$$

Negative sign indicates that second principal point H_2 lies at a distance of 6 cm to the left of eye lens L_2.

Position of Nodal Points

As the medium on either side of the eye piece is same (air), the nodal points N_1, N2 coincide with the principal points H_1, H_2 respectively. The position of cardinal points for Huygen's eyepiece can be plotted as

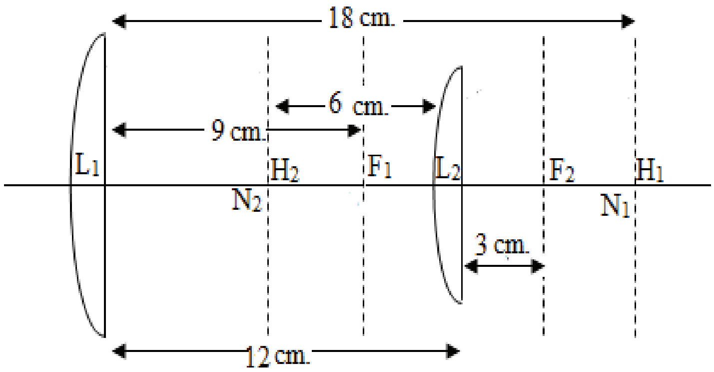


Figure 14.12

Position of Image: Given that object is situated at a distance 9 cm in front of the field lens. If u and v are the distance of the object O and image I from the first and second principal point H_1 and H_2 .

Then u = - H_1O = - (H_1 L_1+ L_1O) = - (18+9) = - 27 cm, F = 9 cm.

If u is the distance of I_1 from the field lens, than from lens formula

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{F}$$

$$\frac{1}{9} = \frac{1}{v} - \frac{1}{(-24)} \quad or \; \frac{1}{v} = \frac{1}{9} - \frac{1}{27} = \frac{3-1}{27} \quad or \; v = \frac{27}{2} = 13.5 \; cm.$$

Hence, $L_2I = H_2I - H_2L_2 = 13.5 - 6 = 7.5$ cm.

Thus the image lies at a distance of 7.5 cm to the right of eye lens L_2.

$\boldsymbol{L_1F_1 = 9\ cm}$., $\boldsymbol{L_2F_2 = 3\,cm}$., $\boldsymbol{L_1H_1 = L_1N_1 = 18\ cm}$., $\boldsymbol{L_2H_2 = L_2N_2 = -6\ cm}$.,

Position of image formed by eyepiece, therefore, lies at a distance of 7.5 cm to the right of eye lens L_2.

14.8 GAUSSIAN EYEPIECE

14.8.1 Construction

It is a modification of Ramsden’s eyepiece. Like the Ramsden’s eyepiece, it consists of a field lens and eye lens having equal focal length f separated by a distance equal to two third of focal length f. A thin plane glass plate G inclined at an angle 45^0 to the axis of the lens system is placed between the field lens and eye lens to illuminate the field of view. The light from the source S enter the tubes from opening given for the purpose and reflects from the glass plate G along the axis towards the objective of the telescope and illuminates the field of view. The cross wire C is kept at a distance f/4 in front of the field lens like Ramsden’s eyepiece.

14.8.2 Uses

The eye piece is often used in spectrometer telescopes and is very helpful in focusing the telescope and collimator of the spectrometer for parallel rays and adjusting the axis of telescope and microscope perpendicular to the axis of the instrument.

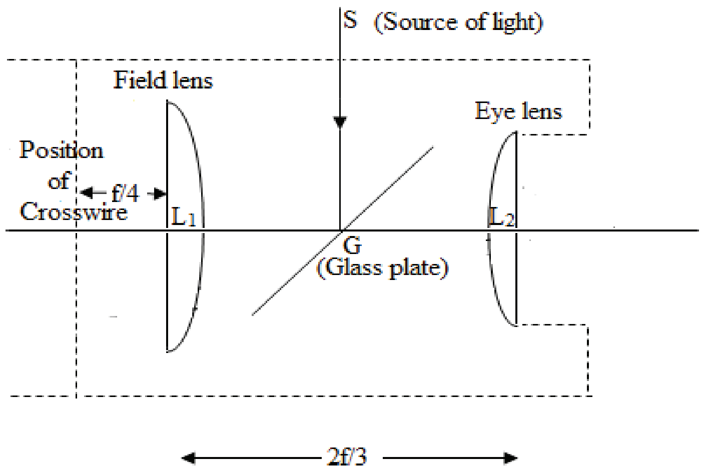


Figure 14.13

(i) To Focus the Telescope and Collimator of the Spectrometer for Parallel Rays

To adjust the axis of the telescope perpendicular to the axis of the spectrometer the telescope is first focused for parallel rays, the turn table (with the reflecting glass surface) is rotated through 180^0. Now the leveling screws of the table are adjusted until the image of the cross wires are at the same place as before rotating the table. Now the inclination of the telescope is altered until the image of the cross wires coincides with cross wires. At this position the axis of the telescope is perpendicular to the axis of the spectrometer.

To focus the telescope for parallel rays the eye-piece is first focused on the cross wires. Then a plane reflecting surface is placed on the turn table in front of the objective. Now the distance between the objective and cross wires is so adjusted that their image and cross wires are in sharp focus without parallax. In this position the cross wires are in the focal plane of the objectives means telescope is focused for parallel rays.

Now to focus the collimator for parallel rays the slit of the collimator is illuminated and a telescope focused for parallel rays is brought along the line of the collimator. Now the distance between the slit and crosswire is so adjusted that the image of the slit is formed on the cross wires without parallax. In this position collimator is focused for parallel rays.

(ii) To Adjust the Axis of the Telescope and Microscope Perpendicular to the Axis of the Spectrometer

To adjust the axis of the collimator perpendicular to the axis of the spectrometer the inclination of the collimator is altered until the image of the slit is formed on the cross wires in such a way that half of the image is up the intersection of cross wires and half is down the intersection of the cross wires. At this position the axis of the collimator is perpendicular to the axis of the spectrometer.

14. 9 COMPARISON OF HUYGEN'S AND RAMSDEN'S EYEPIECES

S.N.	Huygen's Eyepiece	Ramsden's Eyepiece
1.	Construction: a) Lens Type: Two planoconvex lenses made of same material with their convex side facing the incident light. b) Ratio of focal length of field lens and eye lens is 3:1. c) Distance between field lens and eye lens is 2f where f is focal length of eye lens.	Construction: a) Lens Type: Two planoconvex lenses made of same material with their convex side facing each other inwards. b) Ratio of focal length of field lens and eye lens is 1:1. c) Distance between field lens and eye lens is 2/3f where f is focal length of eye lens.
2.	Huygen's eyepiece is a negative eye piece as the image formed by objective lens of the instrument lies between field lens and eye lens.	Ramsden's eyepiece is a positive eye piece as the image formed by objective lens of the instrument lies in front of the field lens.
3.	In this eyepiece generally crosswire cannot be used. Cross wire if used is magnified by eye lens; while the image of the object is magnified by both field lens and eye lens hence magnification is different for two.	In this eyepiece generally crosswire can be used. Cross wire and the image of the object is magnified by both field lens and eye lens hence magnification is same for two.
4.	As the cross wires are put outside the eyepiece it involves no mechanical difficulty.	Cross wires if used has to be placed between the field lens and eye lens which causes mechanical difficulty to fit them.
5.	The condition for minimum spherical aberration ($d=f_1-f_2$) is completely satisfied.	The condition for minimum spherical aberration ($d=f_1-f_2$) is not satisfied. But it is reduced by using the planoconvex lenses with their convex surface facing each other.
6.	The condition for minimum chromatic aberration [$d = (f_1+f_2) / 2$] is completely satisfied. Therefore, can be used for white colour.	The condition for minimum chromatic aberration ($d= (f_1+f_2)/2$) is not satisfied. Therefore generally used for monochromatic (single) colour. Even if used for heterogeneous light measurement

		is made for a particular colour at a time.
7.	It exhibits other type of aberration like coma and distortion.	Other types of aberration are better eliminated. Coma is absent and distortion is less than Huygen's eyepiece.
8.	It is used for qualitative purposes in microscopes and telescopes.	It is used in microscopes and telescopes for accurate quantitative measurements.
9.	It cannot be used as a simple magnifier.	It can be used as a simple magnifier.

Self Assessment Questions

6. What is an eyepiece?
7. What is the condition of minimum spherical aberration in Huygens' eyepiece?
8. A Huygens eyepiece has an eyelens of 4cm focal length. What will be the focal length of field lens and the distance between the two lenses.
9. If eyepiece is Ramsden's then?
10. Discuss in brief the specific arrangement of lenses in Ramsden's eyepiece.
11. Write the conditions of achromatism and minimum spherical aberration in Ramsden's eyepiece.

14.10 SPECTROMETER

A spectrometer is a measuring device that collects light waves. It uses these light waves to determine the material that emitted the energy, or to create a frequency spectrum. Astronomers make the most frequent use of spectrometers to determine the makeup of stars or other celestial bodies. The concept of a spectrometer now encompasses instruments that particles, atoms, and molecules by their mass, momentum, or energy. These types of spectrometers are used in chemical analysis and particle physics.

When objects are hot enough, they emit visible light at a given point or points on the electromagnetic spectrum. Spectrometers split the incoming light wave into its component colors. Using this, they can determine what material created the light.

14.10.1 General Layout of a Spectrometer

The most basic design of a modern spectrometer is an assembly of a slitted screen, a diffraction grating and a photo detector. The screen allows a beam of light into the interior of the spectrometer, where the light passes through the diffraction grating. The grating splits the light into a beam of its component colors, similar to a prism. Many spectrometers also have a collimating mirror that makes the light waves parallel and coherent, thus making it more focused. This applies especially to spectrometers used in telescopes. The light then reflects onto a detector that picks up individual wavelengths.

14.10.2 Types of spectrometer

Optical Spectrometer

Optical spectrometers (often simply called spectrometers), in particular, show the intensity of light as a function of wavelength or of frequency. The deflection is produced either by refraction in a prism or by diffraction in a diffraction grating. It consists of a slit, a collimator to make light rays parallel to the axis, a table in which prism or grating mounted and a detector (e.g. telescope).

These spectrometers utilize the phenomenon of optical dispersion. The light from a source can consist of a continuous spectrum, an emission spectrum (bright lines), or an absorption spectrum (dark lines). Because each element leaves its spectral signature in the pattern of lines observed, a spectral analysis can reveal the composition of the object being analyzed.

Mass Spectrometer

A mass spectrometer is an analytical instrument that is used to identify the amount and type of chemicals present in a sample by measuring the mass-to-charge ratio and abundance of gas-phase ions.

Time-of-Flight Spectrometer

The energy spectrum of particles of known mass can also be measured by determining the time of flight between two detectors (and hence, the velocity) in a time-of-flight spectrometer. Alternatively, if the velocity is known, masses can be determined in a time-of-flight mass spectrometer.

14.11 ELECTRON MICROSCOPE

The electron microscope is a powerful microscope that uses a beam of electrons instead of light beam and an electron detector instead of our eyes to create an image of the specimen. It is capable of much higher magnifications and has a greater resolving power than a light microscope, allowing it to see much smaller objects in finer detail. They are large, expensive

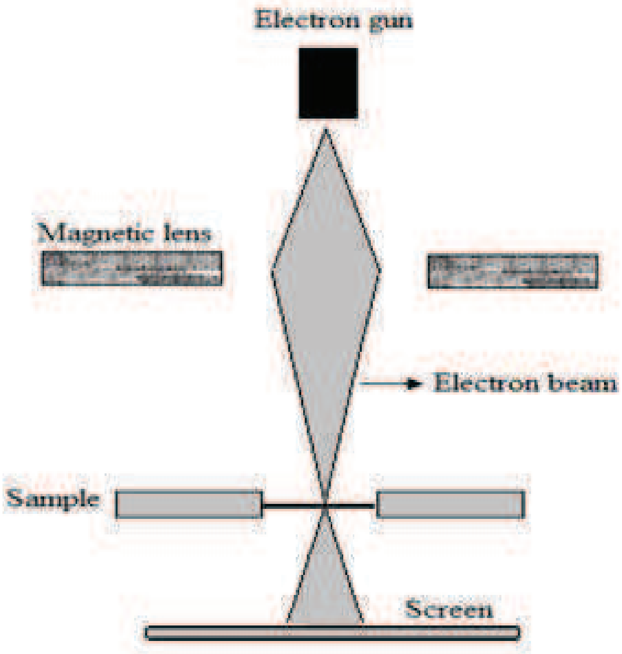


Figure 14.14

pieces of equipment, generally standing alone in a small, specially designed room and requiring trained personnel. An electron has the properties of a wave with a wavelength that is much smaller than visible light (a few trillionths of a meter!). With this wavelength we can distinguish features down to a fraction of a nanometer.

14.11.1 Types of Electron Microscopes

Scanning Electron Microscope (SEM)

In a scanning electron microscope or SEM, a beam of electrons scans the surface of a sample. The electrons interact with the material in a way that triggers the emission of secondary electrons. These secondary electrons are captured by a detector, which forms an image of the surface of the sample. The direction of the emission of the secondary electrons depends on the orientation of the features of the surface. There, the image formed will reflect the characteristic feature of the region of the surface that was exposed to the electron beam.

Transmission Electron Microscope (TEM)

In a transmission electron microscope or TEM, Transmission electron microscopy (TEM) involves a high voltage electron beam emitted by a cathode and formed by magnetic lenses hits a very thin sample (≤100 nm thick). The electrons are transmitted through the sample. The spatial variation in this information (the "image") is then magnified by a series of magnetic lenses until it is recorded by hitting a fluorescent screen, photographic plate, or light sensitive sensor such as a CCD (charge-coupled device) camera. The image detected by the CCD may be displayed in real time on a monitor or computer.

This process is similar to working of movie projector. In a projector, a film has the negative image that will be projected. The projector shines white light on the negative and the light transmitted forms the image contained in the negative. Transmission electron microscopes produce two-dimensional, black and white images.

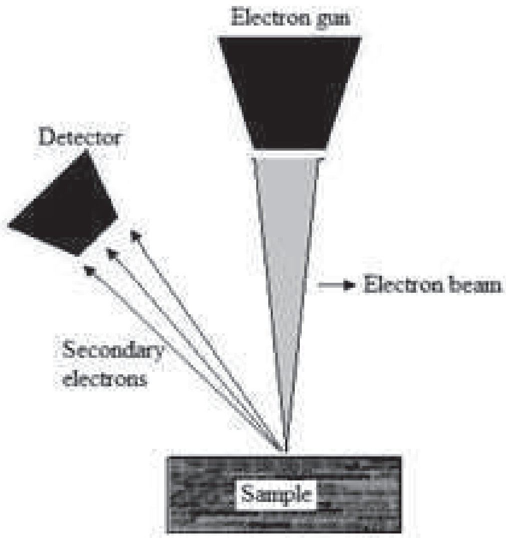


Figure 14.15

Scanning Transmission Electron Microscope (STEM)

A scanning transmission electron microscope or STEM combines the capabilities of both an SEM and a TEM. The electron beam is transmitted across the sample to create an image (TEM) while it also scans a small region on the sample (SEM). The ability to scan the electron beams allows the user to analyze the sample with various techniques such as Electron Energy Loss Spectroscopy (EELS) and Energy Dispersive X-ray (EDX) Spectroscopy which are useful tools to understand the nature of the materials in the sample.

14.11.2 Use of Electron Microscope

Electron microscope is used to examine biological materials (such as microorganisms and cells), a variety of large molecules, medical biopsy samples, metals and crystalline structures, and the characteristics of various surfaces. Outside research, they can be used in the fabrication of silicon chips, or within forensics laboratories for looking at samples such as gunshot residues. They are also used for fault diagnosis and quality control, e.g., they can be used to look for stress lines in engine parts or simply to check the ratio of air to solids in ice cream.

14.12 SUMMARY

Optical instruments are devices which process light waves so that the image of object is enhanced as well as become clearer, e.g., microscope, telescope etc. Our eyes are not only vital for seeing the world around us but also an essential part of all optical instruments. Most of the refraction of light entering the eye occurs at the outer surface of the cornea. Refraction at the cornea and the surfaces of the lens produces a real and inverted image of the object on the retina. The optic nerve sends a signal to the brain.

Field of view and aberration are two main concerns for optical instruments. Field of view of an optical instrument may be defined as the extent or range of its visual area. The basic purpose of optical instrument is to produce a magnified image free from aberrations and enlarged field of view. An eyepiece is used in place of single eye lens for enlarged field of view and reduces aberrations in final image formed by eyepieces.

Ramsden's Eyepiece consists of two plano convex lenses made of same material with their curved (convex) side facing each other inwards. In this type of eyepiece focal length of field lens and eye lens is same and distance between field lens and eye lens is $\frac{2}{3}f$ where f is focal length of eye lens.

Huygens's eyepiece consist of a combination of two coaxial planoconvex lenses having focal length in the ratio 3:1 separated by the distance equal to the difference in their focal length. The focal length and the positions of the two lenses are such that the system is free from chromatic as well as spherical aberrations. It is a modification of Ramsden's eyepiece. It consists of a field lens and eye lens having equal focal length f separated by a distance equal to two third of focal length f.

The eye piece is often used in spectrometer telescopes and is very helpful in focusing the telescope and collimator of the spectrometer for parallel rays and adjusting the axis of telescope and microscope perpendicular to the axis of the instrument.

A spectrometer is a measuring device that collects light waves. It uses these light waves to determine the material that emitted the energy, or to create a frequency spectrum. Astronomers make the most frequent use of spectrometers to determine the makeup of stars or other celestial bodies.

The electron microscope is a powerful microscope that uses a beam of electrons instead of light beam and an electron detector instead of our eyes to create an image of the specimen. It is capable of much higher magnifications and has a greater resolving power than a light microscope, allowing it to see much smaller objects in finer detail.

14.13 GLOSSARY

Optical Instruments: Optical instruments are devices which process light waves so that image of object is enhanced as well as clearer.

Field of View: Field of view of an optical instrument may be defined as the extent or range of its visual area.

Eyepiece: An eyepiece is an optical instrument consisting of two lenses i.e. field lens and eye lens, made by suitable material. It is preferred in place of single lens. It enlarges field of view and reduces chromatic aberration and spherical aberrations.

Ramsden's Eyepiece: Ramsden's Eyepiece consists of two plano convex lenses made of same material and same focal length with their curved (convex) sides facing each other. Distance between field lens and eye lens is 2/3 of focal length of either lens. It is an example of positive eye piece and we can use cross wire in it.

Huygen's Eyepiece: Huygens's eyepiece consists of a combination of two coaxial plano convex lenses having focal length in the ratio 3:1. The distance between them is equal to the difference in their focal length. The focal length and the positions of the two lenses are such that the system is free from chromatic as well as spherical aberrations. The field and eye lenses are placed with their convex surface towards the incident ray. It is an example of negative eye piece and we generally don't use cross wire in it.

Gaussian eyepiece: It is a modification of Ramsden's eyepiece. It consists of a field lens and eye lens having equal focal lengths f separated by a distance equal to two third of f. A thin plane glass plate G inclined at an angle 45^0 to the axis of the lens system is placed between the field lens and eye lens to illuminate the field of view. The cross wire C is kept at a distance f/4 in front of the field lens.

14.14 REFERENCE BOOKS

1. Optics by Ajoy Ghatak

2. A textbook of Optics by Brij Lal and Dr. N.Subrahmnyam

3. Optics by Dr. S.P. Singh and Dr. J.P. Agarwal

14.15 SUGGESTED READINGS

1. Fundamental of Optics by F. A. Jenkins and H. E. White.

2. The Feynman Lectures on Physics by Richard Feynman

3. Optics by Eugene Hecht

14.16 TERMINAL QUESTIONS:

14.16.1 Short Answer Type Questions

1. What is field of view? Why we need multiple lens eye piece.
2. Give the name and construction which satisfies the condition for achromatism?
3. Give the name and construction of eyepiece in which cross wire is used.
4. 4. What types of eyepieces should be used in a) spectrometer b) low power microscope?
5. Give the construction and working of electron microscope.

14.16.2 Long Answer Type Questions

1. Explain the construction and working of Huygen's eyepiece. How chromatic and spherical aberrations minimized in the eyepiece? Calculate the position of cardinal points and indicate them in a diagram. Why crosswire is not used in Huygen's eyepiece?
2. Explain the construction and working of Ramsden's eyepiece. How chromatic and spherical aberrations minimized in the eyepiece? Calculate the position of cardinal points and indicate them in a diagram. How it is modified in Gaussian eyepiece?

3. Explain the sectional view and working of different parts of human eye.

4. Write short notes on

a) Spectrometer b) Comparison of eye piece

c) Positive and negative eyepiece d) Gaussian eyepiece

14.16.3 Numerical Questions

3. In Huygen's eyepiece the focal length of lenses are 4 cm and 12 cm. Find and plot the position of cardinal points in a diagram. If an object is situated at 6 cm in front of the field lens, what will be the position of the image formed by the eyepiece? **(**Ans. $L_1H_1 = L_1N_1 = +12$cm, $L_2H_2 = L_2N_2 = +0.4$cm, $L_1F_1 = 6$ cm, $L_2F_2 = +2$cm, image will be at 0.5 cm behind eye lens**)**

4. A Ramsden's eyepiece designed using two plano convex lens of focal length 4 cm. each. Find the equivalent focal length and position of cardinal points. **(**Ans. equivalent focal length = 3 cm, $L_1H_1 = L_1N_1 = +2$cm, $L_2H_2 = L_2N_2 = -2$cm, $L_1F_1 = -1$ cm, $L_2F_2 = +1$cm)

5. The equivalent focal length of Huygen's eyepiece is 5 cm. Calculate the focal length and distance between field lens and eye lens. Also locate the position of cardinal points. (Ans. focal length of field lens and eye lens = +10 cm and +10/3 cm and distance between field lens and eye lens = +20/3cm, $L_1H_1 = L_1N_1 = +10/3$ cm, $L_2H_2 = L_2N_2 = -10/3$ cm, $L_1F_1 = +5$ cm, $L_2F_2 = +5/3$ cm)

6. The focal length of each lens of a Ramsden's eyepiece is 10 cm. Calculate the equivalent focal length of the eyepiece.

7. The focal length of the more convergent lens of an Huygens' eyepiece is 0.5cm. Calculate the focal length of the eyepiece and locate on a diagram the positions of its focal points.

14.16.4 Objective Questions

1. Huygen's eye piece consists of

a. Two plano convex lens of focal length F and 3F separated by a distance 2F.
b. Two plano convex lens of equal focal length F separated by a distance 2/3F.
c. Two plano convex lens of equal focal length F separated by a distance F.
d. Two plano convex lens of focal length F and 3F separated by a distance 2/3F.

Ans. Option **'a'**

2. Ramsden's eye piece is

a. Negative eyepiece b. Positive eye piece
c. Null eye piece d. Can't say

Ans. Option **'b'**

3. Spectrometer consists of

a. A collimator b. Telescope
c. Prism Table d. All of the above

Ans. Option **'d'**

4. The conditions of achromatism and minimum spherical aberration are fully satisfied in:

a. Ramsden's eye piece b. Huygen's eyepiece
c. both Ramsden's and Huygen's eyepieces

d. Neither Ramden’s nor Huygen’s eyepiece.

Ans. Option **‘b’**

5. In Ramsden’s eyepiece fitted in a telescope, the position of the equivalent lens is:

a. before the objective
b. between the field lens and the eye lens
c. between the objective and the field lens
d. behind the eye lens

Ans. Option **‘b’**

UNIT 15: ABERRATIONS

CONTANTS

15.1 INTRODUCTION

The purpose of using a lens or a system of lenses is to obtain an image that is exact magnified replica of object. But in general, it is not possible due to various reasons. These reasons will be discussed in detail, in this unit. Any deviation produced in image from object is called aberrations in the image. The focal length of the lens depends on refractive index and refractive index is different for different wavelengths. Thus, if the object is illuminated by white light then due to prismatic action of the lens after refraction through the lens, image looks coloured and blurred. Even if we use monochromatic light to illuminate object, image is not completely free from defects because generally during formulae derivation for object and image distances and magnifications produced by a lens, we assumed that a point object gives a point image, the aperture of lens is small also the angles made by light rays with the principal axis are small. But in actual practice the objects are bigger and lens is used to form a perfect, bright and magnified image of points situated off the axis also. For this we have to consider wide angle rays passing through the extremes of lens (known as peripheral or marginal ray) as well as the rays passing near the axis (known as paraxial rays). Generally the image formed by these two kinds of rays does not meet at a single point after refraction through the lens causing imperfection in image.

15.2 OBJECTIVES

After study of unit the student will be able

- To understand the concept of aberration in images.
- To explain chromatic aberration, it's cause and methods to remove them.
- To explain different types of monochromatic aberration and their elimination
- To describe construction and working of spherical mirrors
- To explain construction and use of Schmidt corrector plate
- To explain construction and use of oil immersion lens

15.3 ABERRATIONS IN IMAGES

Any departure of real images from the objects in respects to shape, orientation, colour etc. is called aberrations. It is not caused by faulty construction of lens or impurity in lens material but an inherent shortcoming arises due to the failure of lens to behave precisely according to the formulae derived.

Aberrations are derived broadly into two categories. The aberrations produced due to variation of refractive index, wavelength or dispersion of light (i.e., object illuminated by white light) is called chromatic aberrations. While the defects due to wide angle incidence and peripheral incidence (due to marginal ray) even if illuminated with monochromatic light is called monochromatic aberrations. Monochromatic aberrations are again classified as

1) Spherical Aberration

2) Coma

3) Astigmatism

4) Curvature

5) Distortion

15.4 CHROMATIC ABERRATION OF LENS

The image of an object illuminated by white light made by a lens is generally coloured and blurred this is called chromatic aberration. The chromatic aberration arises due to prismatic action of lens material. If a parallel beam of white light is incident on a convex lens, because of prismatic action it splits into its constituent colours and focus separately. Hence the position as well as size of image for different colours will be different for different constituent colours.

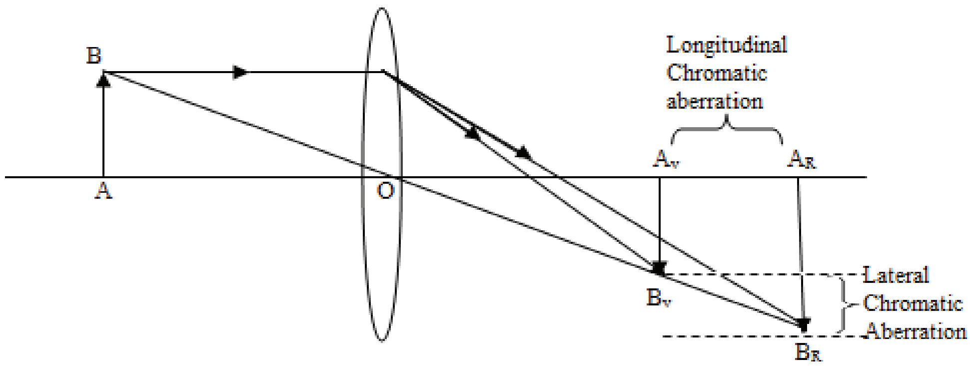


Figure15.1

It is generally of two types.

1. Longitudinal or axial chromatic aberration

2. Lateral or transverse chromatic aberration

15.4.1. Longitudinal or Axial Chromatic Aberration

Since the deviation for the red colour is minimum and for the violet colour is maximum, the focal length of lens for red colour will be different from that due to violet colour. Hence the images for different colours will be formed at different position along the axis. The formation of images of different colours at different positions along the axis is called longitudinal chromatic aberration. If the object is placed at infinity longitudinal chromatic aberration will be equal to the difference in focal lengths for red and violet rays. The difference $f_r - f_v$ measures axial or longitudinal chromatic aberration.

Expression of Longitudinal Chromatic Aberration for Object Situated at Infinity

The focal length of a lens of radii of curvature R_1 and R_2 is given by $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

Let the focal lengths and refractive indices of the lens for red, yellow, and violet colours are $f_{r,}\, f_y \; and \; f_v$ and $\mu_{r,}\, \mu_y \; and \; \mu_v$ respectively.

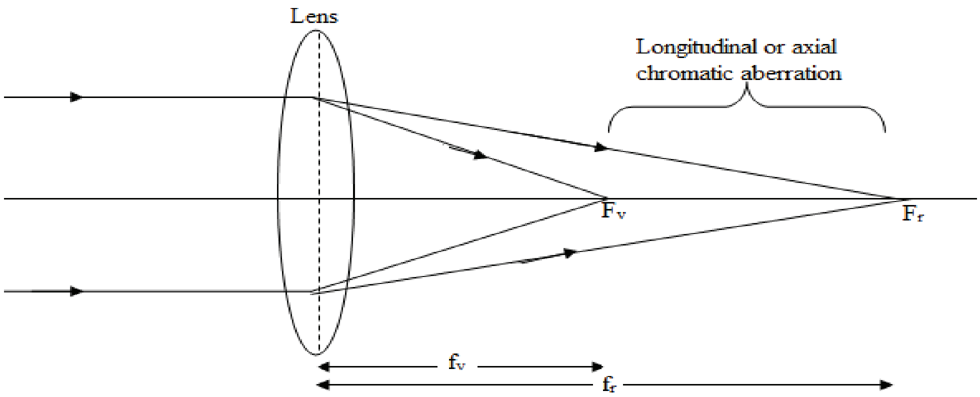


Figure 15.2

So above equation takes the form for different colours as follows

$$\frac{1}{f_r} = (\mu_r - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \text{.......} \quad (15.1)$$

$$\frac{1}{f_y} = \left(\mu_y - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \text{.......} \quad (15.2)$$

$$\frac{1}{f_v} = (\mu_v - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \text{.......} \quad (15.3)$$

Subtracting equation (15.1) from (15.3), we get

$$\frac{1}{f_v} - \frac{1}{f_r} = (\mu_v - \mu_r)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{f_r - f_v}{f_v f_r} = \frac{(\mu_v - \mu_r)}{(\mu_y - 1)}\left(\mu_y - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{f_r - f_v}{{f_y}^2} = \omega.\frac{1}{f_y} \qquad \text{.......} \quad (15.4)$$

Where $\omega = \frac{(\mu_v - \mu_r)}{(\mu_y - 1)}$ is the dispersive power of lens material.

or $$f_r - f_v = \omega f_y \qquad \text{.......} \quad (15.5)$$

Expression of Longitudinal Chromatic Aberration for Object Situated at Finite Distance

Lens formula for thin lens is $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ where symbols have their usual meanings. Let us take the object is at fix point i.e. u is constant will vary with f. Differentiating, we get,

$$\frac{dv}{v^2} = \frac{df}{f^2} \qquad \text{.......} \quad (15.6)$$

Let the focal length and image distance for red and violet ray be $f_{r,}\, f_v,\, f_y$ and $v_{r,}\; v_v,\; v_y$ respectively than $dv = v_r - v_v$ and $df = f_r - f_v$. Therefore,

$$\frac{v_r - v_v}{v^2} = \frac{f_r - f_v}{f^2}$$

Now $f_r - f_v = \omega f_y$

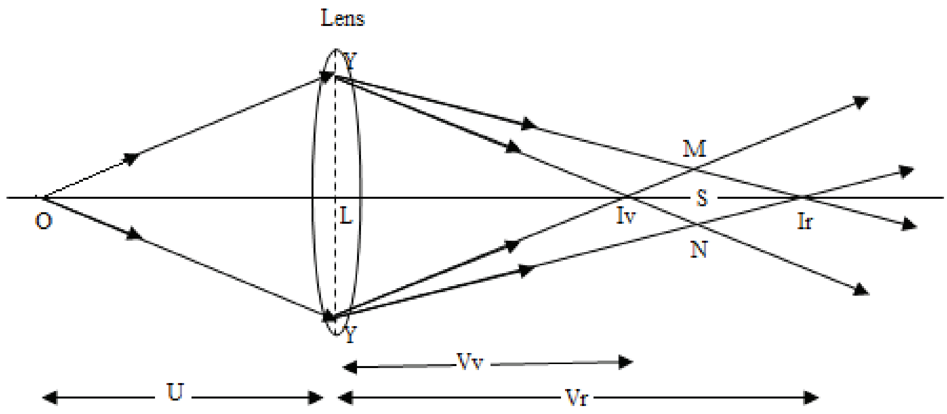


Figure 15.3

$$\frac{v_r - v_v}{{v_y}^2} = \frac{\omega f_y}{{f_y}^2} = \frac{\omega}{f_y}$$

or $$v_r - v_v = \frac{\omega}{f_y} \quad \text{....... (15.7)}$$

This shows that the longitudinal chromatic aberration depends upon the focal length and image distance for mean ray and the dispersive power of the lens material.

15.4.2 Lateral or Transverse Chromatic Aberration

The magnification $\left[m = \left(\frac{f}{u+f}\right)\right]$ produced by a lens depends on the focal length and focal length of the lens is different for different colours. Hence magnification of lens will also vary with colour. As a result for a white light illuminated object the size of different colours will be different. The difference of sizes of images for violet and red colours is a measure of lateral chromatic aberration.

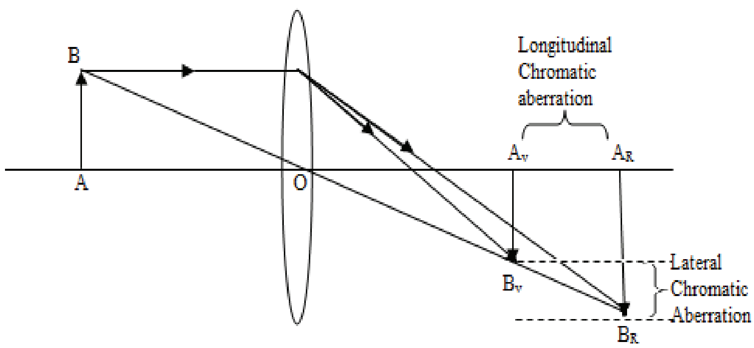


Figure 15.4

Analytically chromatic aberration is expressed as the rate of change of size of image with wavelength. Thus if x and y represents the axial and transverse distance then

Longitudinal chromatic aberration = $\frac{dx}{d\lambda}$

Transverse chromatic aberration = $\frac{dy}{d\lambda}$

15.5 ACHROMATIC COMBINATION OF LENSES

As we discussed earlier image of a white object formed by a single lens is generally coloured and blurred this defect is called chromatic aberration. The removal or minimize chromatic aberration is called achromatism. Generally it is achieved for two prominent colours by following way

A) A combination of two lenses one concave and other convex made from suitable different material placed in contact to each other. This combination is called achromatic doublet. Convex lens of crown and concave lens of flint glass are popularly used.

B) By a combination of two convex lenses made by same material separated by a distance equal to average of focal length of two lenses.

A) Achromatic Doublet: The focal length for a thin lens having radii of curvature R_1, R_2 is given by relation

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Now focal length *f* varies with μ, differentiating above equation, we get,

$$\frac{-\partial f}{f^2} = \partial\mu\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{\partial\mu}{(\mu-1)}(\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{\partial\mu}{(\mu-1)}\frac{1}{f}$$

But $\frac{\partial\mu}{(\mu-1)} = \omega$ = dispersive power of lens

$$\therefore \qquad \frac{-\partial f}{f} = \omega \qquad .... (15.8)$$

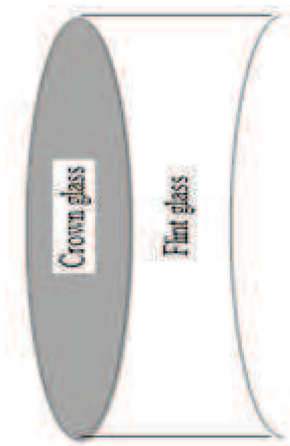


Figure 15.5

Let f_1 and f_2 be the focal length and ω_1 and ω_2 of constituent lens of achromatic doublet.

Equivalent focal length for lens combination in contact is given by

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

Differentiating it $\quad \frac{-\partial F}{F^2} = -\frac{\partial f_1}{f_1^{\,2}} - \frac{\partial f_2}{f_2^{\,2}} = -\frac{\partial f_1}{f_1}\cdot\frac{1}{f_1} - \frac{\partial f_2}{f_2}\cdot\frac{1}{f_2} = \frac{\omega_1}{f_1} + \frac{\omega_2}{f_2}$

For achromatism the focal length of the combination for all colours must be the same i.e. $\partial \mathrm{F} = 0$

$$\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0 \quad \text{or} \quad \frac{\omega_1}{f_1} = -\frac{\omega_2}{f_2} \quad \text{or} \quad \frac{f_1}{f_2} = -\frac{\omega_1}{\omega_2} \qquad \text{.......} \ (15.9)$$

Let us discuss the condition

(i) As ω_1 and ω_2 both are positive therefore f_1 and f_2 must have opposite sign to satisfy condition for achromatism. Thus if one lens is convex other should be concave.

(ii) If $\omega_1 = \omega_2$ i.e. both lenses are made of same material then $\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0$ becomes $\frac{1}{f_1} + \frac{1}{f_2} = 0$

or $\quad \frac{1}{F} = 0 \quad$ or $\quad F = \infty$

It means that the achromatic doublet will behave like a plane glass plate and not as a lens hence for an achromatic doublet material of both the lenses should be different.

(iii) If the combination is to behave like a convergent lens, then power of convex lens should be greater than that of concave lens. But, $Power\ of\ a\ lens \propto \frac{1}{Focal\ length\ of\ lens}$

Therefore focal length of convex lens should be smaller than the focal length of the convex lens. From equation $\frac{f_1}{f_2} = -\frac{\omega_1}{\omega_2}$ if $f_1 < f_2$ than ω_1 should also be less than ω_2. It means that convex lens should be made of crown lens and concave lens should be made of flint glass.

(iv) This type of combination is perfectly aromatic for two specified colours. If a number of thin lenses are placed in contact to form an achromatic lens, the condition of achromatism is given by

$$\sum \frac{\omega}{f} = 0 \qquad \text{.......} \ (15.10)$$

B) Achromatism by Combination of Two Convex Lenses Made By Same Material Separated By a Suitable Distance: Let f_1 and f_2 be the focal length of two convex lens made from same material separated by a distance d.

If F be the focal length of combination then $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$

Differentiating it, we get,

$$\frac{-\partial F}{F^2} = -\frac{\partial f_1}{{f_1}^2} - \frac{\partial f_2}{{f_2}^2} - d\left[-\frac{\mathbf{1}}{\mathbf{f_2}}.\frac{\partial f_1}{{f_1}^2} - \frac{\mathbf{1}}{\mathbf{f_1}}.\frac{\partial f_2}{{f_2}^2}\right]$$

$$= -\frac{\partial f_1}{f_1}.\frac{\mathbf{1}}{f_1} - \frac{\partial f_2}{f_2}.\frac{\mathbf{1}}{f_2} - \frac{d}{f_1 f_2}\left[\frac{\partial f_1}{f_1} - \frac{\partial f_2}{f_2}\right]$$

But $-\frac{\partial f}{f} = \omega$, the dispersive power of the lens medium. Here we have, $-\frac{\partial f_1}{f_1} = \frac{\partial f_2}{f_2} = \omega$ as both the lenses are made by same material.

Hence, $$\frac{-\partial F}{F^2} = \frac{\omega}{f_1} + \frac{\omega}{f_2} - \frac{d}{f_1 f_2}[\omega + \omega]$$

$$= \frac{\omega}{f_1} + \frac{\omega}{f_2} - \frac{2\omega d}{f_1 f_2}$$

For chromatic aberration to be minimum the focal length of the combination for all colour must be the same, i.e., $\partial F = 0$

$$\therefore \quad \frac{\omega}{f_1} + \frac{\omega}{f_2} - \frac{2\omega d}{f_1 f_2} = 0 \text{ or } d = \frac{f_1+f_2}{2} \qquad \text{.......} \ (15.11)$$

Thus the distance between two lenses must be equal to the average of their individual focal lengths.

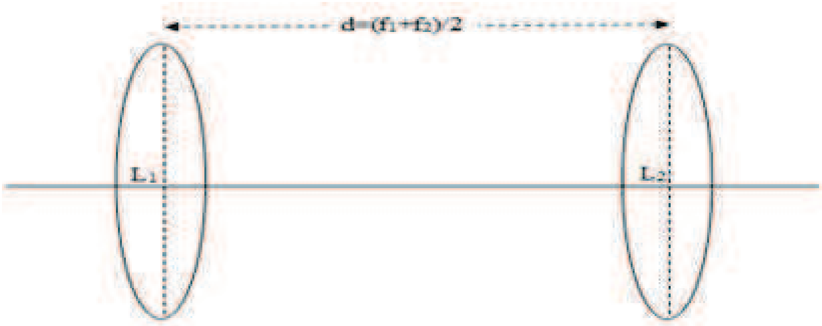


Figure 15.6

i) Since d is always positive $f_1 + f_2$ must be positive. It means either both the lenses or the one with greater focal length are convex.

ii) The achromatic doublet has the same focal length for all colours as the expression for d is independent of ω.

However the condition $d = (f_1 + f_2)/2$ does not mean the coincidence of focal points but simply means the focal length for different colours is same. However, the deviation produced by lens will be same as focal length for all the colours is same therefore the combination is apparently free from lateral chromatic aberration.

Example 15.1: A converging achromatic doublet of focal length 50 cm is to be constructed out of thin crown and flint glass lens. The radius of curvature of the surface is in contact is 15 cm.Find the radius of curvature of the second surface of each lens. (Given that $\omega_{crown} = 0.015, \omega_{flint} = 0.030$ and $\mu_{crown} = 1.5,\ \mu_{flint} = 1.7$)

Solution: For an achromatic doublet $\frac{\omega_{crown}}{f_{crown}} + \frac{\omega_{flint}}{f_{flint}} = 0$ or $\frac{\omega_{crown}}{\omega_{flint}} = -\frac{f_{crown}}{f_{flint}}$

Given that, $\omega_{crown} = 0.015,\ \omega_{flint} = 0.030$ and $\mu_{crown} = 1.5, \mu_{flint} = 1.7$

$$\therefore \quad \frac{f_{crown}}{f_{flint}} = -\frac{0.015}{0.030} = -\frac{1}{2} \quad \text{or} \quad f_{crown} = -\frac{1}{2} f_{flint}$$

Also equivalent focal length of two lenses in contact $\frac{1}{f_{crown}} + \frac{1}{f_{flint}} = \frac{1}{F}$

Here focal length of combination is= 30 cm

$$\therefore \quad \frac{1}{f_{crown}} + \frac{1}{f_{flint}} = \frac{1}{50} \quad \text{or} \quad -\frac{2}{f_{flint}} + \frac{1}{f_{flint}} = \frac{1}{30} \quad \text{as} \quad f_{flint} = -30$$

$\therefore \qquad f_{crown} = -\frac{1}{2} f_{flint} \quad = 15\ cm$

For the crown lens, $\qquad \frac{1}{f_{crown}} = (\mu_{crown} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

or $\frac{1}{15} = (1.5 - 1)\left(\frac{1}{R_1} - \frac{1}{-15}\right)$ as $\mu_{crown} = 1.5$, $f_{crown} = 15\ cm$ and $R_2 = -15cm$

or $\qquad \frac{1}{R_1} = \frac{1}{7.5} - \frac{1}{15} = \frac{2-1}{15} = \frac{1}{15}$, i.e., $\quad R_1 = 15\ cm$

Similarly for flint glass $\qquad \frac{1}{f_{flint}} = \left(\mu_{flint} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

or $\qquad \frac{1}{-30} = (1.7 - 1)\left(\frac{1}{-15} - \frac{1}{R_2}\right)$ as $\mu_{flint} = 1.7$, $f_{flint} = -30\ cm$ and $R_2 = -15cm$

or $\qquad \frac{1}{R_2} = \frac{1}{21} + \frac{1}{-15} = \frac{5-7}{105} = -\frac{2}{105}$, i.e., $R_2 = -52.5\ cm$

Thus the radius of curvature of the second surface of crown lens is = 15 cm and the radius of curvature of the second surface of flint lens is $-52.5\ cm$.

Example 15.2: An achromatic converging combination of focal length 30 cm is made by two lenses having dispersive power in the ratio 3:5. What are the focal lengths of the lenses?

Solution: For an achromatic doublet $\qquad \frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0 \qquad$ or $\qquad \frac{\omega_1}{\omega_2} = -\frac{f_1}{f_2}$

Given, $\qquad \frac{\omega_1}{\omega_2} = \frac{3}{5}$. $\qquad$ Hence, $\qquad \frac{f_1}{f_2} = -\frac{3}{5} \qquad$ or $\qquad f_1 = -\frac{3}{5} f_2$

Also equivalent focal length of two lenses in contact $\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{F}$

Here focal length of combination is = 30 cm

$\therefore \qquad \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{30} \qquad$ or $\qquad -\frac{3}{5f_2} + \frac{1}{f_2} = \frac{1}{30} \qquad$ as $\qquad f_1 = -\frac{3}{5} f_2$

$\qquad \frac{2}{4f_2} = \frac{1}{40} \qquad$ or $\qquad f_2 = 20\ cm$, $\quad$ hence $\ f_1 = -\frac{3}{5} \times\ 20 = -12\ cm$

So the focal lengths of the lenses are $f_1 = -12\ cm$ and $f_2 = 20\ cm$

Self-Assessment Questions (SAQ)

1. Chromatic aberration is due to the prismatic action of lens. True/False

2. Image of an object illuminated by white light is generally coloured and blurred. This is called mono chromatic aberration. True/False

3. For Achromatism the distance between two lenses must be equal to the sum of their individual focal lengths. True/False

4. For achromatism the focal length of the combination for all colours must be the same.

True/False

15.6 MONOCHROMATIC ABERRATION

Chromatic aberration is automatically removed if monochromatic light is used to illuminate an object even than the image formed by lenses is not free from other defects. Such kinds of defects are called monochromatic aberrations. Prominent spherical aberrations are

1) Spherical aberration

2) Coma

3) Astigmatism

4) Curvature of field

5) Distortion

15.7 SPHERICAL ABERRATION AND ITS ELIMINATION

15.7.1 Spherical Aberration

It is an optical defect observed in spherical surfaces (lens, mirror etc.) having large aperture. In spherical aberration parallel light ray that pass through the central region or near the axis of the lens (known as paraxial ray) focus at different point to the rays that passes to the edge or periphery of the lens (known as marginal ray).

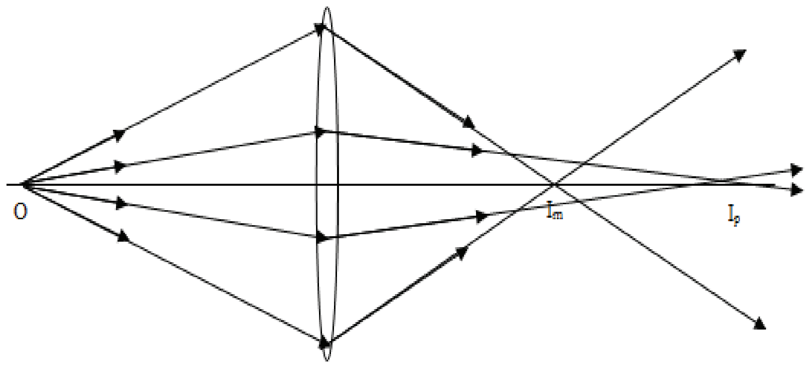


Figure 15.7

As shown in figure 15.7 a point object O on the axis is imaged as I_m and I_p where I_m and I_p are the image formed by marginal and paraxial ray. I_m I_p is the measure of axial or longitudinal spherical aberration.

15.7.2 Reduction of Spherical Aberration

Generally spherical aberration is minimum for lenses having small aperture or the total deviation produced by the system is equally divided on all refracting surfaces. Based on these facts different methods are used to minimize spherical aberrations.

1) By using plano convex lens

2) By crossed lens

3) By using stops: In this method the aperture of the lens is limited by using a stop to cut off marginal or paraxial ray. We can use a circular shutter or blackened the marginal or paraxial parts.

By Using a Convex and Concave Doublet: The image due to marginal ray is closer to the lens than the image due to paraxial rays while for the concave lens the image due to paraxial rays is closer to the lens than the image due to marginal ray. Thus by taking a suitable combination the image due to paraxial rays coincide to the image due to marginal ray to minimize spherical aberration. To form a real image the doublet must behave as convex.

By Using Aplanetic Lens: The image formed by aplanetic lenses exhibit low aberration. If the distance of point object on the axis from centre of curvature is $\frac{R}{\mu}$ than the distance of the point image on the axis will be μR.Such two points are called aplanetic points.

By Using Two Convex Lenses Separated By Suitable Distance: Let two convex lenses of focal length f_1 and f_2 are separated by a distance d. A ray of light AB, parallel to the principal axis, is incident on the first lens L_1 at a height h_1.after deviating though a small angle δ from the first lens the ray is incident on the second lens at height h_2 on the second lens L_2 after deviating by an angle δ_2

From the second lens again the ray meets the principal axis at F_2, The second focus of the system. If the second lens is not there the ray AB should meet the principal axis at F_1 after deviation from L_1 hence L_1F_1 is equal to the focal length of the first lens i.e.f_1.

The deviation produced by single lens is given by $\delta = \frac{h}{f}$

Hence deviation produced by lens L_1 is given by $\delta_1 = \frac{h_1}{f_1}$

And deviation produced by lens L_2 is given by $\delta_2 = \frac{h_2}{f_2}$

Now for minimum spherical aberration the deviation produced by two lenses must be equal, i.e.,

$$\delta_1 = \delta_2 \quad or \quad \frac{h_1}{f_1} = \frac{h_2}{f_2} \quad \text{or} \quad \frac{h_1}{h_2} = \frac{f_1}{f_2} \qquad .... (15.12)$$

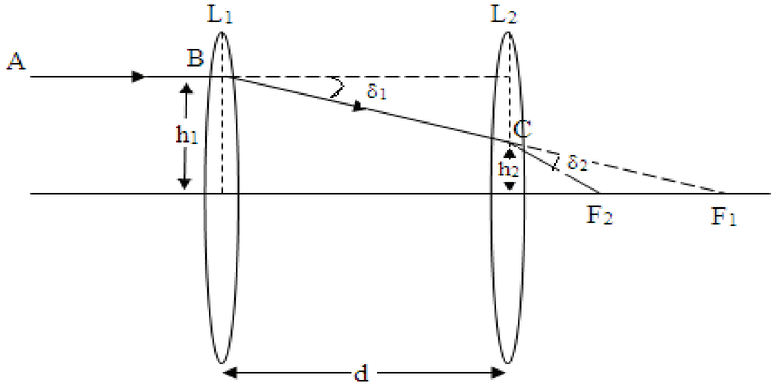


Figure 15.8

From similar triangle BL_1F_1 and C L_2F_2, we have

$$\frac{h_1}{h_2} = \frac{L_1F_1}{L_2F_1} = \frac{L_1F_1}{L_1F_1 - L_1L_2} = \frac{f_1}{f_1 - d} \qquad \text{....... (15.13)}$$

From equations 15.12 and 15.13, we have, $\frac{f_1}{f_2} = \frac{f_1}{f_1 - d}$ or $f_2 = f_1 - d$

$$d = f_1 - f_2 \qquad \text{....... (15.14)}$$

That is for minimum spherical aberration the distance between the two lenses should be equal to the difference of their focal length.

Example 15.3: A convergent doublet of lenses separated by a distance 5 cm. suitable for minimum spherical aberration. The equivalent focal length of the system is 15 cm. Find out the focal length of lenses.

Solution: Let the focal length of convergent doublet of lenses are $f_1 and\, f_2$ separated by distance d.

For minimum spherical aberration $d = f_1 - f_2$

Equivalent focal length of two lenses of focal length $f_1 and f_2$ separated by a distance d is

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1f_2} \qquad \text{or} \qquad F = \frac{f_1f_2}{f_1+f_2-d}$$

For minimum spherical aberration, $d = f_1 - f_2$ so $f_2 = f_1 - d$

$$\therefore \qquad F = \frac{f_1f_2}{f_1+f_2-d} = \frac{f_1f_2}{f_2+f_2} = \frac{f_1f_2}{2f_2} = \frac{f_1}{2} \qquad \text{or} \qquad f_1 = 2F$$

Given focal length of combination is F = 15 cm, hence, $f_1 = 2 \times 15 = 30$ cm and $f_2 = f_1 - d = 30 - 15 = 15\, cm.$

Example 15.4: A combination of two lenses made by same material and of focal length $f_1 and f_2$ separated by a distance d satisfies the condition for no chromatic aberration and minimum spherical aberration .If the equivalent focal length of combination is 30 cm find the focal length of both the lenses as well as distance between them.

Solution: For no chromatic aberration, $d = \frac{f_1+f_2}{2}$

And for minimum spherical aberration, $d = f_1 - f_2$

Solving these two equations, we get, $f_1 = \frac{3d}{2}$ and $f_2 = \frac{d}{2}$

Equivalent focal length of two lenses of focal length $f_1 and f_2$ separated by a distance d is

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1f_2}$$

Here the focal length of combination is 30 cm

$$\text{or} \qquad 30 = \frac{f_1f_2}{f_1+f_2-d} = \frac{\frac{3d}{2} \times \frac{d}{2}}{\frac{3d}{2} + \frac{d}{2} - d} = \frac{3d}{4} \qquad \text{or} \qquad d = 40\ cm.$$

Hence, $f_1 = 60\, cm$ and $f_2 = 20\, cm$

15.8 OTHER MONOCHROMATIC ABERRATION AND THEIR ELIMINATION

15.8.1 Coma

The image of a point object lying just away from the axis is not a point but it looks like a comet. Hence the aberration is called coma. Coma is the result of asymmetrical zones of the lens for the points away from the axis and varying magnification for rays refracted through different zones of the lens. Because of asymmetrical zones for a point object lying away from the axis is imaged in the form of discs by different zones. As shown in Figure 15.9 Object point A is imaged as Q by paraxial ray 2, 2, as R by paraxial ray 3, 3 and as S by paraxial ray 4, and so on. As we pass from paraxial to marginal zones the disc shaped image becomes wider and wider because the focal length for paraxial zones is greater than marginal.

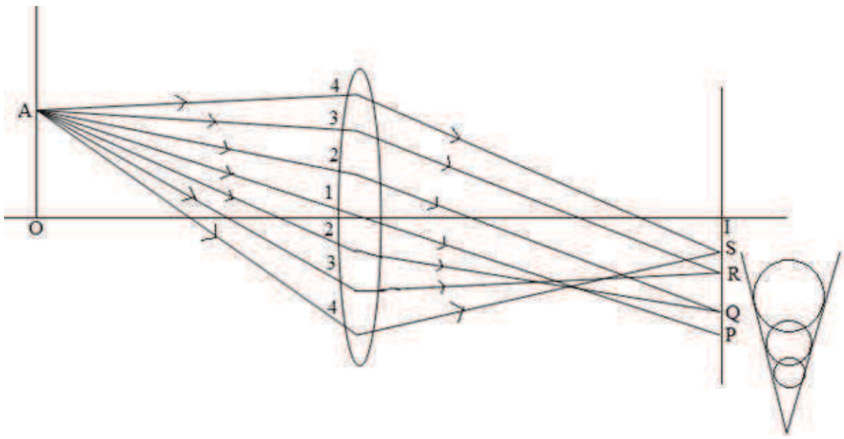


Figure 15.9

Coma is said to be positive if the magnification of the image due to outer zone is larger than the inner zones and if the magnification of the image due to outer zone is smaller than the inner zones it is called negative.

Removal of Coma: A lens satisfying the Abbe's sine condition $\mu_1 y_1 \sin\theta_1 = \mu_2 y_2 \sin\theta_2$ will be free from coma. Where μ_1, y_1, θ_1 and μ_2, y_2, θ_2 refer to the refractive index, height of the object above the axis, and the slope of the incident ray and refracted ray of light respectively.

Now as The magnification of the image is given by $\frac{y_2}{y_1}$. Hence from above equation

$$\frac{y_2}{y_1} = \frac{\mu_1 \sin\theta_1}{\mu_2 \sin\theta_2}$$

Now for elimination of coma transverse magnification should be constant for all rays of light as $\frac{\mu_1}{\mu_2}$ is constant so for $\frac{y_2}{y_1}$ will be constant only if $\frac{\sin\theta_1}{\sin\theta_2}$ = constant.

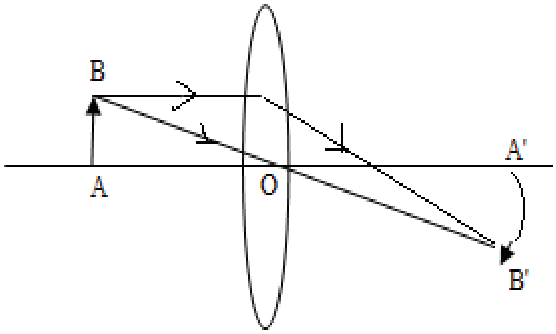
For distant object $u \to \infty$ and $\sin\theta \to h$. So $\frac{h}{\sin\theta_2}$ = constant

A lens that satisfies the above condition is called aplanetic lens. So use of aplanetic lens not only removes spherical aberration but also coma.

15.8.2 Curvature of the Field

Generally the image of a point object made by a single lens free from other monochromatic aberration is a point. But as the paraxial focal length is greater than the marginal focal length the image of an extended or flat plane object in front of lens is curved one instead of flat surface.

The central portion nearer to the axis is in focus but same is not true for the outer portion. As shown in figure 15.10 the real images formed by convex lens curves towards the lens while a virtual image curves away from the lens.



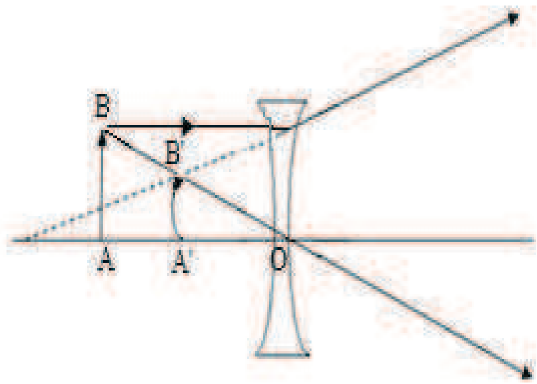


Figure 15.10

Removal of Curvature of the Field

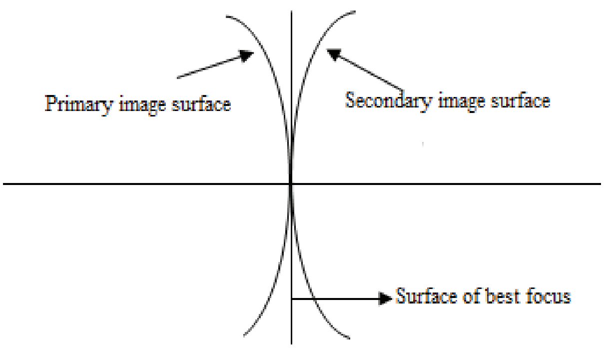
a) It can be eliminated by using stops in the suitable position in front of single thin lens.

b) For a system of thin lens the curvature of image is theoretically given by

$$\frac{1}{R} = \sum \frac{1}{\mu_n f_n}$$

Where R is the radius of curvature of the final image, μ_n and f_n are the refractive index and focal length of n^{th} lens. $\sum \frac{1}{\mu_n f_n}$ is called the Petzwal sum and depends only upon the refractive indices and radii of curvature of the surfaces of the lenses. It does not depend on the thickness and separation of the constituent lens. For the image to be flat, R must be infinity i.e.

$$\frac{1}{R} = \sum \frac{1}{\mu_n f_n} = \frac{1}{\infty} = 0$$



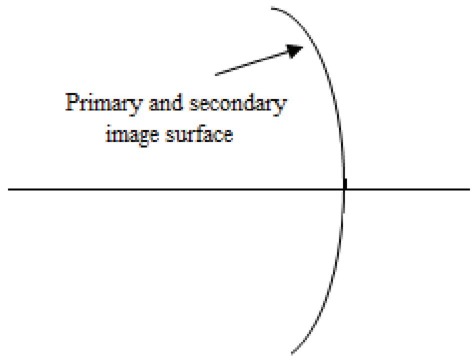


Figure 15.11

Therefore if lens system consists of two lenses placed in air, the condition for no curvature of field will be $\frac{1}{\mu_1 f_1} + \frac{1}{\mu_2 f_2} = 0$

This is known as Petzwal condition for no curvature. The condition holds good for the lenses in contact as well as separated by a distance. As $\mu_1$and μ_2 are positive this condition will be satisfied only if f_1 and f_2 are of opposite signs, i.e., for eliminating curvature if one lens is convex, the other must be concave. A suitable combination of convex and concave lens of different materials can be used to eliminate the curvature of field.

15.8.3 Astigmatism

Like coma, if a point object is far off the axis, its image consists of two mutually perpendicular lines separated by a finite distance and lying in perpendicular planes. This defect of the image is called as astigmatism.

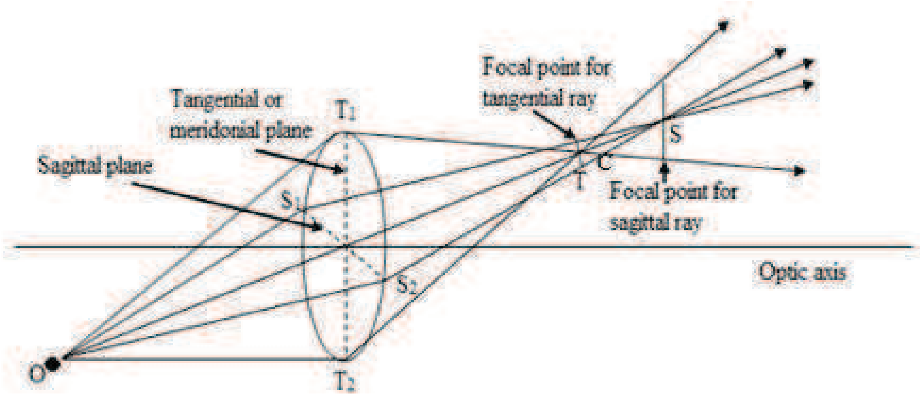


Figure 15.12

As shown in figure 15.12, let O is an object point away from the principal axis. A plane T_1T_2 passing through O and the principal axis is known as tangential or meridonial plane another plane S_1S_2 perpendicular to meridonial plane and passing through the object point O is known as sagittal plane. The rays passing through these two planes do not meet at the same point. Thus a line image T is obtained passing through meridonal plane and other image S is obtained by rays passing through sagittal plane. If screen is moved from image S due to sagittal plane towards T, a circle of least confusion will be observed at a place where the diverging beam from first and converging beam to second focal lines intersect. The distance between these two lines is called astigmatic difference. In fig curved focal lines corresponding to tangential and sagittal planes are shown as T and S.The astigmatism is said positive if T is before S, If S is before T it is called negative. On axis evidently astigmatism is zero.

Removal of Astigmatism: In case of convex lens the astigmatism is positive while it is negative in case of concave lens.Therfore a suitable convex and concave lens separated by a proper distance may be used to remove astigmatigm.Such combination of two lenses is called anastigmat.

15.8.4 Distortion

Any variation in shape of the object is called distortion. Magnification produced by a lens for different axial distances is different because of this generally image of a square is not perfect square as every point on the square is imaged by different zone of the lens and therefore differently magnified. This causes the distortion. It is generally of two types: 1) Barrel shaped, 2) Cushion shaped.

In barrel shaped distortion, the magnification decreases with the increase in axial distance. The image of the square looks as barrel shaped (figure 15.13 B). In cushion shaped distortion, the magnification increases with the increase in axial distance and the image of the square appears as cushion shaped (Figure 15.13 C)

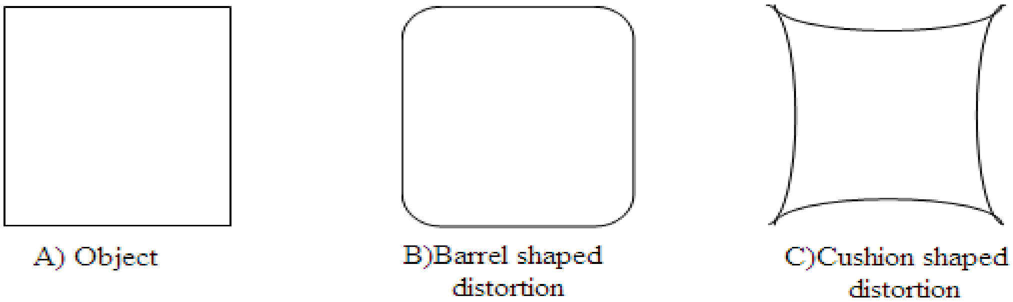


Figure 15.13

However a single thin lens exhibits no distortion the presence of stops limits the cone of rays or light striking the lens thus causing distortion. If a stop is placed before the lens the distortion is barrel shaped and if a stop is placed after the lens the distortion is cushion type. so use of single thin lens without stops is prefeered.however as stops are useful to reduce other type of monochromatic aberration therefore to eliminate distortion stop is placed in between two symmetrical lenses. So that cushion type distortion produced by first lens will be compensated by the barrel shaped distortion produced by second lens. As the optical instruments intended mainly form visual observation it is difficult to remove all kind of aberrations simultaneously but every effort should be made to eliminate distortion in photographic camera lens. In general a compromise is made as per requirement and use of the optical instruments.

SAQ 5: Magnification produced by a lens for different axial distances is different therfore causes variation in shape of the object. This is called Coma. True/False

SAQ 6: For minimum spherical aberration the distance between the two lenses should be equal to the difference of their focal length. True/False

SAQ7: $\frac{1}{\mu_1 f_1} + \frac{1}{\mu_2 f_2} = 0$is known as Petzwal condition for no curvature. True/False

SAQ 8: Chromatic aberration is an optical defect observed in spherical surfaces (lens, mirror etc.) having large aperture. True/False

15. 9 SPHERICAL MIRRORS

A spherical mirror is a mirror which has the shape of a piece cut out of a spherical surface. There are two types of spherical mirrors: *concave* and *convex* (Figure 15.14). The most commonly occurring examples of concave mirrors are shaving mirrors and makeup mirrors. As is well-known, these types of mirrors magnify objects placed close to them. The most commonly occurring examples of convex mirrors are the passenger-side wing mirrors of cars. These type of mirrors have wider fields of view than equivalent flat mirrors, but objects which appear in them generally look smaller (and, therefore, farther away) than they actually are.

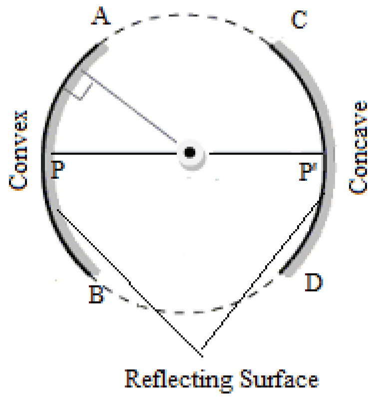


Figure 15.14

15.10 SCHMIDT CORRECTOR PLATE

In case of large telescopes, large spherical mirrors are used to form images and the images are recorded on a curved film. In such cases although the image is free from astigmatism and coma but because of large aperture of lens spherical aberration occurs. To remove spherical aberration in such large spherical mirror or lens Schmidt designed a special system in 1929 known as Schmidt corrector plate.

Construction and Working

A thin glass plate having a shallow toroidal surface known as Schmidt corrector plate (Figure 15.15) before the spherical mirror could help minimizing the spherical aberration in a mirror. Schmidt corrector plate is used with large aperture reflector telescopes. The surfaces of the corrector plate facing large spherical aperture is plane and the second surface is convex towards the axis and concave towards marginal ends.

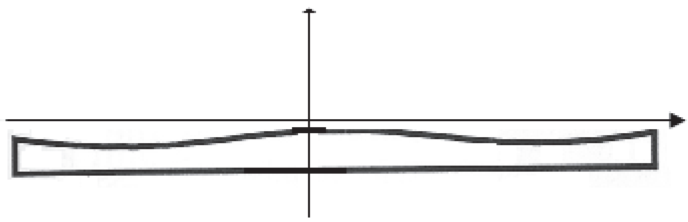


Figure 15.15

We know that the marginal rays are focused nearer than the paraxial rays for large aperture reflecting surfaces hence causing spherical aberration by behaving as a concave lens for the marginal rays and as a convex lens for the paraxial rays. The paraxial rays are refracted towards the axis and the marginal rays are refracted outwards. All the refracted rays (paraxial as well as marginal) from spherical surface come to focus at F and spherical aberration is minimized. The Schmidt plate is generally placed at the curvature of the reflecting surface so that working of plate is not affected.

15.11 OIL IMMERSION LENS

In light microscopy, oil immersion is a technique used to increase the resolving power of a microscope. This is achieved by immersing both the objective lens and the specimen in a transparent oil of high refractive index, thereby increasing the numerical aperture of the objective lens of microscope.

Why Use Microscope Immersion Oil?

When light passes from a material of one refractive index to another (for example: from glass to air), it bends. In the space between the microscope objective lens and the slide (where air is), light is refracted, the light scatters and it is lost. The refractive index of air is approximately 1.0, while the refractive index of glass is approximately 1.5. When light passes through both glass and air it is refracted. Light of different wavelengths bend at different angles, so as objects are magnified more, images become less distinct. Basically when using lower magnification microscope objective lenses (4x, 10x, 40x) the light refraction is not usually noticeable. However, once you use the 100x objective lens, the light refraction when using a dry lens is noticeable. If you can reduce the amount of light refraction, more light passing through the microscope slide will be directed through the very narrow diameter of a higher power objective lens. In microscopy, more light = clear and crisp images. By placing a substance such as immersion oil with a refractive index equal to that of the glass slide in the space filled with air, more light is directed through the objective and a clearer image is observed.

Oil immersion lenses make use of aplanatic surfaces for obtaining a very wide angled pencil of light from each point of the object. In case of single spherical surface these points are at distance R/μ and $R\mu$ from the optic centre. If object is placed at one aplanetic centre its image will be formed at other aplanatic point also the image will be free from spherical aberration and coma.

15.12 SUMMARY

The purpose of using a lens or a system of lens is to obtain an image exact magnified replica of object but in general it is not possible due to various reasons. Any deviation produced in image from object is called aberrations in the image. Aberrations are derived broadly into two categories, chromatic aberrations and mochromatic aberrations. The image of an object illuminated by white light made by a lens is generally coloured and blurred this is called chromatic aberration. It arises basically because of the prismatic action of lens. To remove or minimize chromatic aberration is called achromatism.

Generally it is achieved either by using achromatic doublet or by a combination of two convex lenses made by same material separated by a distance equal to average of focal length of two lenses. Even if monochromatic light is used to illuminate an object image is not free from defects. Such aberration is called monochromatic aberrations. Prominent spherical aberrations are spherical aberration, coma, astigmatism, curvature of field and distortion. In spherical aberration paraxial rays focus at different point to the marginal rays while in coma the image of a point object lying just away from the axis has a comet like structure. Also if a point object is far off the axis, its image made by sagital plane and tangential plane which are perpendicular to each other are not same it is called astigmatism.

The image of an extended or flat plane object in front of lens is curved one instead of flat surface due to the aberration known as curvature of field. While any variation in shape of the image with respect to object. It is generally of two types; barrel shaped and cushion shaped.

15.13 GLOSSARY

Aberration: Any departure of actual image with respect to size, shape, position, colour with respect to ideal image is called aberration.

Chromatic Aberration: The image of an object illuminated by white light made by a lens is generally coloured and blurred this is called chromatic aberration.

Achromatism: To remove or minimize chromatic aberration is called achromatism.Generally it is achieved either by using achromatic doublet or by a combination of two convex lenses made by same material separated by a distance equal to average of focal length of two lenses.

Monochromatic Aberration: Kinds of defects occurred in image even if monochromatic light is used to illuminate an object are called monochromatic aberrations. Prominent spherical aberrations are spherical aberration, coma, astigmatism, curvature of field and distortion

Spherical Aberration: It is an optical defect observed in spherical surfaces (lens, mirror etc.) having large aperture. In spherical aberration paraxial rays focus at different point to the marginal rays.

Coma The image of a point object lying just away from the axis looks like a comet such aberration is called coma.

Astigmatism: A point object is far off the axis, its image made by sagittal plane and maridonal plane is not same. Such defect is called astigmatism.

Curvature of Field: Generally the image of a point object made by a single lens free from other monochromatic aberration is a point. But as the paraxial focal length is greater than the marginal focal length the image of an extended or flat plane object in front of lens is curved one instead of flat surface.

Distortion: Any variation in shape of the image with respect to object. It is generally of two types: 1) Barrel shaped, 2) Cushion shaped.

15.14 ANSWERS OF SAQ'S

1. True, 2.False, 3. False, 4. True, 5. False, 6. True, 7. True, 8. False

15.15 REFERENCE BOOKS

1. Optics by Ajoy Ghatak

2. A textbook of Optics by Brij Lal and Dr. N. Subhrahmnyam

3. Optics by Dr. S.P. Singh and Dr. J.P. Agarwal

15.16 SUGGESTED READINGS

1. Fundamental of Optics by F. A. Jenkins and H. E. White.

2. The Feynman Lectures on Physics by Richard Feynman

3. Optics by Eugene Hecht

15.17 TERMINAL QUESTIONS

15.17.1 Short Answer Type Questions

1. What is meant by monochromatic aberration? What are different types of monochromatic aberration?
2. With the suitable diagram explain longitudinal and lateral spherical aberration in lens.
3. What is the difference between coma and astigmatism?
4. Define astigmatism, distortion and radius of curvature.
5. What do you mean by meniscus lens?

15.17.2 Long Answer Type Questions

1. Explain Chromatic aberration, its type and expression for its magnitude. how two lenses may be combined for making achromatic doublet?
2. What do you mean by aberration? Discuss its type and the way for their elimination.
3. Explain spherical aberration and various ways for its removal.
4. Discuss aplanetism. What is an aplanetic lens? Find the aplanetic foci for a spherical refracting surface? How they are utilized for the construction of oil immersion objective lens
5. Write short notes on

   a) Spherical mirrors b) Oil immersion lens

   c) Schmidt corrector plate d) Aplanetic points

15.17.3 Numerical Questions

1. A double convex lens has radii of curvature of 20 cm and 5 cm. Find the longitudinal chromatic aberration of an object at infinity? Given that $\mu_v = 1.52$ and $\mu_r = 1.51$

2. An achromatic doublet made by crown glass lens and flint glass lens of 40 cm focal length have a common radius of 25 cm for the surfaces in contact. Calculate the radius of curvature of the second face of each lens. (Given that $\omega_{crown} = 0.017, \omega_{flint} = 0.034$ and $\mu_{crown} = 1.5, \mu_{flint} = 1.7$) (Ans. The radius of curvature of the second surface of crown glass lens is = 16.67 cm and the radius of curvature of the second surface of flint glass lens is $-233.33\ cm$)

3. The ratio of dispersive power of a converging achromatic doublet is in the ratio 1:2. If the equivalent focal length of the doublet is 40 cm. Find the focal length of each lens. (Ans. f_1 = 20 cm and f_2 = 40 cm)

4. An oil immersion objective has lens of refractive index 1.67. Find out the semi angle of the incident and emergent rays. (Ans. 59^0, 31^0)

5. Two thin lenses separated by a distance 5 cm form a combination free from spherical aberration. If the system has an equivalent focal length of 14 cm, calculate the focal length of its component lenses. (Ans. f_1 = 28 cm and f_2 = 23cm)

15.17.4 Objective Questions

1. Which of the following is Monochromatic aberration?

a. Spherical aberration b. Astigmatism
c. Coma d. All of the above

Ans. Option **'d'**

2. Two plano convex lens of focal length f_1 and f_2 separated by a distance d. The condition of achromatism is

a. $d= f_1 - f_2$ b. $d= f_1 + f_2$
c. $d= (f_1 + f_2)/2$ d. $d= f_1 / f_2$

Ans. Option **'c'**

3. Spherical mirrors are free from

a. Chromatic aberration b. Spherical aberration
c. both chromatic and spherical aberration d. None of these

Ans. Option **'a'**

4. Both chromatic and spherical aberration are minimized for two plano convex lens of focal length f_1 and f_2 separated by a distance d .then the ratio of their focal length will be

a. 2:1 b. 3:1
c. 1:2 d. 1:4

Ans. Option **'b'**